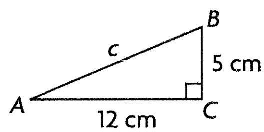
CHAPTER 5
Trigonometric Ratios

NOTE: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 274

1. a)



By the Pythagorean theorem, $c^2 = a^2 + b^2$.

$c^2 = 5^2 + 12^2$

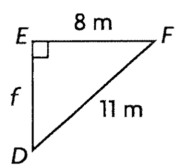
$c^2 = 25 + 144$

$c^2 = 169$

$c = \sqrt{169}$

$= 13$ m

b)



By the Pythagorean theorem, $e^2 = d^2 + f^2$.

$11^2 = 8^2 + f^2$

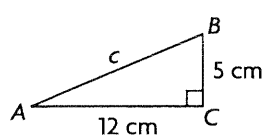
$121 = 64 + f^2$

$f^2 = 121 - 64$

$f^2 = 57$

$f = \sqrt{57}$ m

2. a)



From number 1, $c = 13$ m.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin A = \frac{a}{c}$

$\sin A = \frac{5}{13}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos A = \frac{b}{c}$

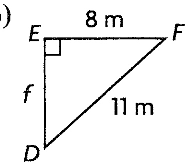
$\cos A = \frac{12}{13}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan A = \frac{a}{b}$

$\tan A = \frac{5}{12}$

b)



From number 1, $f = \sqrt{57}$ m.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin D = \frac{d}{e}$

$\sin D = \frac{8}{11}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos D = \frac{f}{e}$

$\cos D = \frac{\sqrt{57}}{11}$

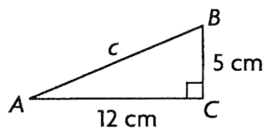
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan D = \frac{d}{f}$

$\tan D = \frac{8}{\sqrt{57}}$

$\tan D = \frac{8\sqrt{57}}{57}$

3. a)



From number 1, $c = 13$ m.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

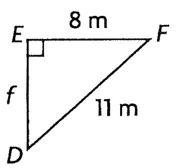
$$\sin B = \frac{b}{c}$$

$$\sin B = \frac{12}{13}$$

$$B = \sin^{-1}\frac{12}{13}$$

$$= 67°$$

b)



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos F = \frac{d}{e}$$

$$\cos F = \frac{8}{11}$$

$$F = \sin^{-1}\frac{8}{11}$$

$$= 43°$$

4. a) $\sin 31° = 0.515$

b) $\cos 70° = 0.342$

5. a) $\cos\theta = 0.3312$

$$\theta = \cos^{-1}0.3312$$

$$\doteq 71°$$

b) $\sin\theta = 0.7113$

$$\theta = \sin^{-1}0.7113$$

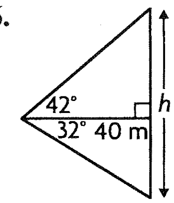
$$\doteq 45°$$

c) $\tan\theta = 1.1145$

$$\theta = \tan^{-1}1.1145$$

$$\doteq 48°$$

6.



As shown, the angles of sight to the bottom of the tower and the top of the tower form two right triangles. The distance from the basket of the repair truck to the tower, 40 m, is the adjacent side to both given angles. If you split h into two heights, h_1 and h_2, then; $h = h_1 + h_2$.

$$\tan 42° = \frac{h_1}{40}$$

$$h_1 = 40 \times \tan 42$$

$$h_1 = 36 \text{ m}$$

$$\tan 32° = \frac{h_2}{40}$$

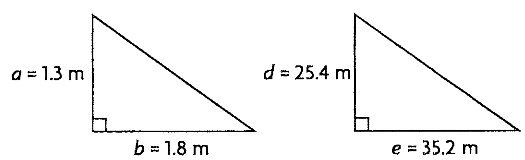
$$h_2 = 40 \times \tan 32$$

$$h_2 = 25 \text{ m}$$

$$h = 36 + 25$$

$$= 61 \text{ m}$$

7.



The tower and its shadow and the parking meter and its shadow form two similar right triangles. The angle of the sun relative to the earth is equal to angle B.

$$\tan B = \frac{a}{b}$$

$$\tan B = \frac{1.3}{1.8}$$

$$B = \tan^{-1}\frac{1.3}{1.8}$$

$$= 35.8°$$

Since the triangles are similar, $\angle D = \angle B$.

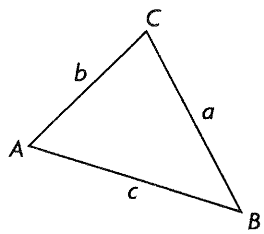
$$\tan D = \frac{d}{e}$$

$$\tan 35.8° = \frac{d}{35.2}$$

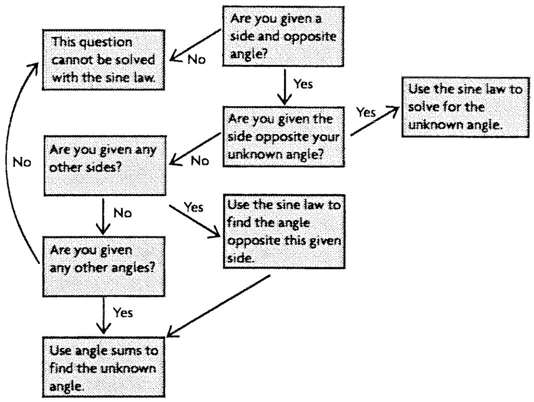
$$d = 35.2 \times \tan 35.8°$$

$$= 25.4$$

8.

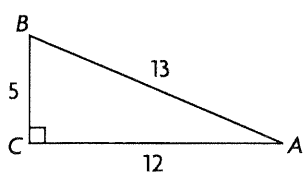


The Sine Law states, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.



5.1 Trigonometric Ratios of Acute Angles, pp. 280–282

1.



$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin A = \dfrac{a}{c}$

$\sin A = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos A = \dfrac{b}{c}$

$\cos A = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan A = \dfrac{a}{b}$

$\tan A = \dfrac{5}{12}$

$\csc A = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\csc A = \dfrac{c}{a}$

$\csc A = \dfrac{13}{5}$

$\sec A = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\sec A = \dfrac{c}{a}$

$\sec A = \dfrac{13}{12}$

$\cot A = \dfrac{\text{adjacent}}{\text{opposite}}$

$\cot A = \dfrac{b}{a}$

$\cot A = \dfrac{12}{5}$

2. $\csc\theta = \dfrac{1}{\sin\theta}$

$\csc\theta = \dfrac{17}{8}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\sec\theta = \dfrac{17}{15}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{15}{8}$

3. a) $\csc\theta = \dfrac{1}{\sin\theta}$

$\csc\theta = \dfrac{2}{1}$

$= 2.00$

b) $\sec\theta = \dfrac{1}{\cos\theta}$

$\sec\theta = \dfrac{4}{3}$

$= 1.33$

c) $\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{2}{3}$

$= 0.67$

d) $\cot \theta = \dfrac{1}{\tan \theta}$

$\cot \theta = \dfrac{4}{1}$

$= 4.00$

4. a) $\cos 34 = 0.83$

b) $\sec \theta = \dfrac{1}{\cos \theta}$

$\sec 10° = \dfrac{1}{\cos 10°}$

$\sec 10° = \dfrac{1}{0.98}$

$= 1.02$

c) $\cot \theta = \dfrac{1}{\tan \theta}$

$\cot 75° = \dfrac{1}{\tan 75°}$

$\cot 75° = \dfrac{1}{3.73}$

$= 0.27$

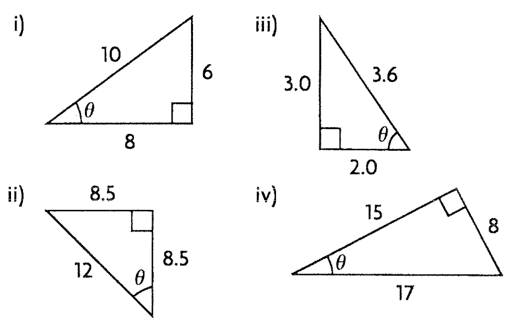
d) $\csc \theta = \dfrac{1}{\sin \theta}$

$\csc 45° = \dfrac{1}{\sin 45°}$

$\csc 45° = \dfrac{1}{0.71}$

$= 1.41$

5.



a) $\csc \theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\sec \theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\cot \theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

i) $\csc \theta = \dfrac{10}{6}$

$= \dfrac{5}{3}$

$\sec \theta = \dfrac{10}{8}$

$= \dfrac{5}{4}$

$\cot \theta = \dfrac{8}{6}$

$= \dfrac{4}{3}$

ii) $\csc \theta = \dfrac{12}{8.5}$

$\sec \theta = \dfrac{12}{8.5}$

$\cot \theta = \dfrac{8.5}{8.5}$

$= 1.0$

iii) $\csc \theta = \dfrac{3.6}{3.0}$

$= 1.2$

$\sec \theta = \dfrac{3.6}{2.0}$

$= 1.8$

$\cot \theta = \dfrac{2.0}{3.0}$

$= \dfrac{2}{3}$

iv) $\csc \theta = \dfrac{17}{8}$

$\sec \theta = \dfrac{17}{15}$

$\cot \theta = \dfrac{15}{8}$

b) i) From part **a**, $\csc \theta = \dfrac{5}{3}$, therefore:

$\csc \theta = 1.67$

$\sin \theta = \dfrac{1}{1.67}$

$\theta = \sin^{-1} \dfrac{1}{1.67}$

$= 37°$

ii) From part **a**, $\csc \theta = \dfrac{12}{8.5}$, therefore:

$\csc \theta = 1.41$

$\sin \theta = \dfrac{1}{1.41}$

$\theta = \sin^{-1} \dfrac{1}{1.41}$

$= 45°$

iii) From part **a**, sec $\theta = \frac{3.6}{2.0}$, therefore:

$$\sec\theta = 1.8$$
$$\cos\theta = \frac{1}{1.8}$$
$$\theta = \cos^{-1}\frac{1}{1.8}$$
$$= 56°$$

iv) From part **a**, cot $\theta = \frac{15}{8}$, therefore:

$$\cot\theta = 1.88$$
$$\tan\theta = \frac{1}{1.88}$$
$$\theta = \tan^{-1}\frac{1}{1.88}$$
$$= 28°$$

6. a) $\cot\theta = \frac{1}{\tan\theta}$

$$\cot\theta = 3.24$$
$$3.24 = \frac{1}{\tan\theta}$$
$$\tan\theta = \frac{1}{3.24}$$
$$\tan\theta = 0.31$$
$$\theta = \tan^{-1}0.31$$
$$= 17°$$

b) $\csc\theta = \frac{1}{\sin\theta}$

$$\csc\theta = 1.2711$$
$$1.2711 = \frac{1}{\sin\theta}$$
$$\sin\theta = \frac{1}{1.2711}$$
$$\sin\theta = 0.79$$
$$\theta = \sin^{-1}0.79$$
$$= 52°$$

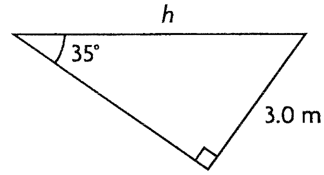
c) $\sec\theta = \frac{1}{\cos\theta}$

$$\sec\theta = 1.4536$$
$$1.4536 = \frac{1}{\cos\theta}$$
$$\cos\theta = \frac{1}{1.4536}$$
$$\cos\theta = 0.69$$
$$\theta = \cos^{-1}0.69$$
$$= 46°$$

d) $\cot\theta = \frac{1}{\tan\theta}$

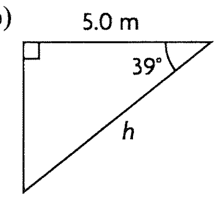
$$\cot\theta = 0.5814$$
$$0.5814 = \frac{1}{\tan\theta}$$
$$\tan\theta = \frac{1}{0.5814}$$
$$\tan\theta = 1.72$$
$$\theta = \tan^{-1}1.72$$
$$= 60°$$

7. a)



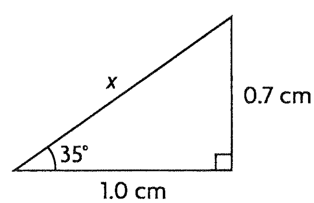
$$\sin 35° = \frac{3.0}{h}$$
$$h = \frac{3.0}{\sin 35°}$$
$$= \frac{3.0}{0.57}$$
$$= 5.2 \text{ m}$$

b)



$$\cos 39° = \frac{5.0}{h}$$
$$h = \frac{5.0}{\cos 39°}$$
$$= \frac{5.0}{0.78}$$
$$= 6.4 \text{ m}$$

8. a)



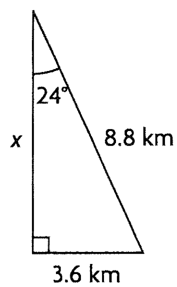
Method 1, for example:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 35° = \frac{0.7}{x}$$

$$x = \frac{0.7}{\sin 35°}$$
$$x = \frac{0.7}{0.57}$$
$$= 1.2 \text{ cm}$$

Method 2, for example:
By the Pythagorean theorem,
$x^2 = 0.7^2 + 1.0^2$
$x^2 = 0.49 + 1.00$
$x^2 = 1.49$
$x = \sqrt{1.49}$
$= 1.2$ cm

b)



Method 1, for example:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 24° = \frac{3.6}{x}$$
$$x = \frac{3.6}{\tan 24°}$$
$$= \frac{3.6}{0.45}$$
$$= 8.0 \text{ km}$$

Method 2, for example:
By the Pythagorean theorem,
$8.8^2 = x^2 + 3.6^2$
$x^2 = 8.8^2 - 3.6^2$
$x^2 = 77.44 - 12.96$
$x^2 = 64.48$
$x = \sqrt{64.48}$
$= 8.0$ cm

9. a) For any right triangle with acute angle θ,

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}.$$

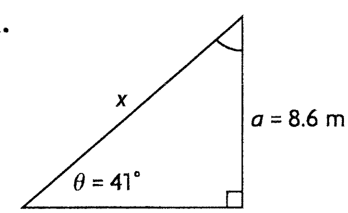
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\csc\theta > 1$.
Case 2: If the adjacent side is reduced to zero, each time you calculate $\csc\theta$, you get a smaller and smaller value until $\csc\theta = 1$.
Case 3: If the opposite side is reduced to zero, each time you calculate $\csc\theta$, you get a greater and greater value until you reach infinity. So for all possible cases in a right triangle, cosecant is always greater than or equal to 1.
b) For any right triangle with acute angle θ,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}.$$

Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\cos\theta < 1$.
Case 2: If the opposite side is reduced to zero, each time you calculate $\cos\theta$, you get a greater and greater value until $\cos\theta = 1$.
Case 3: If the adjacent side is reduced to zero, each time you calculate $\cos\theta$, you get a smaller and smaller value until $\cos\theta = 0$. So for all possible cases in a right triangle, cosine is always less than or equal to 1.

10. $\theta = 45°$ and adjacent side = opposite side

11.



The kite, string, and ground form a right triangle. The length of the string is the hypotenuse of the right triangle and the height above ground the opposite side of the triangle, therefore:

a)
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 41° = \frac{8.6}{x}$$
$$x = \frac{8.6}{\sin 41°}$$
$$= \frac{8.6}{0.65}$$
$$= 13.1 \text{ m}$$

b)
$$\csc\theta = \frac{1}{\sin\theta}$$
$$\csc\theta = \frac{x}{8.6}$$
$$\csc 41° = \frac{1}{\sin 41°}$$

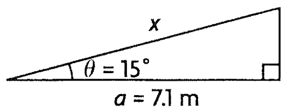
$$\frac{1}{\sin 41°} = \frac{x}{8.6}$$

$$\frac{1}{0.66} = \frac{x}{8.6}$$

$$x = \frac{8.6}{0.66}$$

$$= 13.1 \text{ m}$$

12.



The wheelchair ramp and the ground form a right triangle. The length of the ramp is the hypotenuse of the right triangle and the distance from the door is the adjacent side of the triangle, therefore:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 15° = \frac{7.1}{x}$$

$$x = \frac{7.1}{\cos 15°}$$

$$= \frac{7.1}{0.97}$$

$$= 7.36 \text{ m}$$

13. a) $\sec A = \dfrac{1}{\cos A}$

$$\sec A = 1.7105$$

$$\cos A = \frac{1}{1.7105}$$

$$A = \cos^{-1} 0.5846$$

$$= 54°$$

b) $\cos A = 0.7512$

$$A = \cos^{-1} 0.7512$$

$$= 41°$$

c) $\csc A = \dfrac{1}{\sin A}$

$$\csc A = 2.2703$$

$$\sin A = \frac{1}{2.2703}$$

$$A = \sin^{-1} 0.4405$$

$$= 26°$$

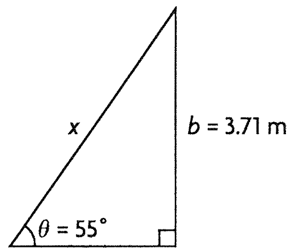
d) $\sin A = 0.1515$

$$A = \sin^{-1} 0.1515$$

$$= 9°$$

a Since b) has its angle A closest to 45°, it will have the greatest area of all the triangles (a 45°–45°–90° right triangle would have the largest possible area).

14.



The TV antenna, guy wire, and ground form a right triangle. The length of the guy wire is the hypotenuse of the right triangle and the height that the guy wire is attached is the opposite side of the triangle, therefore:

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\csc\theta = \frac{x}{3.71}$$

$$\csc 55° = \frac{1}{\sin 55°}$$

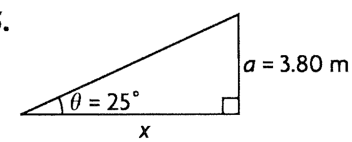
$$\frac{1}{\sin 55°} = \frac{x}{3.71}$$

$$\frac{1}{0.82} = \frac{x}{3.71}$$

$$x = \frac{3.71}{0.82}$$

$$= 4.5 \text{ m}$$

15.



Julie and the flagpole form a right triangle from Julies head, horizontally to the flag pole, and the tip of the flag pole to Julies head. If the angle from the top of Julies head to the top of the flagpole is 25°, then the opposite side of the triangle is 5.35 − 1.55 or 3.80 m. The adjacent side of the triangle is equal to the distance between Julie and the flagpole.

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

$$\cot\theta = \frac{x}{3.8}$$

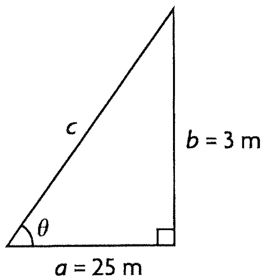
$$\cot 25° = \frac{1}{\tan 25°}$$

$$\frac{1}{\tan 25°} = \frac{x}{3.8}$$

$$\frac{1}{0.466} = \frac{x}{3.8}$$

$$x = \frac{3.8}{0.466}$$
$$= 8.15 \text{ m}$$

16.



A 12% slope has a ratio of $\frac{12}{100}$ and can be represented at a right triangle with one side of 12 and one side 100. A similar triangle with sides 3 and 25, respectively, would have the same angles.

a) E.g., 10°

b) $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$= \tan\theta = \frac{b}{a}$

$\tan\theta = \frac{3}{25}$

$\theta = \tan^{-1}0.12$

$= 7°$

c) By the Pythagorean theorem, $c^2 = a^2 + b^2$.

$c^2 = 25^2 + 3^2$

$x^2 = 625 + 9$

$x^2 = 634$

$x = \sqrt{634}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin\theta = \frac{a}{c}$

$\sin\theta = \frac{3}{\sqrt{634}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos\theta = \frac{b}{c}$

$\cos\theta = \frac{25}{\sqrt{634}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \frac{a}{b}$

$\tan\theta = \frac{3}{25}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\csc\theta = \frac{c}{a}$

$\csc\theta = \frac{\sqrt{634}}{3}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\sec\theta = \frac{c}{a}$

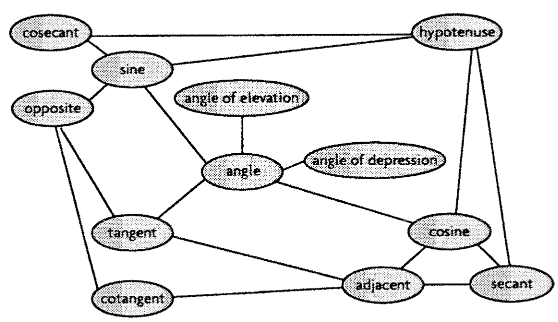
$\sec\theta = \frac{\sqrt{634}}{25}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

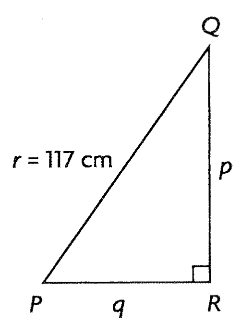
$\cot\theta = \frac{b}{a}$

$\cot\theta = \frac{25}{3}$

17. For example:



18.



As given, $\tan P = 0.51$, therefore:

$\angle P = \tan^{-1}0.51$

$\angle P = 27°$

Since this is a right triangle, $\angle R = 90°$ and:

$\angle Q = 90° - \angle P$

$\angle Q = 90° - 27°$

$= 63°$

$$\sin P = \frac{p}{r}$$

$$\sin 27° = \frac{p}{117}$$

$$p = 117 \times \sin 27° = 53 \text{ cm}$$

$$\sin Q = \frac{q}{r}$$

$$\sin 63° = \frac{q}{117}$$

$$q = 117 \times \sin 63° = 104 \text{ cm}$$

19. Since $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$, the adjacent side must be the smallest side.

20. $\csc\theta = \frac{1}{\sin\theta}$

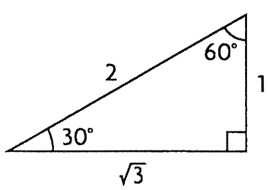
$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\csc\theta$ is undefined when $\sin\theta = 0$.
$\theta = \sin^{-1} 0$
$\theta = 0°$
$\sec\theta$ is undefined when $\cos\theta = 0$.
$\theta = \cos^{-1} 0$
$\theta = 90°$
$\cot\theta$ is undefined when $\tan\theta = 0$.
$\theta = \tan^{-1} 0$
$\theta = 0°$

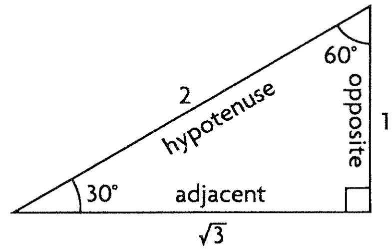
5.2 Evaluating Trigonometric Ratios for Special Angles, pp. 286–288

1. a)

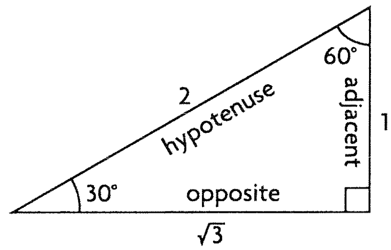


For example: The side opposite the 30° angle will be the smallest of the three sides, and is therefore the side of length 1. The hypotenuse should be the longest side, and since $\sqrt{3} < 2$, the hypotenuse is length 2. That leaves the adjacent side as length $\sqrt{3}$

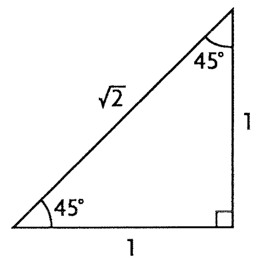
b)



c)

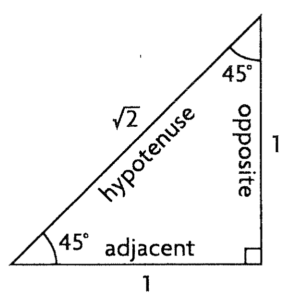


2. a)



For example: Since the triangle is a right triangle and one of the angles = 45° the other angle will also be 45°. Therefore, the two side of the right triangle will also be equal, so they are both 1. This leaves the hypotenuse as length $\sqrt{2}$.

b)



3. a) Using the special triangle 30°– 60°– 90°,

$$\sin 60° = \frac{\sqrt{3}}{2}$$

b) Using the special triangle 30°– 60°– 90°,

$$\cos 30° = \frac{\sqrt{3}}{2}$$

c) Using the special triangle 45°– 45°– 90°,

$$\tan 45° = \frac{1}{1}$$
$$= 1$$

d) Using the special triangle 45°– 45°– 90°,

$$\cos 45° = \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

4. a) Using the special triangle 30°– 60°– 90°,

$\sin 30° = \frac{1}{2}$, $\tan 60° = \frac{\sqrt{3}}{1}$, and $\cos 30° = \frac{\sqrt{3}}{2}$, therefore:

$$\sin 30° \times \tan 60° - \cos 30° = \frac{1}{2} \times \frac{\sqrt{3}}{1} - \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$
$$= 0$$

b) Using the special triangle 45°– 45°– 90°,

$\cos 45° = \frac{1}{1}$ and $\sin 45° = \frac{1}{\sqrt{2}}$, therefore:

$$2 \cos 45° \times \sin 45° = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$
$$= \frac{2}{1}$$

c) Using the special triangles 30°– 60°– 90° and 45°– 45°– 90°, $\tan 30° = \frac{1}{\sqrt{3}}$ and

$\cos 45° = \frac{1}{\sqrt{2}}$, therefore:

$$\tan^2 30° - \cos^2 45° = \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{1}{3} - \frac{1}{2}$$
$$= \frac{2}{6} - \frac{3}{6}$$
$$= -\frac{1}{6}$$

d) Using the special triangle 45°– 45°– 90°,

$\sin 45° = \frac{1}{\sqrt{2}}$ and $\cos 45° = \frac{1}{\sqrt{2}}$, therefore:

$$1 - \frac{\sin 45°}{\cos 45°} = 1 - \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}}$$
$$= 1 - \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1}$$
$$= 1 - 1$$
$$= 0$$

5. a) Using the special triangle 30°– 60°– 90°,

$\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$, therefore:

$$\sin^2 30° + \cos^2 30° = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= 1$$

b) Using the special triangle 45°– 45°– 90°,

$\sin 45° = \frac{1}{\sqrt{2}}$ and $\cos 45° = \frac{1}{\sqrt{2}}$, therefore:

$$\sin^2 45° + \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$

c) Using the special triangle 30°– 60°– 90°,

$\sin 60° = \frac{\sqrt{3}}{2}$ and $\cos 60° = \frac{1}{2}$, therefore:

$$\sin^2 60° + \cos^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$= \frac{3}{4} + \frac{1}{4}$$
$$= 1$$

6. a) Using the special triangle 30°– 60°– 90°,

$\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, and $\tan 30° = \frac{1}{\sqrt{3}}$, therefore:

$$\frac{\sin 30°}{\cos 30°} = \frac{1}{2} \div \frac{\sqrt{3}}{2}$$
$$= \frac{1}{2} \times \frac{2}{\sqrt{3}}$$
$$= \frac{2}{2 \times \sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}$$

b) Using the special triangle 45°– 45°– 90°,

$\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, and $\tan 45° = 1$, therefore:

$$\frac{\sin 45°}{\cos 45°} = \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1}$$
$$= 1$$

c) Using the special triangle 30°– 60°– 90°,

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, and $\tan 60° = \sqrt{3}$,

therefore:

$$\frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}}{2} \div \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2} \times \frac{2}{1}$$
$$= \frac{2\sqrt{3}}{2}$$
$$= \sqrt{3}$$

7. a) $\sin \theta = \frac{\sqrt{3}}{2}$

Using the special triangle 30°– 60°– 90°,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\theta = 60°$$

b) $\sqrt{3} \times \tan \theta = 1$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

Using the special triangle 30°– 60°– 90°,

$$\text{Tan } \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\theta = 30°$$

c) $2\sqrt{2} \times \cos \theta = 2$

$$\sqrt{2} \times \cos \theta = \frac{2}{2}$$
$$\cos \theta = \frac{1}{\sqrt{2}}$$

Using the special triangle 45°– 45°– 90°,

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\theta = 45°$$

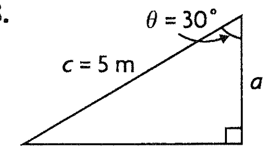
d) $2 \cos \theta = \sqrt{3}$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

Using the special triangle 45°– 45°– 90°

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\theta = 30°$$

8.



Assuming that the wall is perpendicular to the floor:

$$\cos 30° = \frac{a}{5}$$

Using the special triangle 30°– 60°– 90°,

$\cos 30° = \frac{\sqrt{3}}{2}$, therefore:

$$\frac{\sqrt{3}}{2} = \frac{a}{5}$$
$$a = 5 \times \frac{\sqrt{3}}{2}$$
$$= \frac{5\sqrt{3}}{2} \text{ m}$$

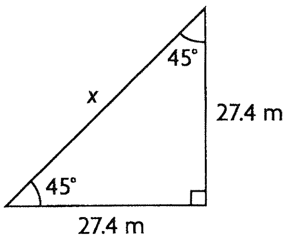
9. $\tan 30° + \frac{1}{\tan 30°} = \frac{1}{\sqrt{3}} + \left(1 \div \frac{1}{\sqrt{3}}\right)$

$$= \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}$$
$$= \frac{1}{\sqrt{3}} + \frac{3}{\sqrt{3}}$$
$$= \frac{4}{\sqrt{3}}$$
$$= \frac{4\sqrt{3}}{3}$$

$$\frac{1}{\sin 30° \cos 30°} = \frac{1}{\frac{1}{2} \times \frac{\sqrt{3}}{2}}$$
$$= 1 \div \frac{\sqrt{3}}{4}$$
$$= \frac{4}{\sqrt{3}}$$
$$= \frac{4\sqrt{3}}{3}$$

10. a) Use the proportions of the special triangle 45°–45°–90°, given that the two smaller sides are 27.4 m.

b)



$\sin 45° = \dfrac{27.4}{x}$

Using the special triangle 45°–45°–90°,

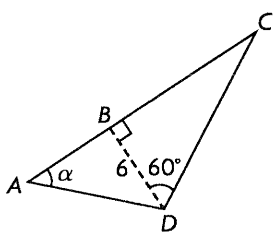
$\sin 45° = \dfrac{1}{\sqrt{2}}$, therefore:

$\dfrac{27.4}{x} = \dfrac{1}{\sqrt{2}}$

$x = 27.4 \times 0.71$

$= 38.7$ m

11. a)



Given $\tan \alpha = 1$, line $AB = BD = 6$

Area of triangle $ABD = \dfrac{1}{2} \times 6 \times 6 = 18$

Using the special triangle, $\tan 60° = \sqrt{3}$,

$\tan 60° = \dfrac{C}{6}$, therefore:

$\sqrt{3} = \dfrac{C}{6}$

$C = 6\sqrt{3}$

Area of triangle

$BCD = \dfrac{1}{2} \times 6\sqrt{3} \times 6$

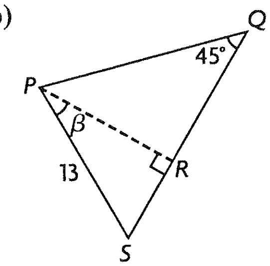
$= 18\sqrt{3}$

Area of triangle

$ACD = 18 + 18\sqrt{3}$

$= 3(6 + 6\sqrt{3})$ square units

b)



Given $\cos\beta = \dfrac{\sqrt{3}}{2}$, $\sin\beta = \dfrac{1}{2}$, and $\tan\beta = \dfrac{1}{\sqrt{3}}$

$\cos\beta = \dfrac{PR}{13}$ and $\sin\beta = \dfrac{RS}{13}$, therefore:

$\dfrac{\sqrt{3}}{2} = \dfrac{PR}{13}$

$PR = 13\dfrac{\sqrt{3}}{2}$

$\dfrac{1}{2} = \dfrac{RS}{13}$

$RS = \dfrac{13}{2}$

Area of triangle $PRS = \dfrac{1}{2} \times 13\dfrac{\sqrt{3}}{2} \times \dfrac{13}{2}$

$= \dfrac{169}{8}\sqrt{3}$

Since $\angle Q = 45°$, $PR = RQ = 13\dfrac{\sqrt{3}}{2}$

Area of triangle $PRQ = \dfrac{1}{2} \times 13\dfrac{\sqrt{3}}{2} \times 13\dfrac{\sqrt{3}}{2}$

$= \dfrac{507}{8}$

Area of triangle $PQS = \dfrac{169}{8}\sqrt{3} + \dfrac{507}{8}$

$= \dfrac{169}{8} \times (3 + \sqrt{3})$

square units

12. a) $\sin 45°(1 - \cos 30°)$

$+ 5 \tan 60° (\sin 60° - \tan 30°)$

$= 0.095 + 2.5$

$= 2.595$

b) Using the special triangle 30°–60°–90° and 45° × 45° × 90°, $\sin 45° = \dfrac{1}{\sqrt{2}}$,

$\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 60° = \sqrt{3}$, $\sin 60° = \frac{\sqrt{3}}{2}$, and $\tan 30° = \frac{1}{\sqrt{3}}$, therefore:

$$\sin 45°(1 - \cos 30°) + 5\tan 60°$$
$$(\sin 60° - \tan 30°)$$
$$= \frac{1}{\sqrt{2}} \times \left(1 - \frac{\sqrt{3}}{2}\right) + 5 \times \sqrt{3} \times \left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\right)$$
$$= \left(\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}\right) + 5 \times \left(\frac{3}{2} - \frac{\sqrt{3}}{\sqrt{3}}\right)$$
$$= \left(\frac{2}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}\right) + 5 \times \left(\frac{3}{2} - \frac{2}{2}\right)$$
$$= \frac{2 - \sqrt{3}}{2\sqrt{2}} + \frac{5}{2}$$
$$= \frac{2 - \sqrt{3}}{2\sqrt{2}} + \frac{5\sqrt{2}}{2\sqrt{2}}$$
$$= \frac{2 - \sqrt{3} + 5\sqrt{2}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{2\sqrt{2} - \sqrt{6} + 10}{4}$$

c) Megan didn't use a calculator. Her answer is exact, not rounded off.

13. Given $\cot \alpha = \sqrt{3}$, then by definition, $\tan \alpha = \frac{1}{\sqrt{3}}$ Using the special triangle from questions 1, $\sin \alpha = \frac{1}{2}$ and $\cos \alpha = \frac{\sqrt{3}}{2}$.

$$(\sin \alpha)(\cot \alpha) - \cos^2 \alpha = \frac{1}{2} \times \sqrt{3} - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{\sqrt{3}}{2} - \frac{3}{4}$$
$$= \frac{2\sqrt{3}}{4} - \frac{3}{4}$$
$$= \frac{2\sqrt{3} - 3}{4}$$

14. Given $\csc\beta = 2$, then by definition, $\sin \beta = \frac{1}{2}$. Using the special triangle from questions 1, $\sec \beta = \frac{2}{\sqrt{3}}$, and $\tan \beta = \frac{1}{\sqrt{3}}$.

$$\frac{\tan \beta}{\sec \beta} - \sin^2\beta = \frac{1}{\sqrt{3}} \div \frac{2}{\sqrt{3}} - \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} - \frac{1}{4}$$
$$= \frac{1}{2} - \frac{1}{4}$$
$$= \frac{1}{4}$$

15. a) Given $\theta = 30°$, then $\cot \theta = \frac{-8}{17}$ and $\csc \theta = 2$.

$$1 + \cot^2 30° = 1 + \left(\sqrt{3}\right)^2$$
$$= 4$$
$$\csc^2 \theta = 2^2$$
$$= 4$$

b) Given $\theta = 45°$, then $\cot \theta = 1$ and $\csc \theta = \frac{\sqrt{2}}{1}$.

$$1 + \cot^2 45° = 1 + 1^2$$
$$= 2$$
$$\csc^2 \theta = \left(\frac{\sqrt{2}}{1}\right)^2$$
$$= \frac{2}{1}$$
$$= 2$$

c) Given $\theta = 60°$, then $\cot \theta = \frac{1}{\sqrt{3}}$ and $\csc \theta = \frac{2}{\sqrt{3}}$.

$$1 + \cot^2 60° = 1 + \left(\frac{1}{\sqrt{3}}\right)^2$$
$$= \frac{3}{3} + \frac{1}{3}$$
$$= \frac{4}{3}$$
$$\csc^2 \theta = \left(\frac{2}{\sqrt{3}}\right)^2$$
$$= \frac{4}{3}$$

5.3 Exploring Trigonometric Ratios for Angles Greater than 90°, pp. 292

1. a) By definition, $\sin(180° - \theta) = \sin\theta$, therefore:

$\sin 45° = \sin(180° - 45°)$

$\theta = 135°$

b) By definition, $\cos(180° - \theta) = -\cos\theta$, and $\cos(180° + \theta) = -\cos\theta$, therefore:

$$-\cos(-60°) = \cos(180° - (-60°))$$
$$= \cos(-180° + 60°)$$
$$\theta = 240°, \text{ and}$$
$$-\cos(-60°) = \cos(180° + (-60°))$$
$$= \cos(180° - 60°)$$
$$\theta = 120°$$

c) By definition, $\tan(180° + \theta) = \tan\theta$, therefore:

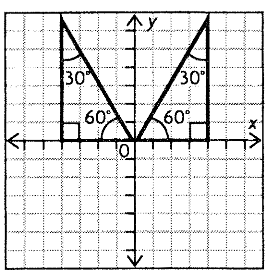
$\tan 30° = \tan(180° + 30°)$

$\theta = 210°$

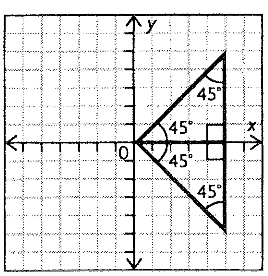
d) By definition, $\tan(180° - \theta) = -\tan\theta$ and $\tan(180° + \theta) = \tan\theta$, therefore:

$$\tan 135° = \tan(180° - \theta) = -\tan\theta$$
$$\tan 45° = -\tan 45°$$
$$\theta = 45°, \text{ and}$$
$$\tan\theta = \tan(180° + 45°)$$
$$\theta = 225°$$

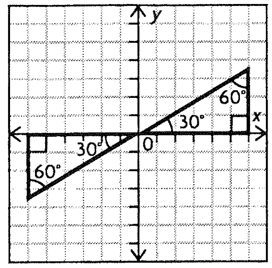
2. a)



b)



c)



3. a) Given $\tan(180° + \theta) = 1$ and $\tan(180° + \theta) = \tan\theta$

$\theta = 45°$

b) Since the triangle is in quadrant 3, $\theta = 225°$.

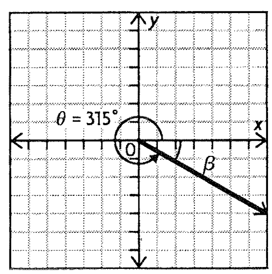
$\tan\theta = 1$, $\cos\theta = -\frac{\sqrt{2}}{2}$, $\sin\theta = -\frac{\sqrt{2}}{2}$

4.

	Quadrant			
Trigonometric Ratio	1	2	3	4
sine	+	+	−	−
cosine	+	−	−	+
tangent	+	−	+	−

5.4 Evaluating Trigonometric Ratios for Any Angle between 0° and 360°, pp. 299-301

1. a)



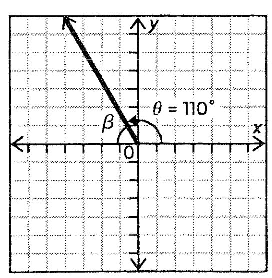
From the sketch, $\sin 315°$ is in quadrant 4. Since $\angle\theta$ is in quadrant 4, $\beta = 360° - \theta$.

$\beta = 360° - 315°$

$= 45°$

$\sin\theta = \frac{y}{r}$, where y is negative and r is positive, therefore $\sin\theta$ is negative.

b)

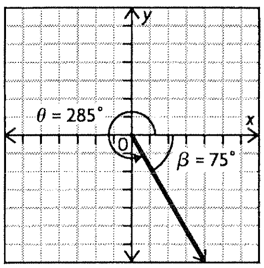


From the sketch, $\tan 110°$ is in quadrant 2.
Since $\angle\theta$ is in quadrant 2, $\beta = 180° - \theta$.
$\beta = 180° - 110°$
$= 70°$

$\tan\theta = \frac{y}{x}$, where y is positive and x is negative,

therefore $\tan\theta$ is negative.

c)

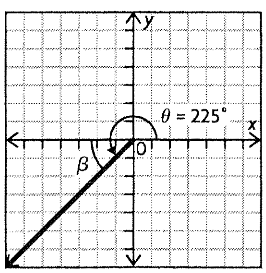


From the sketch, $\cos 285°$ is in quadrant 4.
Since $\angle\theta$ is in quadrant 4, $\beta = 360° - \theta$.
$\beta = 360° - 285°$
$= 75°$

$\cos\theta = \frac{x}{r}$, where x is positive and r is positive,

therefore $\cos\theta$ is positive.

d)

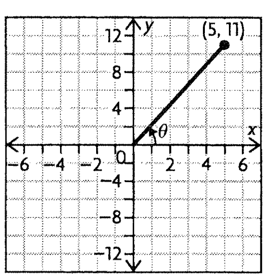


From the sketch, $\tan 225°$ is in quadrant 3.
Since $\angle\theta$ is in quadrant 2, $\beta = \theta - 180°$.
$\beta = 225° - 180°$
$= 45°$

$\tan\theta = \frac{y}{x}$, where y is negative and x is negative,

therefore $\tan\theta$ is positive.

2. a) i)



ii) $r^2 = x^2 + y^2$
$r^2 = 5^2 + 11^2$
$r^2 = 146$
$r = \sqrt{146}$
$= 12.1$

iii) $\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{11}{12.1}$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{5}{12.1}$

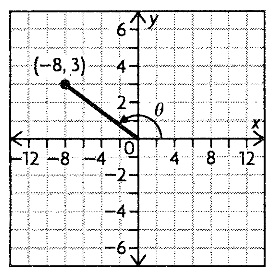
$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{11}{5}$

iv) $\tan\theta = \frac{11}{5}$

$\theta = \tan^{-1}\frac{11}{5}$
$= 66°$

b) i)



ii) $r^2 = x^2 + y^2$
$r^2 = (-8)^2 + 3^2$
$r = \sqrt{73}$
$= 8.5$

iii) $\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{3}{8.5}$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{-8}{8.5}$

$\tan\theta = \frac{y}{x}$

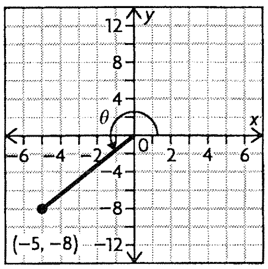
$\tan\theta = \frac{3}{-8}$

iv) $\tan\theta = \dfrac{3}{-8}$

$\theta = 180 - \tan^{-1}\dfrac{3}{|-8|}$

$\theta = 180° - 21°$
$= 159°$

c) i)



ii) $r^2 = x^2 + y^2$
$r^2 = (-5)^2 + (-8)^2$
$r^2 = 89$
$r = \sqrt{89}$
$= 9.4$

iii) $\sin\theta = \dfrac{y}{r}$

$\sin\theta = \dfrac{-8}{9.4}$

$\cos\theta = \dfrac{x}{r}$

$\cos\theta = \dfrac{-5}{9.4}$

$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{-8}{-5}$

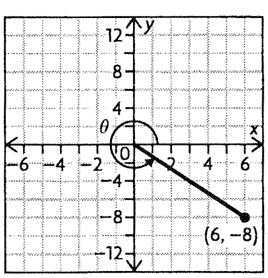
$= \dfrac{8}{5}$

iv) $\tan\theta = \dfrac{8}{5}$

$\theta = 180 + \tan^{-1}\dfrac{8}{5}$

$\theta = 180° + 58°$
$= 238°$

d) i)



ii) $r^2 = x^2 + y^2$
$r^2 = 6^2 + (-8)^2$
$r^2 = 100$
$r = \sqrt{100}$
$= 10$

iii) $\sin\theta = \dfrac{y}{r}$

$\sin\theta = \dfrac{-8}{10}$

$= \dfrac{-4}{5}$

$\cos\theta = \dfrac{x}{r}$

$\cos\theta = \dfrac{6}{10}$

$= \dfrac{3}{5}$

$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{-8}{6}$

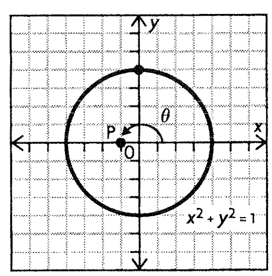
$= \dfrac{-4}{3}$

iv) $\tan\theta = \dfrac{-4}{3}$

$\theta = 180 + \tan^{-1}\dfrac{|-4|}{3}$

$\theta = 360° - 53°$
$= 307°$

3. a)



Since $P(-1, 0)$; $x = -1$, $y = 0$, and $r = 1$. Using the definitions of sine, cosine, and tangent:

$\sin\theta = \dfrac{y}{r}$

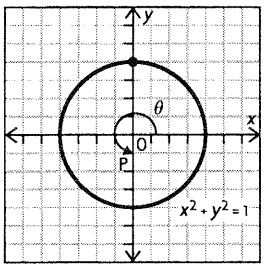
$\sin\theta = \dfrac{0}{1}$

$\sin 180° = 0$

$\cos\theta = \dfrac{x}{r}$

$$\cos\theta = \frac{-1}{1}$$
$$\cos 180° = -1$$
$$\tan\theta = \frac{y}{x}$$
$$\tan\theta = \frac{0}{-1}$$
$$\tan 180° = 0$$

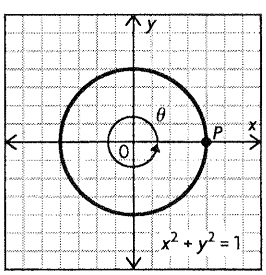
b)



Since $P(0, -1)$; $x = 0$, $y = -1$, and $r = 1$.
Using the definitions of sine, cosine and tangent:

$$\sin\theta = \frac{y}{r}$$
$$\sin\theta = \frac{-1}{1}$$
$$\sin 270° = -1$$
$$\cos\theta = \frac{x}{r}$$
$$\cos\theta = \frac{0}{1}$$
$$\cos 270° = 0$$
$$\tan\theta = \frac{y}{x}$$
$$\tan\theta = \frac{-1}{0}$$
$\tan 270°$ is undefined.

c)



Since $P(1, 0)$; $x = 1$, $y = 0$, and $r = 1$.
Using the definitions of sine, cosine and tangent:

$$\sin\theta = \frac{y}{r}$$
$$\sin\theta = \frac{0}{1}$$
$$\sin 360° = 0$$
$$\cos\theta = \frac{x}{r}$$
$$\cos\theta = \frac{1}{1}$$
$$\cos 360° = 1$$
$$\tan\theta = \frac{y}{x}$$
$$\tan\theta = \frac{0}{1}$$
$$\tan 360° = 0$$

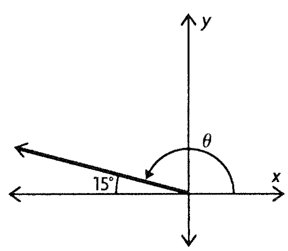
4. a) For example: A related acute angle of $\sin 160°$ is $180° - 160° = 20°$.
$\sin 20°$

b) For example: A related acute angle of $\cos 300°$ is $360° - 300° = 60°$.
$\cos 60°$

c) For example: A related acute angle of $\tan 110°$ is, $180° + 110° = 290°$.
$\tan 290°$

d) For example: A related acute angle of $\sin 350°$ is $360° - 350° + 180° = 190°$.
$\sin 190°$

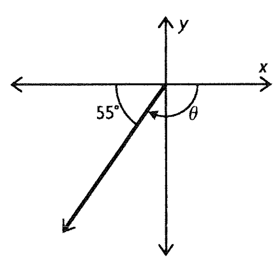
5. a)



i) $\sin\theta$

ii) $\theta = 180° - 15°$
$\theta = 165°$
$\sin 165° = 0.26$
$\cos 165° = -0.97$
$\tan 165° = -0.27$

b)

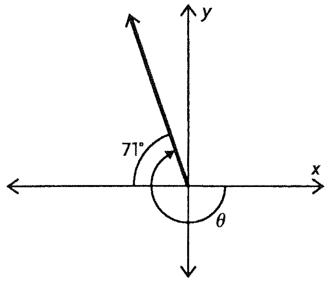


i) $\tan\theta$

ii) $\theta = -180° + 55°$
$\theta = -125°$
$\sin(-125°) = -0.82$

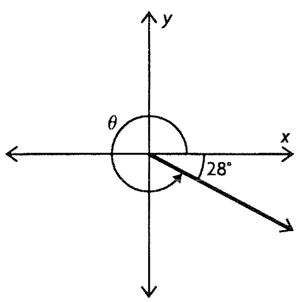
$\cos(-125°) = -0.57$
$\tan(-125°) = 1.43$

c)



i) $\sin\theta$
ii) $\theta = -180° - 71°$
$\theta = -251°$
$\sin(-251°) = 0.95$
$\cos(-251°) = -0.33$
$\tan(-251°) = -2.90$

d)



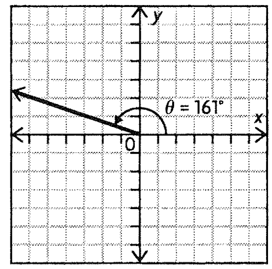
i) $\cos\theta$
ii) $\theta = 360° - 28°$
$= 332°$
$\sin 332° = -0.47$
$\cos 332° = 0.88$
$\tan 332° = -0.53$

6. a) i) Since the angle lies in quadrant 2, x is negative. Therefore:

$\sin\theta = \frac{y}{r}$
$y = 1$
$r = 3$

By the Pythagorean theorem, $r^2 = x^2 + y^2$.
$x^2 = r^2 - y^2$
$x^2 = 3^2 - 1^2$
$x^2 = 9 - 1$
$x^2 = 8$
$x = -\sqrt{8}$
$= -2\sqrt{2}$

ii)



iii) $\sin\beta = \frac{1}{3}$

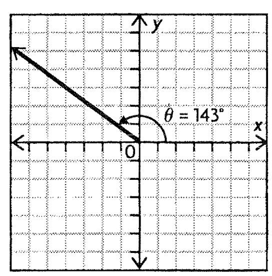
$\beta = \sin^{-1}\frac{1}{3}$
$\beta = 19°$
$\theta = 180° - \beta$
$\theta = 180° - 19°$
$= 161°$

b) i) Since the angle lies in quadrant 2, x is negative. Therefore:

$\cot\theta = \frac{x}{y}$
$x = -4$
$y = 3$

From the Pythagorean theorem,
$r^2 = x^2 + y^2$
$r^2 = (-4^2) + 3^2$
$r^2 = 16 + 9$
$r^2 = 25$
$r = \sqrt{25}$
$= 5$

ii)



iii) $\cot\beta = -\frac{4}{3}$

$\tan\beta = \frac{3}{4}$

$\beta = \tan^{-1}\frac{3}{4}$
$\beta = 37°$
$\theta = 180° - \beta$
$\theta = 180° - 37°$
$= 143°$

c) i) Since the angle lies in quadrant 2, x is negative. Therefore:

$\cos\theta = \frac{x}{r}$

$x = -1$

$r = 4$

By the Pythagorean theorem, $r^2 = x^2 + y^2$.

$y^2 = r^2 - x^2$

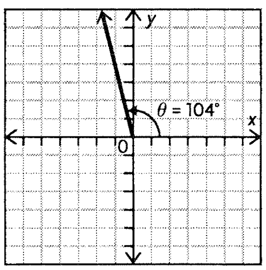
$y^2 = 4^2 - (-1)^2$

$y^2 = 16 - 1$

$y^2 = 15$

$y = \sqrt{15}$

ii)



iii) $\cos\beta = -\frac{1}{4}$

$\beta = \cos^{-1}\frac{1}{4}$

$\beta = 76°$

$\theta = 180° - \beta$

$\theta = 180° - 76°$

$= 104°$

d) i) Since the angle lies in quadrant 2, x is negative. Therefore:

$\csc\theta = \frac{r}{y}$

$\csc\theta = 2.5$

$\csc\theta = \frac{5}{2}$

$y = 2$

$r = 5$

By the Pythagorean theorem, $r^2 = x^2 + y^2$.

$x^2 = r^2 - y^2$

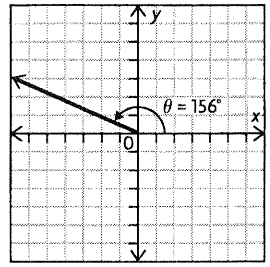
$x^2 = 5^2 - 2^2$

$x^2 = 25 - 4$

$x^2 = 21$

$x = -\sqrt{21}$

ii)



iii) $\csc\beta = \frac{5}{2}$

$\sin\beta = \frac{2}{5}$

$\beta = \sin^{-1}\frac{2}{5}$

$\beta = 24°$

$\theta = 180° - \beta$

$\theta = 180° - 24°$

$= 156°$

e) i) Since the angle lies in quadrant 2, x is negative. Therefore:

$\tan\theta = \frac{y}{x}$

$\tan\theta = -1.1$

$\tan\theta = -\frac{11}{10}$

$x = -10$

$y = 11$

By the Pythagorean theorem, $r^2 = x^2 + y^2$.

$r^2 = x^2 + y^2$

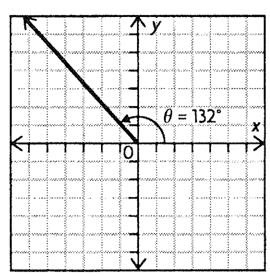
$r^2 = (-10)^2 + 11^2$

$r^2 = 100 + 121$

$r^2 = 221$

$r = \sqrt{221}$

ii)



iii) $\tan\beta = -\frac{11}{10}$

$\beta = \tan^{-1}\frac{11}{10}$

$\beta = 48°$

$\theta = 180° - \beta$

$\theta = 180° - 48°$

$= 132°$

f) i) Since the angle lies in quadrant 2, x is negative. Therefore:

$\sec\theta = \frac{r}{x}$

$\csc\theta = -3.5$

$\csc\theta = -\frac{7}{2}$

$x = -2$

$r = 7$

By the Pythagorean theorem, $r^2 = x^2 + y^2$.

$y^2 = r^2 - x^2$

$y^2 = 7^2 - (-2)^2$

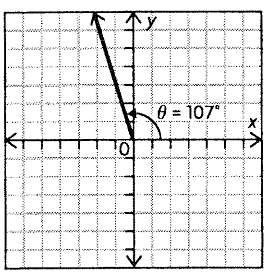
$y^2 = 49 - 4$

$y^2 = 45$

$y = \sqrt{45}$

$= 3\sqrt{5}$

ii)



iii) $\sec\beta = -\frac{7}{2}$

$\cos\beta = -\frac{2}{7}$

$\beta = \cos^{-1}\frac{2}{7}$

$\beta = 73°$

$\theta = 180° - \beta$

$\theta = 180° - 73°$

$= 107°$

7. a) From 6a, $\theta = 161°$

$161° - 360° = -199°$

b) From 6b, $\theta = 143°$

$143° - 360° = -217°$

c) From 6c, $\theta = 104°$

$104° - 360° = -256°$

d) From 6d, $\theta = 156°$

$156° - 360° = -204°$

e) From 6e, $\theta = 132°$

$132° - 360° = -228°$

f) From 6c, $\theta = 107°$

$107° - 360° = -253°$

8. a) $\sin\theta = 0.4815$

$\theta = \sin^{-1}0.4815$

$= 29°$

$\theta = 180° - 29°$

$= 151°$

b) $\tan\theta = -0.1623$

$\theta = \tan^{-1}(-0.1623)$

$= -9°$

$\theta = 360° + (-9°)$

$= 351°$

$\theta = 180° + (-9°)$

$= 171°$

c) $\cos\theta = -0.8722$

$\theta = \cos^{-1}(-0.8722)$

$= 151°$

$\theta = 360° - 151°$

$= 209°$

d) $\cos\theta = 8.1516$

$\tan\theta = \frac{1}{8.1516}$

$\theta = \tan^{-1}(0.1227)$

$= 7°$

$\theta = 180° + (7°)$

$= 187°$

e) $\csc\theta = -2.3424$

$\sin\theta = -\frac{1}{2.3424}$

$\theta = \sin^{-1}(-0.4269)$

$= -25°$

$\theta = 360° + (-25°)$

$= 335°$

$\theta = 180° - (-25°)$

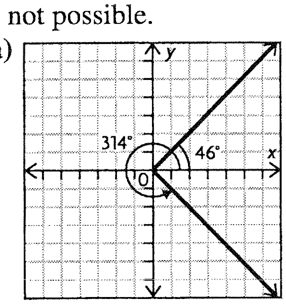
$= 205°$

f) $\sec\theta = 0$

$\cos\theta = \frac{1}{0}$

θ is not possible.

9. a)



$\cos\theta = 0.6951$

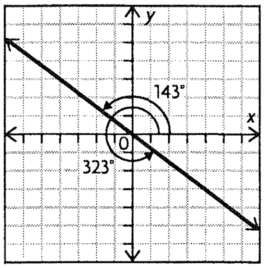
$\theta = \cos^{-1}0.6951$

$= 46°$

$\theta = 360° - 46°$

$= 314°$

b)



$\tan\theta = -0.7571$

$\theta = \tan^{-1}(-0.7571)$

$= -37°$

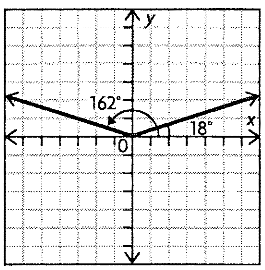
$\theta = 360° + (-37°)$

$= 323°$

$\theta = 180° + (-37°)$

$= 143°$

c)



$\sin\theta = 0.3154$

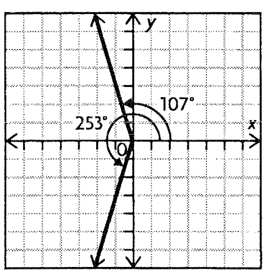
$\theta = \sin^{-1}0.3154$

$= 18°$

$\theta = 180° - 18°$

$= 162°$

d)



$\cos\theta = -0.2882$

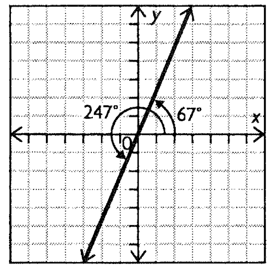
$\theta = \cos^{-1}(-0.2882)$

$\theta = 107°$

$\theta = 360° - 107°$

$= 253°$

e)



$\tan\theta = 2.3151$

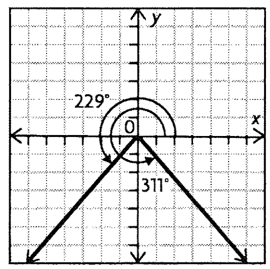
$\theta = \tan^{-1}2.3151$

$= 67°$

$\theta = 180° + 67°$

$= 247°$

f)



$\sin\theta = -0.7503$

$\theta = \sin^{-1}(-0.7503)$

$= -49°$

$\theta = 360° + (-49°)$

$= 311°$

$\theta = 180° - (-49°)$

$= 229°$

10. a) i) $\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{-1}{-1}$

$\theta = 225°$, and

$\theta = 225° - 360°$

$= -135°$

ii) $x = -1$, $y = -1$, and $r = \sqrt{2}$, therefore:

$\tan\theta = \dfrac{y}{r}$

$\sin\theta = \dfrac{-1}{\sqrt{2}}$

$= \dfrac{-\sqrt{2}}{2}$

$\cos\theta = \dfrac{x}{r}$

$\cos\theta = \dfrac{-1}{\sqrt{2}}$

$= \dfrac{-\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-1}{-1}$

$= 1$

b) i) $\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-1}{0}$

$\theta = 270°$, and

$\theta = 270° - 360°$

$\theta = -90°$

ii) $x = 0$, $y = -1$, and $r = 1$, therefore:

$\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{-1}{1}$

$= -1$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{0}{1}$

$= 0$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-1}{0}$

$\tan\theta$ in undefined

c) i) $\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{0}{-1}$

$\theta = 180°$, and

$\theta = 180° - 360°$

$= -180°$

ii) $x = -1$, $y = 0$, and $r = 1$, therefore:

$\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{0}{1}$

$= 0$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{-1}{1}$

$= -1$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{0}{-1}$

$= 0$

d) i) $\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{0}{1}$

$\theta = 0°$, and

$\theta = 0° - 360°$

$= -360°$

ii) $x = 1$, $y = 0$, and $r = 1$, therefore:

$\sin\theta = \frac{y}{r}$

$= 0$

$\sin\theta = \frac{0}{1}$

$= 0$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{1}{1}$

$= 1$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{0}{1}$

$= 0$

11. You can't draw a right triangle if $\theta \geq 90°$.

12. a) Quadrant 2 or 3

b) Given $\cos\theta = -\frac{5}{12}$ and by definition,

$\cos\theta = \frac{x}{r}$, therefore

$x = -5$

$r = 12$

By the Pythagorean theorem, $r^2 = x^2 + y^2$.

$y^2 = r^2 - x^2$

$y^2 = 12^2 - (-5)^2$

$y^2 = 144 - 25$

$y^2 = 119$

$y = \pm\sqrt{119}$

Quadrant 2:

$\sin\theta = \frac{\sqrt{119}}{12}$

$\cos\theta = \frac{-5}{12}$

$\tan\theta = \frac{\sqrt{119}}{-5}$

Quadrant 3:

$\sin\theta = \frac{-\sqrt{119}}{12}$

$\cos\theta = \frac{-5}{12}$

$\tan\theta = \frac{\sqrt{119}}{5}$

c) $\cos\theta = -\frac{5}{12}$

$\theta = \cos^{-1} -\frac{5}{12}$

$\theta = 115°$, and

$\theta = 360° - 115°$

$= 245°$

13. $\alpha = 180°$

For example, since $\cos\theta = \frac{x}{r}$ (will be unique when θ is on the negative x-axis.

14. For example, given $P(x, y)$ on the terminal arm of angle θ, $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$.

15. a) $\cos 2\theta = 0.6420$

$2\theta = \cos^{-1} 0.6420$

$2\theta = 50°$

$\theta = 50° \div 2$

$= 25°$

$2\theta = 360° - 50°$

$\theta = 310° \div 2$

$= 155°$

$2\theta = 360° + 50°$

$\theta = 410° \div 2$

$= 205°$

$2\theta = 720° - 50°$

$\theta = 670° \div 2$

$= 335°$

b) $\sin(\theta + 20°) = 0.2045$

$\theta + 20° = \sin^{-1} 0.2045$

$\theta + 20° = 12°$

$\theta = 12° - 20°$

$\theta = -8°$

$= 352°$

$\theta + 20° = 180° - 12°$

$\theta = 168° - 20°$

$= 148°$

c) $\tan(90° - 2\theta) = 1.6443$

$90° - 2\theta = \tan^{-1}(1.6443)$

$90° - 2\theta = 59°$

$2\theta = 90° - 59°$

$\theta = 31° \div 2$

$= 16°$

$90° - 2\theta = 180° + 59°$

$2\theta = 90° - 239°$

$\theta = -149° \div 2$

$\theta = -74.5°$

$= 286°$

$90° - 2\theta = 360° + 59°$

$2\theta = 90° - 419°$

$\theta = -329° \div 2$

$\theta = -164.5°$

$= 196°$

$90° - 2\theta = 360° + 180° + 59°$

$2\theta = 90° - 599°$

$\theta = -509° \div 2$

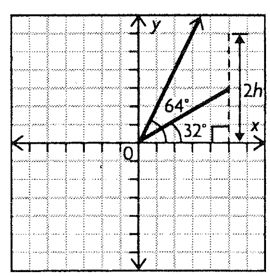
$\theta = -254.5°$

$= 106°$

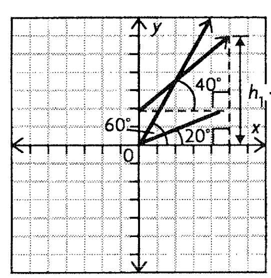
16. a) θ could lie in quadrant 3 or 4. $\theta = 233°$ or $307°$.

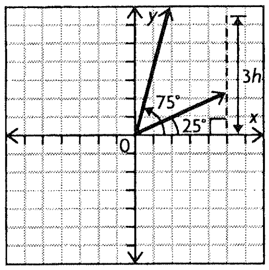
b) θ could lie in quadrant 2 or 3. $\theta = 139°$ or $221°$.

17. a)



b)



c) 

Mid-Chapter Review, p. 304

1. a) $\csc \theta = \frac{1}{\sin \theta}$

$\csc 20° = \frac{1}{\sin 20°}$

$= 2.9238$

b) $\sec \theta = \frac{1}{\cos \theta}$

$\sec 75° = \frac{1}{\cos 75°}$

$= 3.8637$

c) $\cot \theta = \frac{1}{\tan \theta}$

$\cot 10° = \frac{1}{\tan 10°}$

$= 5.6713$

d) $\csc \theta = \frac{1}{\sin \theta}$

$\csc 81° = \frac{1}{\sin 81°}$

$= 1.0125$

2. a) $\cot \theta = \frac{1}{\tan \theta}$

$\cot \theta = 0.8701$

$0.8701 = \frac{1}{\tan \theta}$

$\tan \theta = \frac{1}{0.8701}$

$\theta = \tan^{-1} 1.1493$

$= 49°$

b) $\sec \theta = \frac{1}{\cos \theta}$

$\sec \theta = 4.1011$

$4.1011 = \frac{1}{\cos \theta}$

$\cos \theta = \frac{1}{4.1011}$

$\theta = \cos^{-1} 0.2438$

$= 76°$

c) $\csc \theta = \frac{1}{\sin \theta}$

$\csc \theta = 1.6406$

$1.6406 = \frac{1}{\sin \theta}$

$\sin \theta = \frac{1}{1.6406}$

$\theta = \sin^{-1} 0.6095$

$= 38°$

d) $\sec \theta = \frac{1}{\cos \theta}$

$\sec \theta = 2.4312$

$2.4312 = \frac{1}{\cos \theta}$

$\cos \theta = \frac{1}{2.4312}$

$\theta = \cos^{-1} 0.4113$

$= 66°$

3. Since the hypotenuse must be the longest side, the denominator of $\frac{7}{5}$ cannot be the hypotenuse. Therefore $\frac{7}{5}$ cannot be a ratio for sine or cosine.

$\tan \theta = \frac{7}{5}$

$\theta = \tan^{-1} \frac{7}{5}$

$= 54°$

$\tan 54°$

$\csc \theta = \frac{1}{\sin \theta}$

$\csc \theta = \frac{7}{5}$

$\frac{7}{5} = \frac{1}{\sin \theta}$

$\sin \theta = \frac{5}{7}$

$\theta = \sin^{-1} \frac{5}{7}$

$= 46°$

$\csc 46°$

$\sec \theta = \frac{1}{\cos \theta}$

$$\sec\theta = \frac{7}{5}$$

$$\frac{7}{5} = \frac{1}{\cos\theta}$$

$$\cos\theta = \frac{5}{7}$$

$$\theta = \cos^{-1}\frac{5}{7}$$

$$= 44°$$

sec 44°

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{7}{5}$$

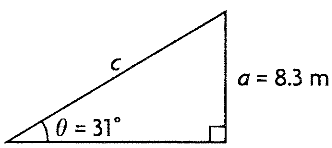
$$\frac{7}{5} = \frac{1}{\tan\theta}$$

$$\theta = \tan^{-1}\frac{5}{7}$$

$$= 36°$$

cot 36°

4.



The rope and mast form a right triangle. Since Claire needs 0.5 m to tie the rope, the length of rope, x, needed is $c + 0.5$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 31° = \frac{8.3}{c}$$

$$c = \frac{8.3}{\sin 31°}$$

$$c = 16.1$$

$$x = c + 0.5$$

$$x = 16.1 + 0.5$$

$$= 16.6$$

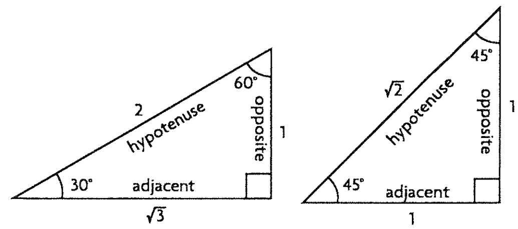
5. Angle θ is acute and therefore $0° < \theta < 90°$

$\csc\theta = \frac{r}{y}$ and $\sec\theta = \frac{r}{x}$

For $\csc\theta < \sec\theta$, $x < y$

$45° < \theta < 90°$

6.



a) Using the special triangle 30°–60°–90°,

$$\sin 60° = \frac{\sqrt{3}}{2}$$

b) Using the special triangle 45°–45°–90°,

$$\tan 45° = \frac{1}{1}$$

$$= 1$$

c) Using the special triangle 30°–60°–90°,

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\csc 30° = \frac{2}{1}$$

$$= 2$$

d) Using the special triangle 45°–45°–90°,

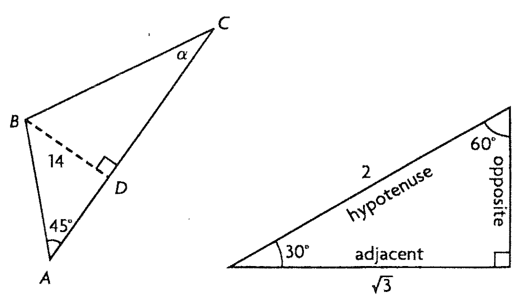
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\sec 45° = \frac{\sqrt{2}}{1}$$

$$= \sqrt{2}$$

7.



a) Since $\sin\alpha = \frac{1}{2}$, triangle BCD is a similar triangle to the 30° special triangle. If $\sin\alpha = \frac{1}{2}$, then $\alpha = 30°$ and $\tan\alpha = \frac{1}{\sqrt{3}}$ and therefore:

$$BC = 2 \times 14$$

$$= 28, \text{ and}$$

$CD = 14\sqrt{3}$

Triangle ABD is also similar to special triangle 45°–45°–90°, with $\sin 45° = \frac{\sqrt{2}}{2}$ Therefore:

$AB = 14\sqrt{2}$, and

$AD = 14$

b) As determined in part a, triangle ABD is similar to a special triangle 45°–45°–90° and triangle BCD is similar to a special triangle 30°–60°–90°. Therefore:

$$\sin A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

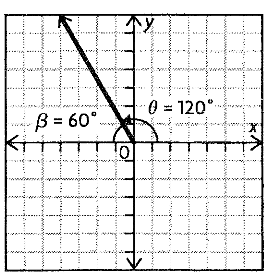
$$\tan A = \frac{1}{1} = 1$$

$$\sin DBC = \frac{\sqrt{3}}{2}$$

$$\cos DBC = \frac{1}{2}$$

$$\tan DBC = \frac{\sqrt{3}}{1} = \sqrt{3}$$

8. a) i)

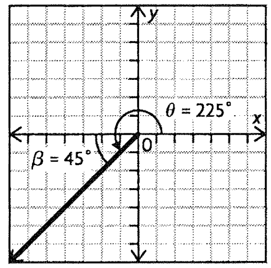


Since the related acute angle $\beta = 60°$ using the special triangle 30°–60°–90°,

$$\sin 120° = \frac{\sqrt{3}}{2}$$

ii) $180° - \theta$
$= 180° - 120°$
$= 60°$

b) i)

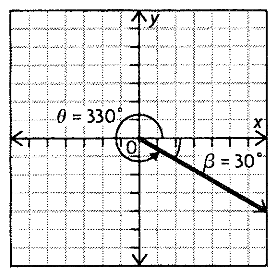


Since the related acute angle $\beta = 45°$, using the special triangle 45°–45°–90°,

$$\cos 225° = \frac{-\sqrt{2}}{2}$$

ii) $360° - \theta$
$= 360° - 225°$
$= 135°$

c) i)

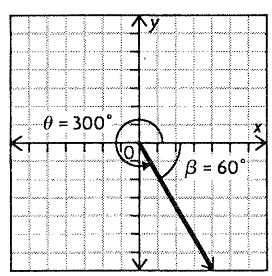


Since the related acute angle $\beta = 30°$, using the special triangle 30°–60°–90°,

$$\tan 330° = \frac{-\sqrt{3}}{3}$$

ii) $180° - \beta$
$= 180° - 30°$
$= 150°$

d) i)

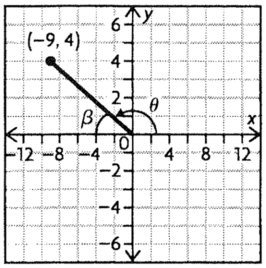


Since the related acute angle $\beta = 60°$, using the special triangle 30°–60°–90°,

$$= \cos 300° = \frac{1}{2}$$

ii) $360° - \theta$
$= 360° - 300°$
$= 60°$

9. a)



b) $\tan \beta = \frac{4}{|-9|}$

$\beta = \tan^{-1}\frac{4}{|-9|}$

$\beta = 24°$

c) $\theta = 180° - \beta$

$\theta = 180° - 24°$

$\theta = 156°$

10. No, the only two possible angles within the given range are 37° and 323°.

11. a) $\tan \theta = \frac{y}{x}$

$\tan \theta = -\frac{15}{8}$

$r^2 = x^2 + y^2$

$r^2 = (-8)^2 + 15^2$

$r^2 = 289$

$r = \sqrt{289}$

$r = 17$

Therefore:

$\sin \theta = \frac{y}{r}$

$\sin \theta = \frac{15}{17}$

$\cos \theta = \frac{x}{r}$

$\cos \theta = \frac{-8}{17}$

$\csc \theta = \frac{r}{y}$

$\csc \theta = \frac{17}{15}$

$\sec \theta = \frac{r}{x}$

$\sec \theta = \frac{17}{-8}$

$\cot \theta = \frac{x}{y}$

$\cot \theta = \frac{-8}{15}$

b) $\tan \theta = -\frac{15}{8}$

$\theta = \tan^{-1}(-1.875)$

$= -62°$

$\theta = 180° + (-62°)$

$= 118°$

12. $\sin \theta = -0.8190$

$\theta = \sin^{-1}(-0.8190)$

$= -55°$

$\theta = 360° + (-55°)$

$= 305°$

$\theta = 180° - (-55°)$

$= 235°$

13. a and **f.** Sine and cosine ratios should be less than or equal to 1.

5.5 Trigonometric Identities, pp. 310–311

1. a) $\cot \theta = \frac{\cos \theta}{\sin \theta}$

$\cot \theta = \frac{x}{y}$

$\cos \theta = \frac{x}{r}$

$\sin \theta = \frac{y}{r}$

$\frac{x}{y} = \frac{x}{r} \div \frac{y}{r}$

$\frac{x}{y} = \frac{x}{r} \times \frac{r}{y}$

$\frac{x}{y} = \frac{x}{y}$, for all angles θ where $0° \leq \theta \leq 360°$ except 0°, 180°, and 360°.

b) $\tan \theta \cos \theta = \sin \theta$

$\tan \theta = \frac{y}{x}$

$\cos \theta = \frac{x}{r}$

$\sin \theta = \frac{y}{r}$

$\frac{y}{x} \times \frac{x}{r} = \frac{y}{r}$

$\frac{y}{r} = \frac{y}{r}$, for all angles θ where $0° \leq \theta \leq 360°$

except 90° and 270°.

c) $\csc\theta = \dfrac{1}{\sin\theta}$

$\csc\theta = \dfrac{r}{y}$

$\sin\theta = \dfrac{y}{r}$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\dfrac{r}{y} = 1 \div \dfrac{y}{r}$

$\dfrac{r}{y} = 1 \times \dfrac{r}{y}$

$\dfrac{r}{y} = \dfrac{r}{y}$, for all angles θ where $0° \leq \theta \leq 360°$ except 0°, 180°, and 360°.

d) $\cos\theta \ \sec\theta = 1$

$\cos\theta = \dfrac{x}{r}$

$\sec\theta = \dfrac{r}{x}$

$\dfrac{x}{r} \times \dfrac{r}{x} = 1$

$\dfrac{x}{x} \times \dfrac{r}{r} = 1$

$1 = 1$, for all angles θ where $0° \leq \theta \leq 360°$ except 90° and 270°.

2. a) $(1 - \sin\alpha)(1 + \sin\alpha)$

$= 1 + \sin\alpha - \sin\alpha - \sin^2\alpha$

$= 1 - \sin^2\alpha$

$= \cos^2\alpha$

b) $\dfrac{\tan\alpha}{\sin\alpha}$

$= \dfrac{y}{x} \div \dfrac{y}{r}$

$= \dfrac{y}{x} \times \dfrac{r}{y}$

$= \dfrac{r}{x}$

$= \sec\alpha$ or $\dfrac{1}{\cos\alpha}$

c) $\cos^2\alpha + \sin^2\alpha$

$= \left(\dfrac{x}{r}\right)^2 + \left(\dfrac{y}{r}\right)^2$

$= \dfrac{x^2}{r^2} + \dfrac{y^2}{r^2}$

$= \dfrac{x^2 + y^2}{r^2}$

$= \dfrac{r^2}{r^2}$

$= 1$

d) $\cos\alpha \ \sin\alpha$

$= \dfrac{x}{y} \times \dfrac{y}{r}$

$= \dfrac{x}{r}$

$= \cos\alpha$

3. a) $1 - \cos^2\theta$

$= (1 - \cos\theta)(1 + \cos\theta)$

b) $\sin^2\theta - \cos^2\theta$

$= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

c) $\sin^2\theta - 2\sin\theta + 1$

$= (\sin\theta - 1)(\sin\theta - 1)$

$= (\sin\theta - 1)^2$

d) $\cos\theta - \cos^2\theta$

$= \cos\theta(1 - \cos\theta)$

4. $\dfrac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta$

$\cos^2\theta = (1 + \sin\theta) \times (1 - \sin\theta)$

$\cos^2\theta = 1 + \sin\theta - \sin\theta - \sin^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = \cos^2\theta$

5. a) $\dfrac{\sin x}{\tan x} = \cos x$

L.S. $= \dfrac{\sin x}{\tan x}$

$= \sin x \div \dfrac{\sin x}{\cos x}$

$= \sin x \times \dfrac{\cos x}{\sin x}$

$= \cos x$

= R.S., for all angles x where $0° \leq x \leq 360°$ except 0°, 90°, 180°, 270°, and 360°.

b) $\dfrac{\tan\theta}{\cos\theta} = \dfrac{\sin\theta}{1 - \sin^2\theta}$

L.S. $= \dfrac{\tan\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \div \cos\theta$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta}$

$= \dfrac{\sin\theta}{1 - \sin^2\theta}$

$=$ R.S., for all angles θ where $0° \le \theta \le 360°$ except 90° and 270°.

c) $\dfrac{1}{\cos\alpha} + \tan\alpha = \dfrac{1 + \sin\alpha}{\cos\alpha}$

L.S. $= \dfrac{1}{\cos\alpha} + \tan\alpha$

$= \dfrac{1}{\cos\alpha} + \dfrac{\sin\alpha}{\cos\alpha}$

$= \dfrac{1 + \sin\alpha}{\cos\alpha}$

$=$ R.S., for all angles α where $0° \le \alpha \le 360°$ except 90° and 270°.

d) $\sin\theta\cos\theta\tan\theta = 1 - \cos^2\theta$

L.S. $= \sin\theta\cos\theta\tan\theta$

$= \dfrac{y}{r} \times \dfrac{x}{r} \times \dfrac{y}{x}$

$= \dfrac{y^2}{r^2}$

$= \sin^2\theta$

$= 1 - \cos^2\theta$

$=$ R.S., for all angles θ where $0° \le \theta \le 360°$ except 90° and 270°.

6. No. You need to prove that the equation is true for all angles specified, not just one.

7. a) $\sin\theta\cot\theta - \sin\theta\cos\theta$

$= \sin\theta \times \dfrac{\cos\theta}{\sin\theta} - \sin\theta\cos\theta$

$= \cos\theta - \sin\theta\cos\theta$

$= \cos\theta(1 - \sin\theta)$

b) $\cos\theta(1 + \sec\theta)(\cos\theta - 1)$

$= \cos\theta\left(1 + \dfrac{1}{\cos\theta}\right)(\cos\theta - 1)$

$= \cos\theta\left(\dfrac{\cos\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)(\cos\theta - 1)$

$= \cos\theta\left(\dfrac{1 + \cos\theta}{\cos\theta}\right)(\cos\theta - 1)$

$= \cos\theta\left(\dfrac{(1 + \cos\theta)(\cos\theta - 1)}{\cos\theta}\right)$

$= (1 + \cos\theta)(\cos\theta - 1)$

$= \cos\theta + \cos^2\theta - 1 - \cos\theta$

$= \cos^2\theta - 1$

$= -\sin^2\theta$

c) $(\sin x + \cos x)(\sin x - \cos x) + 2\cos^2 x$

$= \sin^2 x + \cos x\sin x - \cos x\sin x$
$\quad - \cos^2 x + 2\cos^2 x$

$= \sin^2 x - \cos^2 x + 2\cos^2 x$

$= \sin^2 x + \cos^2 x$

$= 1$

d) $\dfrac{\csc^2\theta - 3\csc\theta + 2}{\csc^2\theta - 1}$

$= \dfrac{(\csc\theta - 2)(\csc\theta - 1)}{(\csc\theta + 1)(\csc\theta - 1)}$

$= \dfrac{\csc\theta - 2}{\csc\theta + 1}$, where $\csc\theta \ne 1$ or–1

8. a) $\dfrac{\sin^2\theta}{1 - \cos\theta} = 1 + \cos\theta$

L.S. $= \dfrac{\sin^2\theta}{1 - \cos\theta}$

$= \dfrac{1 - \cos^2\theta}{1 - \cos\theta}$

$= \dfrac{(1 - \cos\theta)(1 + \cos\theta)}{1 - \cos\theta}$

$= 1 + \cos\theta$

$=$ R.S., where $\cos\theta \ne 1$

b) $\dfrac{\tan^2\alpha}{1 + \tan^2\alpha} = \sin^2\alpha$

L.S. $= \dfrac{\tan^2\alpha}{1 + \tan^2\alpha}$

$= \dfrac{\tan^2\alpha}{\sec^2\alpha}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \div \dfrac{1}{\cos^2\alpha}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \times \cos^2\alpha$

$= \sin^2\alpha$

$=$ R.S., for all angles $0° \le \alpha \le 360°$ except for $\alpha = 90°$ and $\alpha = 270°$.

c) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

R.S. $= (1 - \sin x)(1 + \sin x)$

$= 1 - \sin x + \sin x - \sin^2 x$

$= 1 - \sin^2 x$

$= \cos^2 x$

$=$ L.S.

d) $\sin^2\theta + 2\cos^2\theta - 1 = \cos^2\theta$

L.S. $= \sin^2\theta + 2\cos^2\theta - 1$

$= \sin^2\theta + \cos^2\theta$
$\quad + \cos^2\theta - 1$

$= 1 + \cos^2\theta - 1$

$= \cos^2\theta$

$=$ R.S.

e) $\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$

L.S. $= \sin^4\alpha - \cos^4\alpha$

$= (\sin^2\alpha - \cos^2\alpha)$

$$(\sin^2\alpha + \cos^2\alpha)$$
$$= (\sin^2\alpha - \cos^2\alpha) \times 1$$
$$= \sin^2\alpha - \cos^2\alpha$$
$$= \text{R.S.}$$

f) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$$\text{L.S.} = \tan\theta + \frac{1}{\tan\theta}$$
$$= \frac{\sin\theta}{\cos\theta} + \left(1 \div \frac{\sin\theta}{\cos\theta}\right)$$
$$= \frac{\sin\theta}{\cos\theta} + \left(1 \times \frac{\cos\theta}{\sin\theta}\right)$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

$= \text{R.S.}$, where $\tan\theta \neq 0$, $\sin\theta \neq 0$, and $\cos\theta \neq 0$

9. a) For example: Farah's method only works for equations that don't have a trigonometric ratio in the denominator.

b) For example: If an equation has a trigonometric ratio in the denominator that can't equal zero, Farah's method doesn't work.

10. For example: $\csc^2\theta + \sec^2\theta = 1$ is not an identity; $\csc^2 45° + \sec^2 45° = 4$ shows that it is false.

11. $\sin^2 x\left(1 + \dfrac{1}{\tan^2 x}\right) = 1$

$$\text{L.S.} = \sin^2 x\left(1 + \frac{1}{\tan^2 x}\right)$$
$$= \sin^2 x\left(1 + 1 \div \frac{\sin^2 x}{\cos^2 x}\right)$$
$$= \left(\sin^2 x + \sin^2 x \div \frac{\sin^2 x}{\cos^2 x}\right)$$
$$= \left(\sin^2 x + \sin^2 x \times \frac{\cos^2 x}{\sin^2 x}\right)$$
$$= \sin^2 x + \cos^2 x$$
$$= 1$$

$= \text{R.S.}$, where $\tan x \neq 0$

12. a) $\dfrac{\sin^2\theta + 2\cos\theta - 1}{\sin^2\theta + 3\cos\theta - 3} = \dfrac{\cos^2\theta + \cos\theta}{-\sin^2\theta}$

$$\text{L.S.} = \frac{\sin^2\theta + 2\cos\theta - 1}{\sin^2\theta + 3\cos\theta - 3}$$
$$= \frac{(1 - \cos^2\theta) + 2\cos\theta - 1}{(1 - \cos^2\theta) + 3\cos\theta - 3}$$
$$= \frac{-\cos^2\theta + 2\cos\theta}{-\cos^2\theta + 3\cos\theta - 2}$$
$$= \frac{\cos\theta \times (2 - \cos\theta)}{(2 - \cos\theta)(\cos\theta - 1)}$$
$$= \frac{\cos\theta}{\cos\theta - 1}$$
$$\text{R.S.} = \frac{\cos^2\theta + \cos\theta}{-\sin^2\theta}$$
$$= \frac{\cos^2\theta + \cos\theta}{\cos^2\theta - 1}$$
$$= \frac{\cos\theta \times (\cos\theta + 1)}{(\cos\theta + 1)(\cos\theta - 1)}$$
$$= \frac{\cos\theta}{\cos\theta - 1}$$

$= \text{L.S.}$, where $\sin\theta \neq 0$, $\cos\theta \neq 1$

b) $\sin^2\alpha - \cos^2\alpha - \tan^2\alpha = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1 - \sin^2\alpha}$

$$\text{L.S.} = \sin^2\alpha - \cos^2\alpha - \tan^2\alpha$$
$$= \sin^2\alpha - \cos^2\alpha - \frac{\sin^2\alpha}{\cos^2\alpha}$$
$$= \frac{\cos^2\alpha \times \sin^2\alpha}{\cos^2\alpha} - \frac{\cos^4\alpha}{\cos^2\alpha} - \frac{\sin^2\alpha}{\cos^2\alpha}$$
$$= \frac{\cos^2\alpha \times \sin^2\alpha - \cos^4\alpha - \sin^2\alpha}{1 - \sin^2\alpha}$$
$$= \frac{\sin^2\alpha \times (1 - \sin^2\alpha)}{1 - \sin^2\alpha}$$
$$\frac{-(1 - \sin^2\alpha)^2 - \sin^2\alpha}{1 - \sin^2\alpha}$$
$$= \frac{-\sin^4\alpha - (1 - 2\sin^2\alpha + \sin^4\alpha)}{1 - \sin^2\alpha}$$
$$= \frac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1 - \sin^2\alpha}$$

$= \text{R.S.}$, where $\sin\alpha \neq 1$

13. For example: $\dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$ by multiplying by $\dfrac{\sin\theta}{\cos\theta}$.

14. a) iii) c is not an identity.

iii $\dfrac{\sin\theta\tan\theta}{\sin\theta + \tan\theta} = \sin\theta + \tan\theta$

If $\theta = 45°$, then

$$\frac{\sin\theta\tan\theta}{\sin\theta + \tan\theta}$$
$$= \frac{\sin 45^\circ \tan 45^\circ}{\sin 45^\circ + \tan 45^\circ}$$
$= 0.414$, and
$\sin\theta + \cos\theta$
$= \sin 45^\circ + \cos 45^\circ$
$= 1.414$
$\neq 0.414$

i)
$$(1 - \cos^2 x)(1 - \tan^2 x) = \frac{\sin^2 x - 2\sin^4 x}{1 - \sin^2 x}$$
$$\text{R.S.} = (1 - \cos^2 x)(1 - \tan^2 x)$$
$$= (1 - (1 - \sin^2 x))\left(1 - \frac{\sin^2 x}{\cos^2 x}\right)$$
$$= \sin^2 x \times \left(\frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\right)$$
$$= \sin^2 x \times \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$
$$= \sin^2 x \times \frac{(1 - \sin^2 x) - \sin^2 x}{1 - \sin^2 x}$$
$$= \frac{\sin^2 x \times (1 - 2\sin^2 x)}{1 - \sin^2 x}$$
$$= \frac{\sin^2 x - 2\sin^4 x}{1 - \sin^2 x}$$
$$= \text{L.S.},\ \sin^2 x \neq 1$$

ii) $1 - 2\cos^2\theta = \sin^4\theta - \cos^4\theta$
$$\text{L.S.} = 1 - 2\cos^2\theta$$
$$= 1 - \cos^2\theta - \cos^2\theta$$
$$= 1 - \cos^2\theta - (1 - \sin^2\theta)$$
$$= \sin^2\theta - \cos^2\theta$$
$$\text{R.S.} = \sin^4\theta - \cos^4\theta$$
$$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$
$$= 1 \times (\sin^2\theta - \cos^2\theta)$$
$$= \sin^2\theta - \cos^2\theta$$
= L.S., for all angles θ where
$0^\circ \leq \theta \leq 360^\circ$

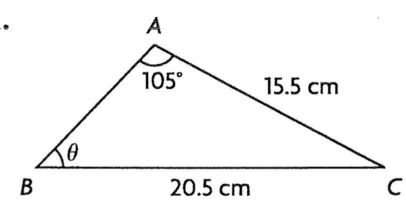
iv) $\dfrac{1 + 2\sin\beta\cos\beta}{\sin\beta + \cos\beta} = \sin\beta + \cos\beta$
$$\text{L.S.} = \frac{1 + 2\sin\beta\cos\beta}{\sin\beta + \cos\beta}$$
$$= \frac{\sin^2\beta + \cos^2\beta + 2\sin\beta\cos\beta}{\sin\beta + \cos\beta}$$
$$= \frac{(\sin\beta + \cos\beta)(\sin\beta + \cos\beta)}{\sin\beta + \cos\beta}$$
$$= \sin\beta + \cos\beta$$
$$= \text{R.S.},\ \sin\beta \neq -\cos\beta$$

v) $\dfrac{1 - \cos\beta}{\sin\beta} = \dfrac{\sin\beta}{1 + \cos\beta}$
$$\text{L.S.} = \frac{1 - \cos\beta}{\sin\beta}$$
$$= \frac{(1 - \cos\beta)(1 + \cos\beta)}{\sin\beta(1 + \cos\beta)}$$
$$= \frac{1 - \cos^2\beta}{\sin\beta(1 + \cos\beta)}$$
$$= \frac{\sin^2\beta}{\sin\beta(1 + \cos\beta)}$$
$$= \frac{\sin^2\beta}{1 + \cos\beta}$$
$$= \text{R.S.},\ \sin\beta \neq 0,\ \cos\beta \neq -1$$

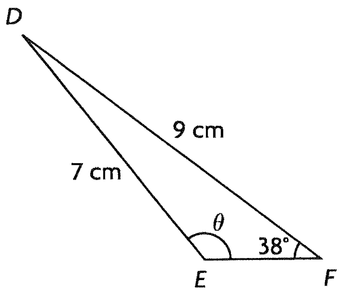
vi) $\dfrac{\sin x}{1 + \cos x} = \csc x - \cot x$
$$\text{R.S.} = \csc x - \cot x$$
$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$
$$= \frac{1 - \cos x}{\sin x}$$
$$= \frac{(1 - \cos x)(1 + \cos x)}{\sin x(1 + \cos x)}$$
$$= \frac{1 - \cos^2 x}{\sin x(1 + \cos x)}$$
$$= \frac{\sin^2 x}{\sin x(1 + \cos x)}$$
$$= \frac{\sin x}{1 + \cos x}$$
$$= \text{L.S.},\ \cos x \neq 1$$

5.6 The Sine Law, pp. 318–320

1.



a) $\dfrac{\sin\theta}{15.5} = \dfrac{\sin 105^\circ}{20.5}$
$$\sin\theta = 15.5\left(\frac{\sin 105^\circ}{20.5}\right)$$
$$\doteq 0.73$$
$$\theta \doteq \sin^{-1}(0.73)$$
$$\doteq 47^\circ$$



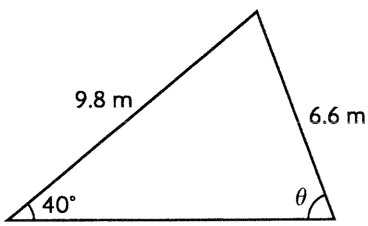
b) $\frac{\sin\theta}{9} = \frac{\sin 38°}{7}$

$$\sin\theta = 9\left(\frac{\sin 38°}{7}\right)$$
$$\doteq 0.79$$

$\sin^{-1}(0.79) \doteq 52°$

Since this is the ambiguous case of the sine law and the angle θ is oblique, θ is $180° - 52°$, or $128°$.

2. a)



b) $h = 9.8(\sin 40°)$
$\doteq 6.3$ m

h is less than both of the two given sides.

c) There are two possible lengths for the third side since $h < 6.6 < 9.8$.

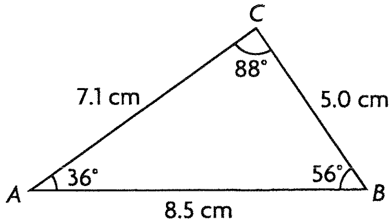
3. a) $h = 5.2(\sin 65°)$
$\doteq 4.7$

Since h is greater than the side measuring 2.8 cm, no triangle exists.

b) $h = 6.7(\sin 63°)$
$\doteq 6$

Since h is greater than the side measuring 2.1 cm, no triangle exists.

c)



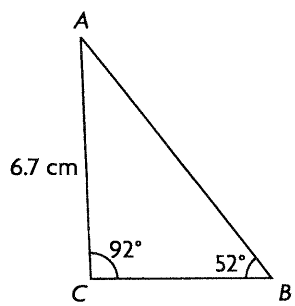
$h = 8.5(\sin 36°)$
$\doteq 4.99$

Since $h < 5.0 < 8.5$, two triangles exists.

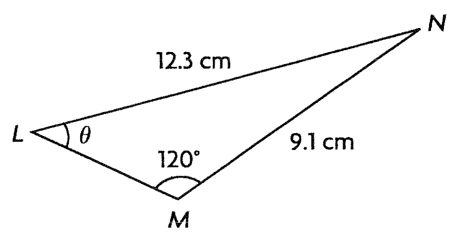
$\frac{\sin C}{8.5} = \frac{\sin 36°}{5}$

$$\sin C = 8.5\left(\frac{\sin 36°}{5}\right)$$
$$\doteq 0.9992$$
$$\angle C \doteq \sin^{-1}(0.9992)$$
$$\doteq 88°, \text{ or}$$
$$\doteq 180° - 88°$$
$$\doteq 92°$$
$$\angle B \doteq 180° - (36° + 92°)$$
$$\doteq 52°, \text{ or}$$
$$\doteq 180° - (36° + 88°)$$
$$\doteq 56°$$

$$b \doteq (\sin 52°)\left(\frac{5}{\sin 36°}\right)$$
$$\doteq 6.7 \text{ cm, or}$$
$$b \doteq (\sin 56°)\left(\frac{5}{\sin 36°}\right)$$
$$\doteq 7.1 \text{ cm}$$

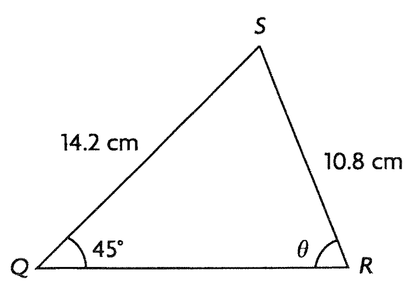


4.



a) $\frac{\sin\theta}{9.1} = \frac{\sin 120°}{12.3}$

$$\sin\theta = 9.1\left(\frac{\sin 120°}{12.3}\right)$$
$$\doteq 0.64$$
$$\theta \doteq \sin^{-1}(0.64)$$
$$\doteq 40°$$

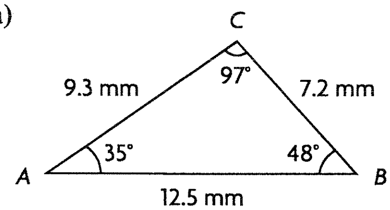


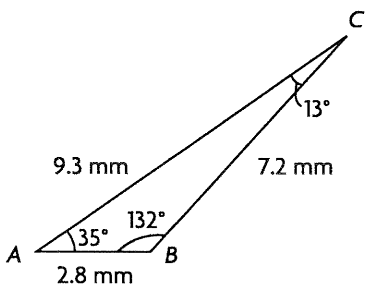
b) $\frac{\sin \theta}{14.2} = \frac{\sin 45°}{10.8}$

$$\sin \theta = 14.2\left(\frac{\sin 45°}{10.8}\right)$$
$$\doteq 0.93$$
$$\theta \doteq \sin^{-1}(0.93)$$
$$\doteq 68°$$

This is an ambiguous case of the sine law and h is $14.2(\sin 45°)$, or about 10.0 cm, which is less than either of the given sides. Therefore, θ can also be equal to $180° - 68°$, or $112°$.

5. a)





This is an ambiguous case of the sine law and h is $9.3(\sin 35°)$, or about 5.3 mm, which is less than either of the given sides. Therefore, there are two possible triangles.

$$\sin B = 9.3\left(\frac{\sin 35°}{7.2}\right)$$
$$\doteq 0.74$$
$$B \doteq \sin^{-1}(0.74)$$
$$\doteq 48°, \text{ or}$$
$$\doteq 180° - 48°$$
$$\doteq 132°$$

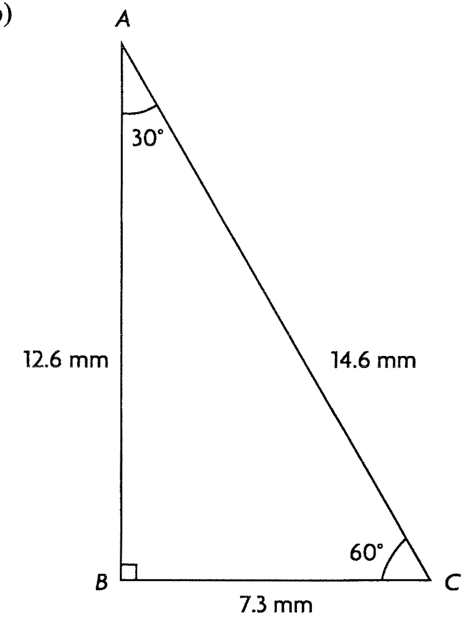
If $\angle B$ is $48°$, then $\angle C$ is $180° - (48° + 35°)$, or $97°$, and c is given by the following equation:

$$\frac{c}{\sin 97°} = \frac{7.2}{\sin 35°}$$
$$c = (\sin 97°)\left(\frac{7.2}{\sin 35°}\right)$$
$$\doteq 12.5 \text{ mm}$$

If $\angle B$ is $132°$, then $\angle C$ is $180° - (132° + 35°)$, or $13°$, and c is given by the following equation:

$$\frac{c}{\sin 13°} = \frac{7.2}{\sin 35°}$$
$$c = (\sin 13°)\left(\frac{7.2}{\sin 35°}\right)$$
$$\doteq 2.8 \text{ mm}$$

b)

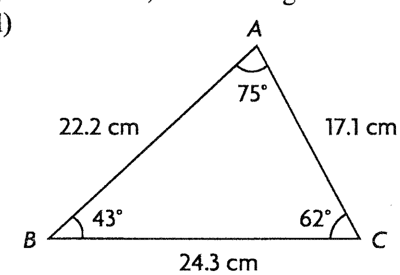


This is not an ambiguous case of the sine law, and h is $14.6(\sin 30°)$, or 7.3, which is equal to a. Therefore, there is only one possible triangle, and it is a right triangle with

$$\angle B = 90°$$
$$\angle C = 180° - (90° + 30°)$$
$$= 60°$$
$$\frac{c}{\sin 60°} = \frac{7.3}{\sin 30°}$$
$$c = (\sin 60°)\left(\frac{7.3}{\sin 30°}\right)$$
$$\doteq 12.6 \text{ m}$$

c) This is an ambiguous case of the sine law, and h is $2.8(\sin 33°)$, or about 1.5 cm, which is greater than a, so no triangle exists.

d)



This is not an ambiguous case of the sine law, so only one triangle exists.

$$\angle C = 180° - (75° + 43°)$$
$$= 62°$$

$$\frac{a}{\sin 75°} = \frac{22.2}{\sin 62°}$$

$$a = (\sin 75°)\left(\frac{22.2}{\sin 62°}\right)$$
$$\doteq 24.3 \text{ cm}$$

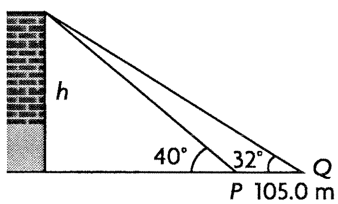
$$\frac{b}{\sin 43°} = \frac{22.2}{\sin 62°}$$

$$b = (\sin 43°)\left(\frac{22.2}{\sin 62°}\right)$$
$$\doteq 17.1 \text{ cm}$$

6. $\frac{\sin\theta}{20} = \frac{\sin 78°}{35}$

$$\sin\theta = 20\left(\frac{\sin 78°}{35}\right)$$
$$\doteq 0.56$$
$$\theta \doteq \sin^{-1}(0.56)$$
$$\doteq 34°$$

7.



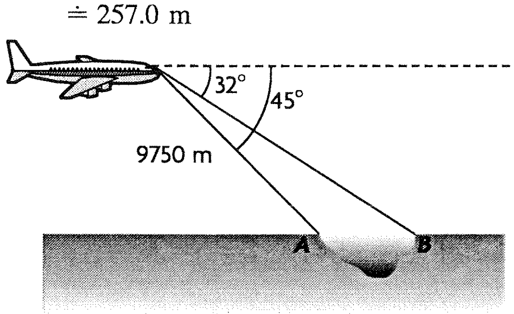
The angle made by P, the top of the building, and Q must be $180° - (32° + 140°)$, or $8°$. Let x be the distance from the top of the building to the point P.

$$\frac{x}{\sin 32°} = \frac{105.0}{\sin 8°}$$

$$x = (\sin 32°)\left(\frac{105.0}{\sin 8°}\right)$$
$$\doteq 399.8 \text{ m}$$

$$\sin 40° \doteq \frac{h}{399.8}$$
$$h \doteq 399.8(\sin 40°)$$
$$\doteq 257.0 \text{ m}$$

8.



In the triangle formed by the plane, shore A, and shore B, the angle opposite the width AB of the lake is equal to $45° - 32°$, or $13°$. By the sine law, the width AB of the lake will be given by the equation $\frac{AB}{\sin 13°} = \frac{9750}{\sin B}$. Since B is the alternate interior angle to the angle of depression measuring $32°$, B is also $32°$.

$$AB = (\sin 13°)\left(\frac{9750}{\sin 32°}\right)$$
$$\doteq 4139 \text{ m}$$

9. Let x be the length of the aqueduct. The closest end will be the one with an angle of depression of $71°$. Following what was done in problem 8,

$$\frac{x}{\sin 17°} = \frac{270.0}{\sin 54°}$$

$$x = (\sin 17°)\left(\frac{270.0}{\sin 54°}\right)$$
$$\doteq 97.6 \text{ m}$$

10. The height of the tower is given by the formula $\cos 34° = \frac{h}{30}$, so h is about 25 m.

11. Assume Carol is on the same side as the person with the $66°$ angle of elevation only and is 11 m from that person.

a) $37(\sin 50°) \doteq 28$ m

b) $x = (\sin 114°)\left(\frac{11}{\sin 16°}\right)$
$\doteq 37$ m

c) $x = (\sin 95°)\left(\frac{37}{\sin 35°}\right)$
$\doteq 64$ m

Assume Carol is on the same side as the person with the $66°$ angle of elevation only and is 11 m away from the other person.

a) $6(\sin 50°) \doteq 5$ m

b) $x = (\sin 35°)\left(\frac{11}{\sin 95°}\right)$
$\doteq 6$ m

c) $x = (\sin 16°)\left(\frac{6}{\sin 114°}\right)$
$\doteq 2$ m

Assume all people are on the same side and Carol is 11 m from the person with the $66°$ angle of elevation.

a) $37(\sin 50°) \doteq 28$ m

b) $x = (\sin 114°)\left(\dfrac{11}{\sin 16°}\right)$

$\doteq 37$ m

c) $x = (\sin 15°)\left(\dfrac{37}{\sin 35°}\right)$

$\doteq 16$ m

Assume all people are on the same side and Carol is 11 m from the person with the 35° angle of elevation.

a) $24.4(\sin 50°) \doteq 19$ m

b) $x = (\sin 35°)\left(\dfrac{11}{\sin 15°}\right)$

$\doteq 24$ m

c) $x = (\sin 16°)\left(\dfrac{24}{\sin 114°}\right)$

$\doteq 7$ m

12. The triangle formed by the tower, the ground, and one's position relative to the tower has one side of 47 m, two unknown sides of length x (since the goal is to be equidistant from the top and bottom of the tower), and two angles measuring 87.2° (since the tower leans at an angle of 2.8°) opposite each side x. That means the remaining angle opposite the 47 m side will be 180° − (2(87.2°)), or 5.6°.

$$\frac{x}{\sin 87.2°} = \frac{47}{\sin 5.6°}$$

$$x = (\sin 87.2°)\left(\frac{47}{\sin 5.6°}\right)$$

$$\doteq 481 \text{ m}$$

13. The 35° angle must be opposite the 430 m side.

$$\frac{\sin\theta}{110} = \frac{\sin 35°}{430}$$

$$\sin\theta = 110\left(\frac{\sin 35°}{430}\right)$$

$$\doteq 0.147$$

$$\theta \doteq \sin^{-1}(0.147)$$

$$\doteq 8°$$

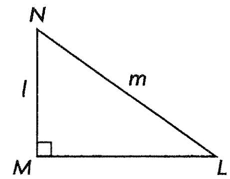
The remaining angle is 180° − (35° + 8°), or 137°, and the remaining side is given by the equation:

$$\frac{x}{\sin 137°} = \frac{110}{\sin 8°}$$

$$x = (\sin 137°)\left(\frac{110}{\sin 8°}\right)$$

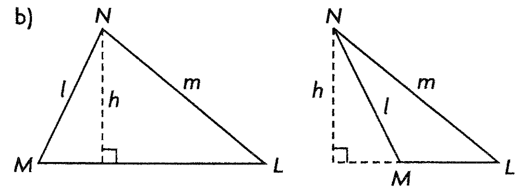
$$\doteq 515 \text{ m}$$

14. a)



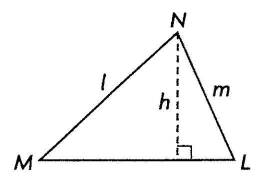
(right triangle) $l < m, \frac{\sin L}{l} = \frac{l}{m}$, (height) $h = l = m\sin L$

b)



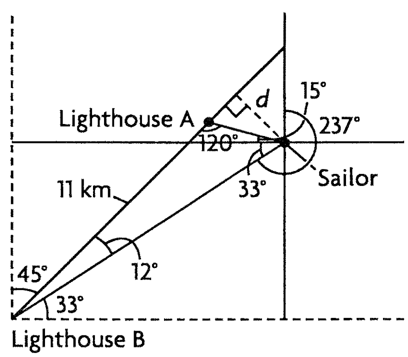
$h < l < m$, (height) $h = m\sin L$

c)



(acute triangle) $l > m$, (height) $h = m\sin L$

15.



a) Since the bearing to lighthouse A is 285° and the bearing to lighthouse B is 237°, from the sailor's point of view, the angle between the lighthouses is 285° − 237° or 48°. So the angle between lighthouse B and the sailor, when viewed from lighthouse A is 180° − 48° − 12° or 120°. Let AS represent the distance from the sailor to lighthouse A and BS represent the distance from the sailor to lighthouse B.

$$\frac{BS}{\sin 120°} = \frac{11}{\sin 48°}$$

$$BS = (\sin 120°)\frac{11}{\sin 48°}$$

$$\doteq 13 \text{ km}$$

$$\frac{AS}{\sin 12°} = \frac{11}{\sin 48°}$$

$$AS = (\sin 12°)\frac{11}{\sin 48°}$$

$$\doteq 3 \text{ km}$$

b) The distance to the shore, represented by d, is the length of a side of a right triangle with BS as its hypotenuse.

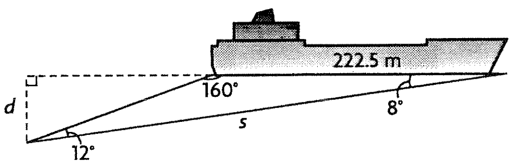
$$\sin 12° = \frac{d}{BS}$$

$$BS(\sin 12°) = d$$

$$d = \left((\sin 120°)\frac{11}{\sin 48°}\right)(\sin 12°)$$

$$d \doteq 2.7 \text{ km}$$

16.



a) Let s represent the distance to the stern.

$$\frac{S}{\sin 160°} = \frac{222.5}{\sin 12°}$$

$$s = (\sin 160°)\frac{222.5}{\sin 12°}$$

$$\doteq 366 \text{ m}$$

b) The shortest distance between the cargo ship and you is represented by d in the figure above. d is the length of a side of a right triangle with s as its hypotenuse.

$$\sin 8° = \frac{d}{s}$$

$$d = s(\sin 8°)$$

$$= \left((\sin 160°)\frac{222.5}{\sin 12°}\right)(\sin 8°)$$

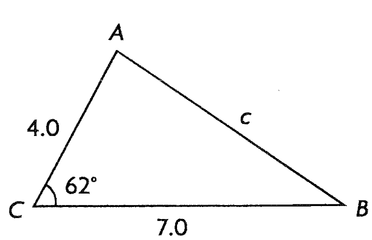
$$\doteq 50.9 \text{ m}$$

Since the *Algomarine* never gets any closer than 50.9 m to you, you won't get swamped.

17. The lower wire has a length of $\frac{155}{\sin 36°}$, or about 264 m. The upper wire has a length of $\frac{350}{\sin 59°}$, or about 408 m.

5.7 The Cosine Law, pp. 325–327

1. a)



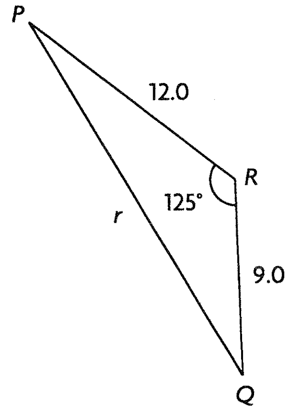
$$c^2 = 4.0^2 + 7.0^2 - 2(4.0)(7.0)(\cos 62°)$$

$$= 38.71$$

$$c = \sqrt{38.71}$$

$$\doteq 6.2$$

b)



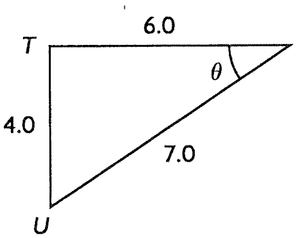
$$r^2 = 12.0^2 + 9.0^2 - 2(9.0)(12.0)(\cos 125°)$$

$$= 348.89$$

$$r = \sqrt{348.89}$$

$$\doteq 18.7$$

2. a)



$$4.0^2 = 6.0^2 + 7.0^2 - 2(6.0)(7.0)(\cos \theta)$$

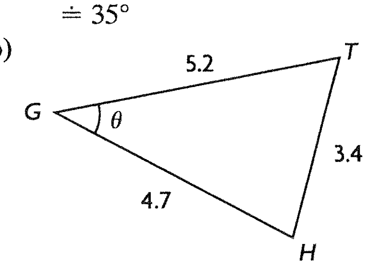
$$\cos \theta = \frac{4.0^2 - 6.0^2 - 7.0^2}{-2(6.0)(7.0)}$$

$$= \frac{23}{38}$$

$$\theta = \cos^{-1}\left(\frac{23}{38}\right)$$

$$\doteq 35°$$

b)



$$3.4^2 = 5.2^2 + 4.7^2 - 2(5.2)(4.7)(\cos \theta)$$

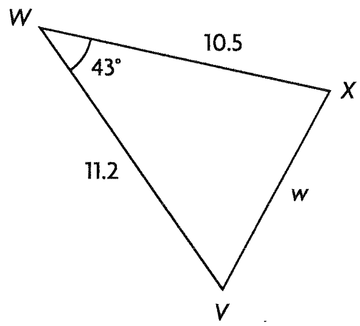
$$\cos \theta = \frac{3.4^2 - 5.2^2 - 4.7^2}{-2(5.2)(4.7)}$$

$$= \frac{289}{376}$$

$$\theta = \cos^{-1}\left(\frac{289}{376}\right)$$

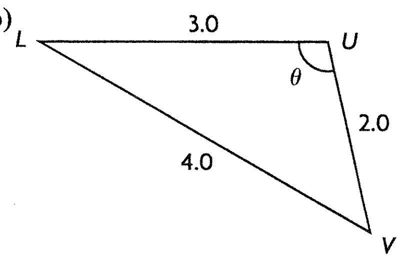
$$\doteq 40°$$

3. a)



$$w^2 = 10.5^2 + 11.2^2 - 2(10.5)(11.2)(\cos 43°)$$
$$= 63.68$$
$$w = \sqrt{63.68}$$
$$\doteq 8.0$$

b)



$$4.0^2 = 2.0^2 + 3.0^2 - 2(2.0)(3.0)(\cos \theta)$$
$$\cos \theta = \frac{4.0^2 - 2.0^2 - 3.0^2}{-2(2.0)(3.0)}$$
$$= \frac{-1}{4}$$
$$\theta = \cos^{-1}\left(\frac{-1}{4}\right)$$
$$\doteq 104°$$

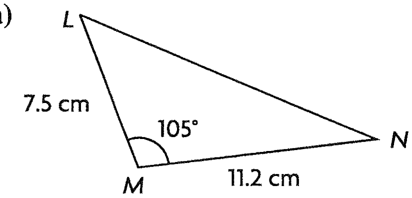
c) $11.5^2 = 8.3^2 + 6.6^2 - 2(8.3)(6.6)(\cos A)$

$$\cos A = \frac{11.5^2 - 8.3^2 - 6.6^2}{-2(8.3)(6.6)}$$
$$= \frac{-15}{83}$$
$$\angle A = \cos^{-1}\left(\frac{-15}{83}\right)$$
$$\doteq 100°$$

d) $p^2 = 25.1^2 + 71.3^2 - 2(25.1)(71.3)\left(\frac{1}{4}\right)$

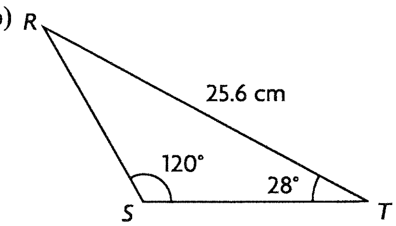
$$= 4818.885$$
$$p = \sqrt{4818.885}$$
$$\doteq 69.4$$

4. a)



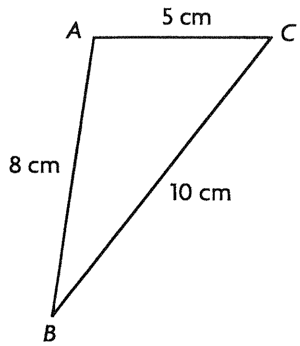
$$m^2 = 7.5^2 + 11.2^2 - 2(7.5)(11.2)(\cos 105°)$$
$$= 225.17$$
$$m = \sqrt{225.17}$$
$$\doteq 15.0 \text{ cm}$$
$$7.5^2 = 11.2^2 + 15.0^2 - 2(11.2)(15.0)(\cos N)$$
$$\cos N = \frac{7.5^2 - 11.2^2 - 15.0^2}{-2(11.2)(15.0)}$$
$$\doteq 0.8756$$
$$\angle N \doteq \cos^{-1}(0.8756)$$
$$\doteq 29°$$
$$\angle L = 180° - (105° + 29°)$$
$$= 180° - 134°$$
$$= 46°$$

b)



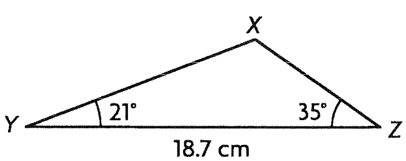
$$\angle R = 180° - (120° + 28°)$$
$$= 180° - 148°$$
$$= 32°$$
$$\frac{r}{\sin 32°} = \frac{25.6}{\sin 120°}$$
$$= 29.56$$
$$r = 29.56(\sin 32°)$$
$$\doteq 15.7 \text{ cm}$$
$$t^2 = 15.7^2 + 25.6^2 - 2(15.7)(25.6)(\cos 28°)$$
$$= 192.10$$
$$t = \sqrt{192.10}$$
$$\doteq 13.9 \text{ cm}$$

c)



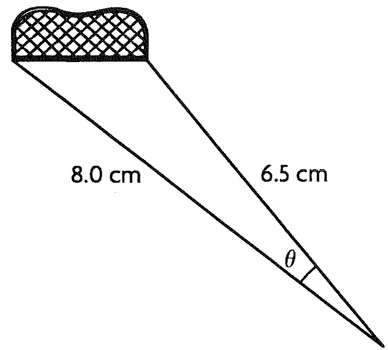
$$5^2 = 8^2 + 10^2 - 2(8)(10)(\cos B)$$
$$\cos B = \frac{5^2 - 8^2 - 10^2}{-2(8)(10)}$$
$$= \frac{139}{160}$$
$$\angle B = \cos^{-1}\left(\frac{139}{160}\right)$$
$$\doteq 30°$$
$$10^2 = 5^2 + 8^2 - 2(5)(8)(\cos A)$$
$$\cos A = \frac{10^2 - 5^2 - 8^2}{-2(5)(8)}$$
$$= \frac{-11}{80}$$
$$\angle A = \cos^{-1}\left(\frac{-11}{80}\right)$$
$$\doteq 98°$$
$$\angle C = 180° - (30° + 98°)$$
$$= 180° - 128°$$
$$= 52°$$

d)



$$\angle X = 180° - (21° + 35°)$$
$$= 180° - 56°$$
$$= 124°$$
$$\frac{z}{\sin 35°} = \frac{18.7}{\sin 124°}$$
$$= 22.56$$
$$z = 22.56(\sin 35°)$$
$$\doteq 12.9 \text{ cm}$$
$$y^2 = 12.9^2 + 18.7^2 - 2(12.9)(18.7)(\cos 21°)$$
$$= 65.68$$
$$y = \sqrt{65.68}$$
$$\doteq 8.1 \text{ cm}$$

5.

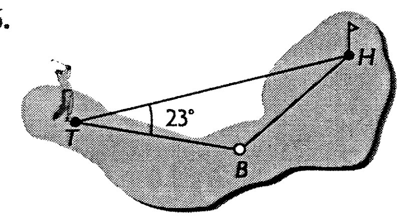


By the cosine law, the angle θ within which the shot must be made is given by

$$2.0^2 = 6.5^2 + 8.0^2 - 2(6.5)(8.0)(\cos \theta).$$
$$\cos \theta = \frac{2.0^2 - 6.5^2 - 8.0^2}{-2(6.5)(8.0)}$$
$$= \frac{409}{416}$$
$$\theta = \cos^{-1}\left(\frac{409}{416}\right)$$
$$\doteq 11°$$

The shot must be made within an angle of about 11°.

6.

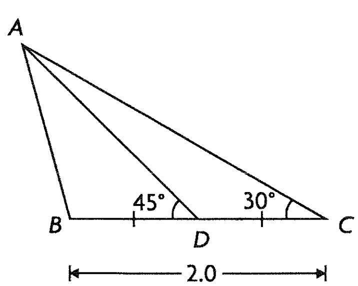


By the cosine law, the distance d from the ball to the hole is given by

$$d^2 = 270^2 + 160^2 - 2(270)(160)(\cos 23°).$$
$$d^2 = 18\,968.38$$
$$d = \sqrt{18\,968.38}$$
$$\doteq 138$$

The distance from the ball to the hole is about 138 m.

7.



Since $\angle ADB$ is 45°, you know that $\angle ADC$ is 180° − 45°, or 135°. Therefore, $\angle CAD$ is 180° − (135° + 30°), or 15°.

By the sine law, $\frac{AC}{\sin 135°}$ is equal to $\frac{1.0}{\sin 15°}$, or about 3.86.

$$AC \doteq 3.86(\sin 135°)$$
$$\doteq 2.73$$

By the cosine law,

$$(AB)^2 = 2.0^2 + 2.73^2 - 2(2.0)(2.73)(\cos 30°)$$
$$\doteq 2$$
$$AB \doteq \sqrt{2}$$
$$\doteq 1.4$$

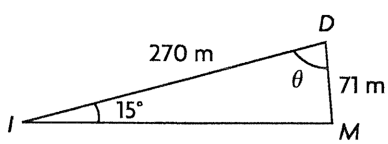
8. Using the information given, tower A, tower B and the fire form a triangle with the following properties (call this triangle ABF): ∠A is 45Y, ∠B is 95Y, ∠F is 40Y and the length of side *AB* is 20.3 km. By the sine law, the distance from tower A to the fire (F) is

$$\frac{AF}{\sin 95Y} = \frac{20.3}{sin\, 40Y}$$
$$AF = \sin 95Y \times \frac{20.3}{\sin 40Y}$$
$$= 31.5 \text{ km}$$

Again using the sine law, the distance from tower B to the fire is

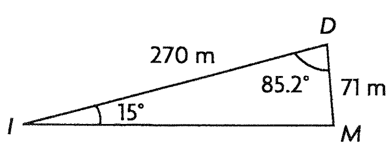
$$\frac{BF}{\sin 45Y} = \frac{20.3}{\sin 40Y}$$
$$BF = \sin 45Y \times \frac{20.3}{\sin 40Y}$$
$$= 22.3 \text{ km}$$

9. a)



Answers may vary. For example, Mike is standing on the other road and is 71 m from Darryl. From Darryl's position, what angle, to the nearest degree, separates the intersection from Mike?

b)



Answers may vary. For example, how far, to the nearest metre, is Mike from the intersection?

$$x^2 = 71^2 + 270^2 - 2(71)(270)(\cos 85.2°)$$
$$= 74\,732.79$$
$$x = \sqrt{74\,732.79}$$
$$\doteq 273 \text{ m}$$

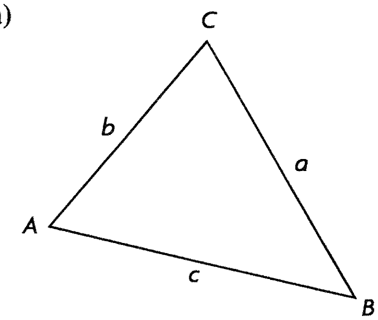
10. Let *d* be the distance from the top of the tower to the top edge of its shadow.

By the cosine law,

$$d^2 = 55.9^2 + 90.0^2 - 2(55.9)(90)(\cos 84.5°).$$
$$d^2 = 10\,275.94$$
$$d = \sqrt{10\,275.94}$$
$$\doteq 101.4$$

The distance from the top of the tower to the top edge of its shadow is about 101.4 m.

11. a)



The minimum information required to use the cosine law is the values of *a*, *b*, and *c* or the values of *b*, *c*, and ∠*A*.

The cosine law is $a^2 = b^2 + c^2 - 2ab(\cos A)$.

b) The minimum information required to use the sine law is the values of *a*, *b*, and ∠*A*.

The sine law is $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

12. The longest side of the triangle will be opposite the 120° angle, and the shortest side (call the length of this side *x*) of the triangle will be opposite the 20° angle, so the unknown side will be opposite the 40° angle.

By the sine law, $\frac{x}{\sin 20°} = \frac{x + 10}{\sin 120°}$.

$$x = (x + 10)\left(\frac{\sin 20°}{\sin 120°}\right)$$
$$= (x + 10)(0.3949)$$
$$= 0.3949x + 3.949$$
$$x - 0.3949x = 3.949$$
$$x(1 - 0.3949) = 3.949$$
$$x = \frac{3.949}{1 - 0.3949}$$
$$\doteq 6.6$$

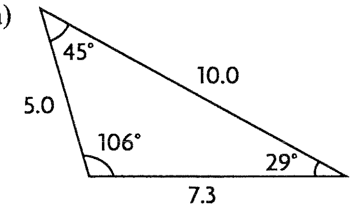
So the shortest side is about 6.6 cm long, and the longest side is about 16.6 cm long.

Call the remaining side y. By the cosine law, y is given by the equation

$$y^2 = 6.6^2 + 16.6^2 - 2(6.6)(16.6)(\cos 40°).$$
$$= 150.5$$
$$y \doteq 12.3$$

So the perimeter of the triangle is about $6.6 + 16.6 + 12.3$, or about 47.4.

13. a)

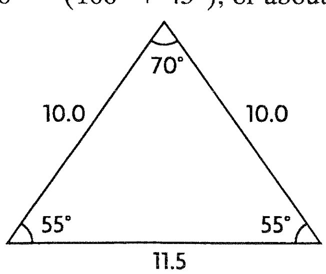


You know that the 45° angle cannot be opposite the side measuring 10.0 cm, or else the other two angles opposite the shorter sides would have to be less than 45°, and the interior angles of the triangle would not add up to 180°. Similarly, the 45° angle cannot be opposite the side measuring 5 cm, or else, by the sine law, the angle opposite the side measuring 10 cm would have to be 90°, and the angle opposite the side measuring 7.4 cm would have to be greater than 45°. However, then the interior angles of the triangle would not add up to 180°. So assume the 45° angle is opposite the side of length 7.4 cm. Let A be the angle opposite the side measuring 10.0 cm. By the cosine law, A is given by the following equation:

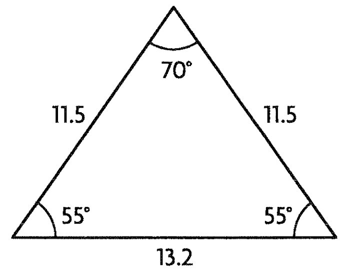
$$10.0^2 = 7.4^2 + 5.0^2 - 2(7.4)(5.0)(\cos A)$$
$$\cos A = \frac{10.0^2 - 7.4^2 - 5.0^2}{-2(7.4)(5.0)}$$
$$= \frac{-253}{925}$$
$$A = \cos^{-1}\left(\frac{-253}{925}\right)$$
$$\doteq 106°$$

The remaining angle opposite the side measuring 5.0 cm will be about $180° - (106° + 45°)$, or about 29°.

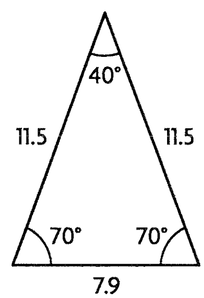
b)



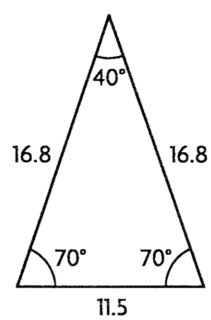
In the first triangle, the side measuring 11.5 is opposite the side of 70°, and there are two sides of $\frac{180° - 70°}{2}$, or 55°. The sides opposite the 55° angle are $(\sin 40°)\frac{11.5}{\sin 70°}$, or about 10.0.



In the second triangle, the sides measuring 11.5 are opposite the 55° angles, and the side opposite the 70° is $\frac{11.5}{\sin 55°}(\sin 70°)$, or about 13.2.

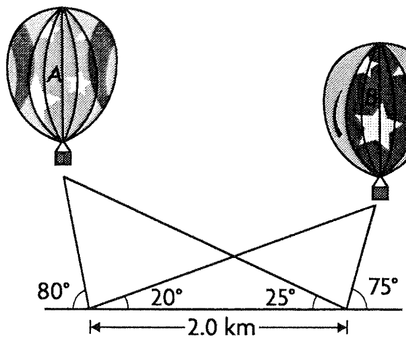


In the third triangle, there are two angles of 70° and one of $180° - 2(70°)$, or 40°. The sides measuring 11.5 are opposite the 70° angle, and the remaining side is $(\sin 40°)\left(\frac{11.5}{\sin 70°}\right)$, or about 7.9



In the last triangle, the side measuring 11.5 is opposite the 40° angle, and the sides opposite the 70° angles are $(\sin 70°)\left(\frac{11.5}{\sin 40°}\right)$, or about 16.8

14. a)



Considering the triangle formed by balloon A, balloon B, and the intersection of the lines from each balloon to the observer on its opposite side, you can find the distances of the various line segments in the above picture. The remaining angle in the middle triangle is $180° - (20 + 25)$, or $135°$. By the law of sines,

$\frac{2}{\sin 135°} = \frac{x}{\sin 25°}$, so x, the bottom segment of the left triangle, is about 1.2 km. Its opposite angle is $180° - (45° + 80°)$, or $55°$. Therefore, the top part of the line segment from balloon A to its opposite observer is

$t = (\sin 80°)\left(\frac{1.2}{\sin 55°}\right)$, or about 1.44 km. The bottom segment of the rightmost triangle, by the sine law, is given by $\frac{2}{\sin 135°} = \frac{x}{\sin 20°}$, so x is about 0.98 km. Since its opposite angle is also $55°$, the top part of the line segment from balloon B to its opposite observer is

$t = (\sin 80°)\left(\frac{0.98}{\sin 55°}\right)$, or about 1.18 km.

Then, by the cosine law, the distance between A and B squared is equal to about $(1.44)^2 + (1.18)^2 - 2(1.44)(1.18)(\cos 135°)$, or about 5.87.

$\sqrt{5.87} \doteq 2.4$

So the distance between A and B is about 2.4 m.

b) By what was done in part a), the distance from balloon A to its opposite observer is about $1.44 + 0.98 = 2.42$ km. The distance from balloon B to the opposite observer is about $1.2 + 1.18 = 2.38$ km.

So the height of balloon A is

$h = 2.42 \times \sin 25Y$

$= 1.02$ km

The height of balloon B is

$h = 2.38 \times \sin 20Y$

$= 0.81$ km

So balloon A is higher by about $1.02 - 0.81 = 0.2$ km

5.8 Solving Three-Dimensional Problems by Using Trigonometry, pp. 332–335

1. Use primary trigonometric ratios to calculate the hypotenuse of each right triangle. Add the results together to get the length of line needed.

2. a) Answers may very. Sine law, given information for angle, side, angle; need to find a side.

b) Answers may vary. For example, use a right triangle with acute angles 40° and 50°. Then, solve:

$$\frac{2.5}{x} = \cos 50°$$

3. a) $90° - 35° = 55°$

$$\frac{DF}{\sin 35°} = \frac{15}{\sin 55°}$$

$$\sin 35° \times \frac{DF}{\sin 35°} = \frac{15}{\sin 55°} \times \sin 35°$$

$$DF = \frac{15}{\sin 55°} \times \sin 35°$$

$$DF = 10.50 \text{ cm}$$

$$\frac{x}{\sin 90°} = \frac{10.50}{\sin 45°}$$

$$\sin 90° \times \frac{x}{\sin 90°} = \frac{10.50}{\sin 45°} \times \sin 90°$$

$$x = \frac{10.50}{\sin 45°} \times \sin 90°$$

$$x = 14.84 \doteq 15 \text{ cm}$$

b) Use the cosine law to find the length of the segment opposite $\angle D$; call this length d. Since there are tick marks on two of the sides, BD also has length 15 cm.

$d^2 = 15^2 + 15^2 - 2(15)(15)\cos 70Y$

$= 296.1$

$d = 17$ cm

Use d to find the value of x.

$$\sin 27Y = \frac{d}{x}$$

$$x = \frac{d}{\sin 27Y}$$

$= 38$ cm

c) Find angles for triangle EBD:

$\angle EBD = 180° - (65° + 55°)$
$\angle EBD = 180° - (120°)$
$\angle EBD = 60°$

Find $\angle DBC$:

$\angle DBC = 180° - (90°+60°)$
$\angle DBC = 180° - (150°)$
$\angle DBC = 30°$

$$\frac{BD}{\sin 115°} = \frac{10}{\sin 30°}$$

$$\sin 115° \times \frac{BD}{\sin 115°} = \frac{10}{\sin 30°} \times \sin 115°$$

$$BD = \frac{10}{\sin 30°} \sin 115°$$

$$BD = 18.13 \text{ cm}$$

$$\frac{EB}{\sin 55°} = \frac{18.13}{\sin 115°}$$

$$\sin 55° \times \frac{EB}{\sin 55°} = \frac{18.13}{\sin 115°} \times \sin 55°$$

$$EB = \frac{18.13}{\sin 115°} \times \sin 55°$$

$$EB = 16.39 \text{ cm}$$

$$\frac{x}{\sin 90°} = \frac{16.39}{\sin 70°}$$

$$\sin 90° = 1$$

$$x = \frac{16.39}{\sin 70°}$$

$$x = 17.44 \doteq 17 \text{ cm}$$

d) $\tan \angle BAD = \frac{18}{15}$

$\tan \angle BAD = 1.2$
$\angle BAD = 50.19 \doteq 50°$

$\tan \angle DAC = \frac{14}{15}$

$\tan \angle DAC = 0.933$
$\angle DAC = 43.03° \doteq 43°$
$\angle BAC = 50° + 43°$
$\angle BAC = 93°$

4. a) $\angle BDC = 180° - (66° + 50°)$

$\angle BDC = 64°$

$$\frac{DC}{\sin 50°} = \frac{175.0}{\sin 64°}$$

$$\sin 50° \times \frac{DC}{\sin 50°} = \frac{175.0}{\sin 64°} \times \sin 50°$$

$$DC = \frac{175.0}{\sin 64°} \times \sin 50°$$

$$DC = 149.2 \text{ m}$$

$$\frac{h}{149.2} = \tan 74°$$

$$\frac{h}{149.2} \times 149.2 = 149.2 \times \tan 74°$$

$$h = 149.2 \times \tan 74°$$

$$h = 520.3 \text{ m}$$

b) Yes, use the sine law.

$\angle DAC = 80° - (90°+74°)$
$\angle DAC = 16°$

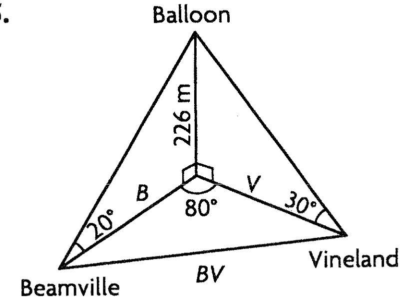
$$\frac{h}{\sin 74} = \frac{149.2}{\sin 16}$$

$$\sin 74° \times \frac{h}{\sin 74°} = \frac{149.2}{\sin 16°} \times \sin 74$$

$$h = \frac{149.2}{\sin 16°} \times \sin 74$$

$$h = 520.3$$

5.



$$\frac{226}{B} = \tan 2°$$

$$B \times \frac{226}{B} = \tan 2° \times B$$

$$226 = \tan 2° \times B$$

$$B = \frac{226}{\tan 2°}$$

$$B = 6471.8 \text{ cm}$$

$$\frac{226}{V} = \tan 3°$$

$$V \times \frac{226}{V} = \tan 3° \times V$$

$$226 = \tan 2° \times V$$

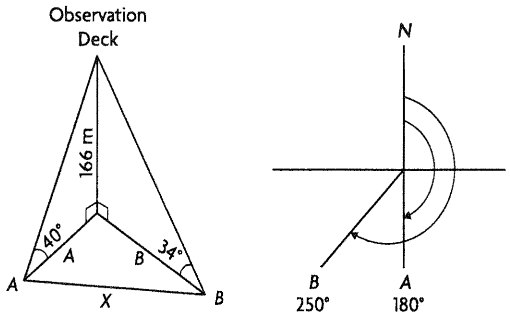
$$V = \frac{226}{\tan 3°}$$

$$V = 4312.3 \text{ m}$$

$$BV^2 = 6471^2 + 4321^2 - 2(6471)(4321)(\cos 80°)$$

$$BV = 7126.6 \doteq 7127 \text{ m}$$

6.



$$\frac{166}{A} = \tan 40°$$

$$A \times \frac{166}{A} = \tan 40° \times A$$

$$166 = \tan 60° \times A$$

$$A = \frac{166}{\tan 40°}$$

$$A = 197.83 \text{ m}$$

$$\frac{166}{B} = \tan 34°$$

$$B \times \frac{166}{B} = \tan 34° \times B$$

$$166 = \tan 34° \times B$$

$$B = \frac{166}{\tan 34°}$$

$$B = 246.11 \text{ m}$$

$$x^2 = 198^2 + 246^2 - 2(198)(246)\cos 70°$$

$$x = 257.69 \doteq 258 \text{ m}$$

7. $\tan 18° = \frac{h}{p}$

$$p = \frac{h}{\tan 18°}$$

$$\tan 20° = \frac{h}{r}$$

$$r = \frac{h}{\tan 20°}$$

$$r^2 + p^2 = 100^2$$

$$\left(\frac{h}{\tan 20°}\right)^2 + \left(\frac{h}{\tan 18°}\right) = 100^2$$

$$\frac{h^2}{(\tan 20°)^2} + \frac{h^2}{(\tan 18°)^2} = 100^2$$

$$\frac{h^2}{0.36^2} + \frac{h^2}{0.32^2} = 100^2$$

$$\frac{h^2}{0.1296} + \frac{h^2}{0.1024} = 100^2$$

$$\frac{0.1024h^2}{0.0133} + \frac{0.1296h^2}{0.0133} = 100^2$$

$$0.1024h^2 + 0.1296h^2 = 133$$

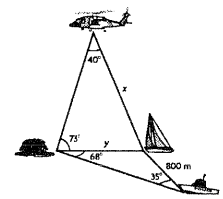
$$0.232h^2 = 133$$

$$h^2 = \frac{133}{0.232}$$

$$h = \sqrt{\frac{133}{0.232}}$$

$$h = 23.9 \doteq 23 \text{ m}$$

8. The figure below reflects all of the given information.



$$\frac{y}{\sin 35Y} = \frac{800}{\sin 68Y}$$

$$y - (\sin 35Y)\left(\frac{800}{\sin 68Y}\right)$$

$$= 494.9 \text{ m}$$

Now use the sine law to find x.

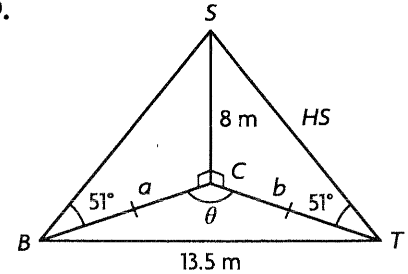
$$\frac{x}{\sin 73Y} = \frac{y}{\sin 40Y}$$

$$x = (\sin 73Y)\left(\frac{494.9}{\sin 40Y}\right)$$

$$= 736 \text{ m}$$

The straight-line distance from the helicopter to the sailboat is about 736 m.

9.



$$a = b$$

$$\frac{a}{8} = \tan 51°$$

$$8 \times \frac{a}{8} = \tan 51° \times 8$$

$$a = 8 \times \tan 51°$$

$$a = b = 9.9 \text{ m}$$

$$a^2 + b^2 - 2ab\cos\theta = 13.5^2$$

$$9.9^2 + 9.9^2 - 2(9.9)(9.9)\cos\theta = 13.5^2$$

$$98.01 + 98.01 - 196.02\cos\theta = 182.25$$

$$196.02 - 196.02\cos\theta = 182.25$$

$$-196.02\cos\theta = -13.77$$

$$\cos\theta = 0.0702$$

$\theta = 94.03°$
$\angle BCT = 180° - 94.03°$
$\angle BCT = 94.03$
$\angle CBT = \angle CTB$
$\angle CBT = \dfrac{94.03}{2}$
$\angle CBT = 47.01 \doteq 47°$

10. For Triangle ABC:
$AB^2 = BC^2 + AC^2 - 2(BC)(AC)\cos 130°$
$AB^2 = 4.5^2 + 2.0^2 - 2(4.5)(2.0)\cos 130°$
$AB^2 = 20.25 + 4.0 - 18.0 \cos 130°$
$AB^2 = 24.25 + 11.57$
$AB^2 = 38.82$
$AB = \sqrt{38.82}$
$AB = 6.0$ m

Triangle *GHI* has all three side dimensions shown

For Triangle *JKL*:
$LK^2 = LJ^2 + JK^2 - 2(LJ)(JK)\cos 42°$
$LK^2 = 4.7^2 + 4.0^2 - 2(4.7)(4.0)\cos 42°$
$LK^2 = 22.09 + 16.0 - 37 \cos 42°$
$LK^2 = 38.09 - 27.94$
$LK^2 = 10.14$
$LK = \sqrt{10.14}$
$LK = 3.2$ m

Find the hypotenuse for each diagonal in each truck.

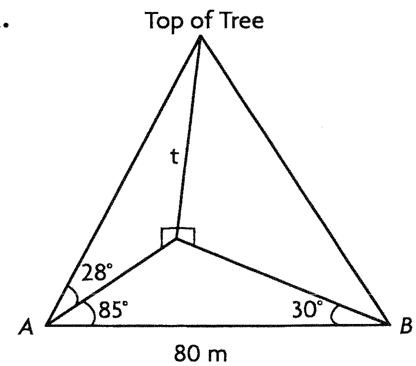
Truck A:
$C^2 = 2.6^2 + 2.1^2$
$C^2 = 6.76 + 4.41$
$C^2 = 11.17$
$C = \sqrt{11.17}$
$C = 3.3$ m
$D^2 = 2.6^2 + 6.0^2$
$D^2 = 6.76 + 36.0$
$D^2 = 42.76$
$D = \sqrt{42.76}$
$D = 6.5$ m
$E^2 = 2.1^2 + 6.0^2$
$E^2 = 4.41 + 36.0$
$E^2 = 40.41$
$E = \sqrt{40.41}$
$E = 6.4$ m

Truck B:
$C^2 = 4.0^2 + 2.1^2$
$C^2 = 16.0 + 4.41$
$C^2 = 20.41$
$C = \sqrt{20.41}$
$C = 4.5$ m
$D^2 = 4.0^2 + 2.5^2$
$D^2 = 16.0 + 6.25$
$D^2 = 22.25$
$D = \sqrt{22.25}$
$D = 4.7$ m
$E^2 = 2.1^2 + 2.5^2$
$E^2 = 4.41 + 6.25$
$E^2 = 10.66$
$E^2 = 40.41$
$E = \sqrt{10.66}$
$E = 3.3$ m

Truck C:
$C^2 = 4.5^2 + 1.6^2$
$C^2 = 20.25 + 2.56$
$C^2 = 22.81$
$C = \sqrt{22.81}$
$C = 4.8$ m
$D^2 = 4.5^2 + 1.8^2$
$D^2 = 20.25 + 3.24$
$D^2 = 23.49$
$D = \sqrt{23.49}$
$D = 4.8$ m
$E^2 = 1.6^2 + 1.8^2$
$E^2 = 2.56 + 3.24$
$E^2 = 5.8$
$E = \sqrt{5.8}$
$E = 2.4$ m

By comparing the side lengths of various triangles to the dimensions and diagonals (that were just found) of various trucks, the following conclusions can be made: Piece number one fits into truck A. Pieces numbers two and three fit into trucks A and B.

11.



$\theta = 180° - (85° + 30°)$
$\theta = 180° - 115°$
$\theta = 65°$

$$\frac{b}{\sin 30°} = \frac{80}{\sin 65°}$$

$$\sin 30° \times \frac{b}{\sin 30°} = \frac{80}{\sin 65°} \times \sin 30°$$

$$b = \frac{80}{\sin 65°} \times \sin 30°$$

$$b = 44.1\,\text{m}$$

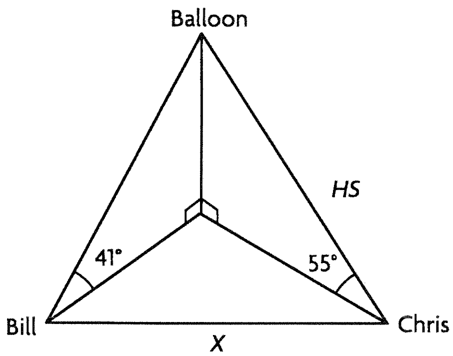
$$\frac{t}{44.1} = \tan 28°$$

$$44.1 \times \frac{t}{44.1} = \tan 28° \times 44.1$$

$$t = \tan 28° \times 44.1$$

$$t = 23.4\text{ m} \doteq 23\text{ m}$$

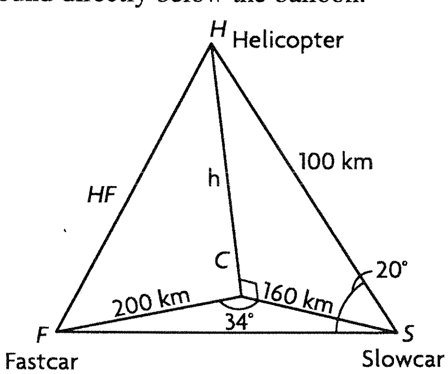
12. a)



Chandra is correct, there is not information to solve the distance.

b) She needs the altitude of the balloon and the angle between Bill and Chris as measured on the ground directly below the balloon.

13. a)



$$\text{Slow car's distance} = 80\frac{\text{km}}{\text{hr}} \times 2\text{ hr}$$

$$\text{Slow car's distance} = 160\text{ km}$$

$$\text{Fast car's distance} = 100\frac{\text{km}}{\text{hr}} \times 2\,\text{hr}$$

$$\text{Fast car's distance} = 200\text{ km}$$

a) To find the distance between the helicopter and the fast car, first find the distance d between the two cars by using the cosine law on the "bottom" triangle (on the ground):

$d^2 = 200^2 + 160^2 - 2(200)(160)\cos 24\text{Y}$

$= 12\,541.6$

$d = 112$ km

Using this information, and the cosine law on the "front" triangle formed by the cars and the helicopter, now find the distance D between the helicopter and the fast car:

$D^2 = 100^2 + 112^2$

$- 2(100)(112)\cos 20\text{Y}$

$= 1494.9$

$D = 39$ km

b) To find the height of the helicopter, notice that the segment labeled h (the height of the helicopter) forms a right triangle in the front right with the 100 km long side as hypotenuse and part of the side between the two cars. So, to find h, notice that

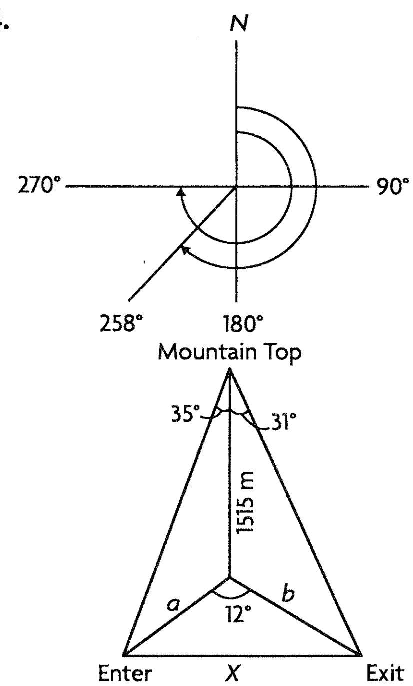
$$\sin 20\text{Y} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{h}{100}$$

$h = 100 \times \sin 20\text{Y}$

$= 34$ km

14.

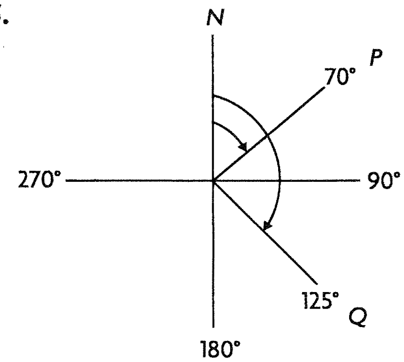


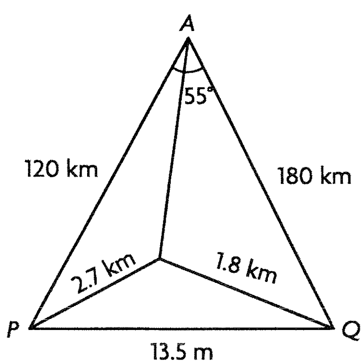
$$\frac{a}{1515} = \tan 35°$$

$$1515 \times \frac{a}{1515} = \tan 35° \times 1515$$

$a = \tan 35° \times 1515$
$a = 1060.8$ m
$\frac{b}{1515} = \tan 31°$
$1515 \times \frac{b}{1515} = \tan 31° \times 1515$
$b = \tan 31° \times 1515$
$b = 910.3$ m
$x^2 = a^2 + b^2 - 2ab\cos\theta$
$x^2 = 1061^2 + 910^2 - 2(1061)(910)\cos 12°$
$x^2 = 64\,998.4$
$x = \sqrt{64\,998.4}$
$x = 254.9 \text{ m} \doteq 255 \text{ m}$

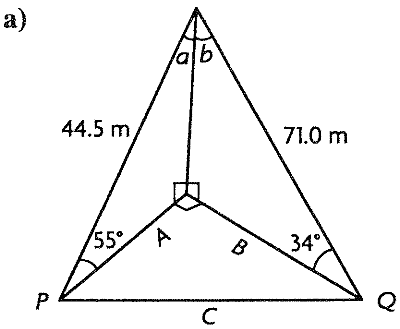
15.





$\theta = 125° - 70°$
$\theta = 55°$
$PQ^2 = AP^2 + AQ^2 - 2(AP)(AQ)\cos\theta$
$PQ^2 = 120^2 + 180^2 - 2(120)(180)\cos 55°$
$PQ^2 = 14\,400 + 32\,400 - 43\,000\cos 55$
$PQ^2 = 46\,800 - 24\,778.5$
$PQ^2 = 22\,021.5$
$PQ = \sqrt{22\,021.5}$
$PQ = 148.4$ km

16. a)



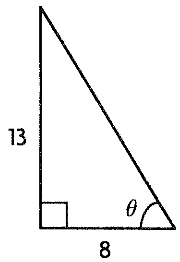
$\angle a = 180° - (90° + 55°)$
$\angle a = 180° - (145°)$
$\angle a = 35°$
$\frac{A}{\sin 35°} = \frac{44.5}{\sin 90°}$
$\sin 35° \times \frac{A}{\sin 35°} = \frac{44.5}{\sin 90°} \times \sin 35°$
$A = \frac{44.5}{\sin 90°} \times \sin 35°$
$A = \frac{44.5}{1} \times \sin 35°$
$A = 44.5 \times \sin 35°$
$A = 25.52$ m
$\angle b = 180° - (90° + 34°)$
$\angle b = 180° - (124°)$
$\angle b = 56°$
$\frac{B}{\sin 56°} = \frac{71.0}{\sin 90°}$
$\sin 56° \times \frac{B}{\sin 56°} = \frac{71.0}{\sin 90°} \times \sin 56°$
$B = \frac{71.0}{\sin 90°} \times \sin 56°$
$B = \frac{71.0}{1} \times \sin 56°$
$B = 71.0 \times \sin 56°$
$B = 58.86$ m

Rope around outside of the building:
$= 25.52 + 58.86$
$= 84.4$ m

b) Straight line between the windows:
$c^2 = a^2 + b^2$
$c^2 = 25.52^2 + 58.86^2$
$c^2 = 651.27 + 3464.50$
$c^2 = 4144.56$
$c = \sqrt{4144.56}$
$c = 64.2$ m

Chapter Review, pp. 338–339

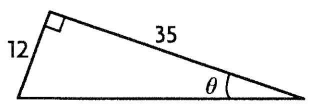
1. i) a)



By the Pythagorean theorem, the hypotenuse of the triangle is equal to $\sqrt{8^2 + 13^2}$, or $\sqrt{233}$, so the reciprocal trigonometric ratios for the angle θ are:

$$\cot\theta = \frac{8}{13},\ \sec\theta = \frac{\sqrt{233}}{8},\ \csc\theta = \frac{\sqrt{233}}{13}.$$

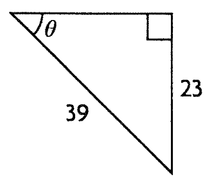
b)



By the Pythagorean theorem, the hypotenuse of the triangle is equal to $\sqrt{12^2 + 35^2}$, or 37, so the reciprocal trigonometric ratios for the angle θ are:

$$\cot\theta = \frac{35}{12},\ \sec\theta = \frac{37}{35},\ \csc\theta = \frac{37}{12}.$$

c)



By the Pythagorean theorem, the unknown side of the triangle is equal to $\sqrt{39^2 - 23^2}$, or $4\sqrt{62}$, so the reciprocal trigonometric ratios for the angle θ are:

$$\cot\theta = \frac{4\sqrt{62}}{23},\ \sec\theta = \frac{39}{4\sqrt{62}},\ \csc\theta = \frac{39}{23}.$$

ii) a) $\theta = \tan^{-1}\left(\frac{13}{8}\right)$

$\doteq 58°$

b) $\theta = \tan^{-1}\left(\frac{12}{35}\right)$

$\doteq 19°$

c) $\theta = \sin^{-1}\left(\frac{23}{39}\right)$

$\doteq 36°$

2. a) $(\sin 45°)(\cos 45°) + (\sin 30°)(\cos 60°)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{2}{4} + \frac{1}{4}$

$= \frac{3}{4}$

b) $(1 - \tan 45°)(\sin 30°)(\cos 30°)(\tan 60°)$

$= (1 - 1)(\sin 30°)(\cos 30°)(\tan 60°)$

$= 0$

c) $\tan 30° + 2(\sin 45°)(\cos 60°)$

$= \frac{\sqrt{3}}{3} + 2\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{3}}{6} + \frac{3\sqrt{2}}{6}$

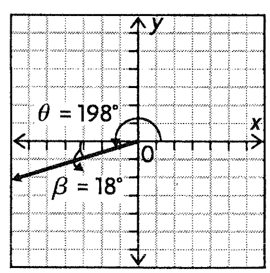
$= \frac{2\sqrt{3} + 3\sqrt{2}}{6}$

3. i) a) positive; $\tan 18° \doteq 0.3249$

b) negative; $\sin 205° \doteq -0.4226$

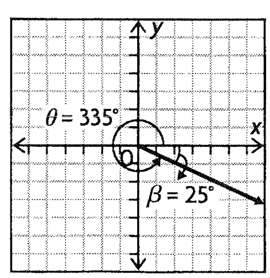
c) positive; $\cos(-55°) \doteq 0.5736$

ii) a) The principle angle of $\tan 18°$ is $\theta = 18°$, and the related acute angle is $\beta = 18°$.



Another angle that has the equivalent ratio is $\theta = 198°$.

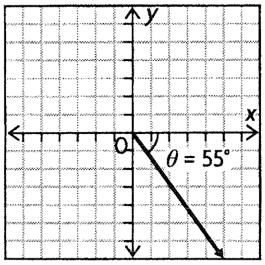
b) The principle angle of $\sin 205°$ is $\theta = 205°$, and the related acute angle is $\beta = 25°$.



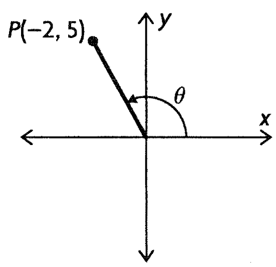
Another angle that has the equivalent ratio is $\theta = 335°$.

c) The principle angle of $\cos(-55°)$ is $\theta = 305°$, and the related acute angle is $\beta = 55°$.

Another angle that has the equivalent ratio is $\theta = 55°$.



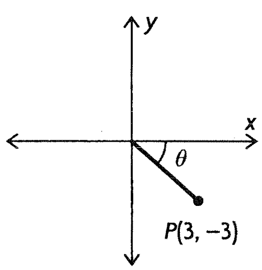
4. a)



By the Pythagorean theorem, the hypotenuse of the triangle is equal to $\sqrt{(-2)^2 + 5^2}$, or $\sqrt{29}$, so the primary trigonometric ratios for the angle θ are:

$$\sin\theta = \frac{5\sqrt{29}}{29},\ \cos\theta = -\frac{2\sqrt{29}}{29},\ \tan\theta = -\frac{5}{2}.$$

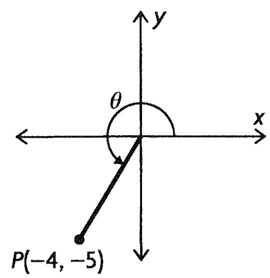
b)



By the Pythagorean theorem, the hypotenuse of the triangle is $\sqrt{3^2 + (-3)^2}$, which is equal to $\sqrt{18}$, or $3\sqrt{2}$, so the primary trigonometric ratios for the angle θ are:

$$\sin\theta = -\frac{\sqrt{2}}{2},\ \cos\theta = \frac{\sqrt{2}}{2},\ \tan\theta = -1.$$

c)



By the Pythagorean theorem, the hypotenuse of the triangle is equal to $\sqrt{(-4)^2 + (-5)^2}$, or $\sqrt{41}$, so the primary trigonometric ratios for the angle θ are:

$$\sin\theta = -\frac{5\sqrt{41}}{41},\ \cos\theta = -\frac{4\sqrt{41}}{41},\ \tan\theta = \frac{5}{4}.$$

5. a) The terminal arm of the angle ϕ lies in either quadrant 2 or 3, since these are the only quadrants in which cosine is negative.

b) Since $\cos\phi$ is $-\frac{7}{\sqrt{53}}$, the remaining side of the triangle is $\sqrt{53 - (-7)^2}$, or $\sqrt{4}$, which is equal to 2 (in quadrant 2) or -2 (in quadrant 3). Assume the terminal arm of ϕ lies in quadrant 2. Then the other five trigonometric ratios for the angle ϕ are:

$$\sin\phi = \frac{2}{\sqrt{53}},\ \tan\phi = \frac{2}{7},\ \cot\phi = \frac{7}{2},$$

$$\sec\phi = -\frac{\sqrt{53}}{7},\ \csc\phi = \frac{\sqrt{53}}{2}.$$

Assume the Terminal arm of ϕ lies in quadrant 3. Then the other five trigonometric ratios are:

$$\sin\phi = -\frac{2}{\sqrt{53}},\ \tan\phi = -\frac{2}{7},\ \cot\phi = -\frac{7}{2},$$

$$\sec\phi = -\frac{\sqrt{53}}{7},\ \csc\phi = -\frac{\sqrt{53}}{2}.$$

c) If ϕ is in quadrant 2, then

$\phi = \cos^{-1}\frac{-7}{\sqrt{53}}$

= 164Y

So the related angle is

β = 180Y − 164Y

= 16Y

So if ϕ is in quadrant 3, then

ϕ = 180Y + 16Y

= 196Y

6. $\cos\beta\cot\beta = \cos\beta\left(\frac{1}{\tan\beta}\right)$

$= \cos\beta\left(\frac{\cos\beta}{\sin\beta}\right)$

$= \frac{\cos^2\beta}{\sin\beta}$

$= \frac{1-\sin^2\beta}{\sin\beta}$

$= \frac{1}{\sin\beta} - \frac{\sin^2\beta}{\sin\beta}$

$= \frac{1}{\sin\beta} - \sin\beta$

So the equation $\cos\beta\cot\beta = \frac{1}{\sin\beta} - \sin\beta$ is an identity.
Angle β cannot be equal to $180°k$ for any k in the integers, since $\sin\beta$ is 0 for any such angle, meaning that $\cot\beta$ is not defined.

7. a) $\tan\alpha\cos\alpha = \left(\frac{\sin\alpha}{\cos\alpha}\right)(\cos\alpha)$

$= \sin\alpha$

α cannot equal 90° or 270°

b) $\frac{1}{\cot\phi} = \tan\phi$

$= \frac{\sin\phi}{\cos\phi}$

$= \sin\phi\left(\frac{1}{\cos\phi}\right)$

$= \sin\phi\sec\phi$

ϕ cannot be equal to 0°, 90°, 180° or 270°.

c) $1 - \cos^2 x = \sin^2 x$

$= \sin^2 x\left(\frac{\cos x}{\cos x}\right)$

$= \sin x\sin x\left(\frac{\cos x}{\cos x}\right)$

$= \sin x\cos x\left(\frac{\sin x}{\cos x}\right)$

$= \sin x\cos x\tan x$

$= \sin x\cos x\left(\frac{1}{\cot x}\right)$

$= \frac{\sin x\cos x}{\cot x}$

x cannot be equal to 0° or 180°.

d) $\sec\theta\cos\theta + \sec\theta\sin\theta$

$= \left(\frac{1}{\cos\theta}\right)\cos\theta + \left(\frac{1}{\cos\theta}\right)\sin\theta$

$= 1 + \frac{\sin\theta}{\cos\theta}$

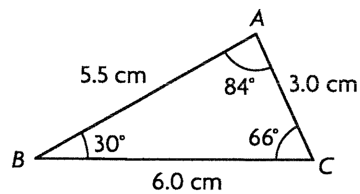
$= 1 + \tan\theta$

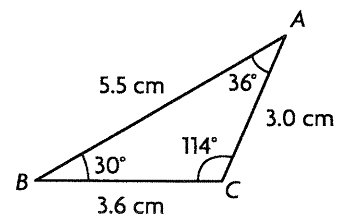
θ cannot be equal to 90° or 180°.

8. a) $h = 5.5(\sin 30°)$

$\doteq 2.75 \text{ cm} < b$

So there are two triangles.

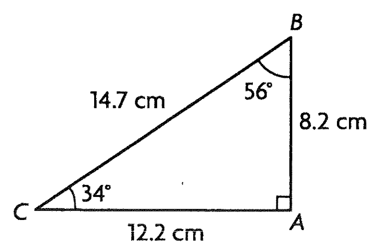


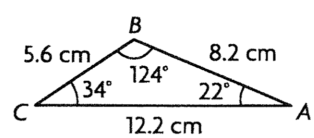


b) $h = 12.2(\sin 34°)$

$\doteq 6.8 \text{ cm} < c$

So there are two triangles.





c) $h = 11.1(\sin 33°)$

$\doteq 6.0 \text{ cm} > c$

So no triangle exists.

9. By the sine law, $\frac{\sin\theta}{20}$ is equal to $\frac{\sin 25°}{15}$, so θ is $\sin^{-1}(0.56)$, or about 34°. So the angle $\angle Q$ is about $180° - (25° + 34°)$, or about 121°.

$x^2 \doteq 20^2 + 15^2 - 2(20)(15)(\cos 121°)$

$\doteq 934.0$

$$x \doteq \sqrt{934.0}$$
$$\doteq 30.6 \text{ km}$$

10. a)
$$j^2 = 11.3^2 + 7.7^2 - 2(11.3)(7.7)(\cos 108°)$$
$$= 240.76$$
$$j = \sqrt{240.76}$$
$$\doteq 15.5$$

b) $c^2 = 6.0^2 + 8.0^2 - 2(6.0)(8.0)(\cos 72°)$
$$= 70.33$$
$$c = \sqrt{70.33}$$
$$\doteq 8.4$$

c) $m^2 = 6.2^2 + 4.5^2 - 2(6.2)(4.5)(\cos 55°)$
$$= 26.68$$
$$m = \sqrt{26.68}$$
$$\doteq 5.2$$

11. The angle between the two angles of elevation measures 180° − (45° + 70°), or 65°. Let x by the hypotenuse of the triangle formed by the observer, the blue spotlight, and the height of the ceiling at the blue spotlight.
$$\frac{x}{\sin 70°} = \frac{6}{\sin 65°}$$
$$x \doteq 6.2$$
$$\sin 45° = \frac{h}{6.2}$$
$$h \doteq 4.4 \text{ m}$$

12. In order to determine the height h, you need to determine first the length BA so that you can use trigonometric ratios to find h. $\angle ABC$ is equal to 180° − (57.5° + 90°), or 32.5°, so by the sine law, we have the following equation:
$$\frac{BA}{\sin 57.5°} = \frac{30}{\sin 32.5°}$$
$$BA = (\sin 57.5°)\left(\frac{30}{\sin 32.5°}\right)$$
$$\doteq 47.1 \text{ m}$$
$$\tan 15.3 \doteq \frac{h}{47.1}$$
$$h \doteq 47.1(\tan 15.3°)$$
$$\doteq 13 \text{ m}$$

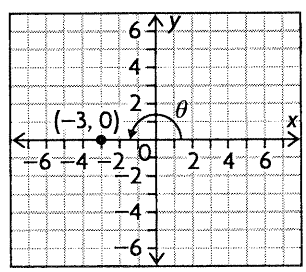
13. First, you determine the unknown interior angles of the triangle formed by Suzie and the right and left base exterior walls of the school.
$$\frac{\sin x}{12.0} = \frac{\sin 39°}{8.9}$$
$$\sin x = 12.0\left(\frac{\sin 39°}{8.9}\right)$$
$$\doteq 0.85$$
$$x \doteq \sin^{-1}(0.85)$$
$$\doteq 58°$$

So the remaining angle of the right exterior wall is about 180° − (39° + 58°), or about 83°. The distance from Suzie to the left base exterior wall is given by the formula
$$\frac{x}{\sin 83°} = \frac{8.9}{\sin 39°}$$
$$x = \sin 83°\left(\frac{8.9}{\sin 39°}\right)$$
$$\doteq 14.0 \text{ m}$$

Now you can determine the angle of elevation θ from Suzie to the top left exterior wall with the formula $\tan\theta = \frac{4.7}{14}$.
$$\theta = \tan^{-1}\left(\frac{4.7}{14}\right)$$
$$\doteq 18.5°$$

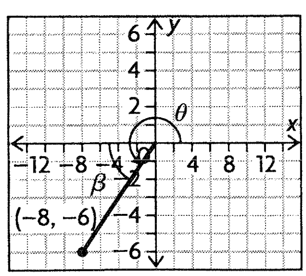
Chapter Self-Test, p. 340

1. a)



i. $\sin\theta = 0$
$\csc\theta$ is undefined
$\cos\theta = -1$
$\sec\theta = -1$
$\tan\theta = 0$
$\cot\theta$ is undefined

ii. The principle angle is 180°, and the related angle is 0°.



b) i. $\sin\theta = \frac{-3}{5}$

$\csc\theta = \frac{5}{-3}$

$\cos\theta = \frac{-4}{5}$

$\sec\theta = \frac{5}{-4}$

$\tan\theta = \frac{3}{4}$

$\cot\theta = \frac{4}{3}$

ii. $\tan^{-1}\left(\frac{3}{4}\right) \doteq 37°$

$180° + 37° = 217°$

The principle angle is 217°, and the related angle is 37°.

2. a) The sine is negative, so you know the angle must lie in either quadrant 3 or quadrant 4.

$$\begin{aligned}\theta &= \sin^{-1}(\sin\theta)\\ &= \sin^{-1}\left(-\frac{1}{2}\right)\\ &= -30°\end{aligned}$$

$$\begin{aligned}-30° &= 360° - 30°\\ &= 330°\end{aligned}$$

So one solution for $0° \le \theta \le 360°$, is $\theta = 330°$. Furthermore, we know that $\sin 30°$ is equal to $\frac{1}{2}$.

Another solution, then, is 30° + 180°, or 210°.

b) You know that cosine is positive in the first and fourth quadrants.

$$\begin{aligned}\theta &= \cos^{-1}(\cos\theta)\\ &= \cos^{-1}\frac{\sqrt{3}}{2}\\ &= 30°\end{aligned}$$

The related acute angle to 30° in quadrant 4 is 330°. So the solutions are 30° and 330°.

c) $\cot\theta = \frac{1}{\tan\theta}$

$$\begin{aligned}\tan\theta &= -\frac{1}{1}\\ &= -1\end{aligned}$$

The tangent is negative in quadrants 2 and 4.

$\tan^{-1}(-1) = -45°$

$$\begin{aligned}-45° &= 360° - 45\\ &= 315°\end{aligned}$$

Another solution is 180° − 45°, 135°.

d) $\sec\theta = \frac{1}{\cos\theta}$

$\frac{1}{\cos\theta} = -2$

$\cos\theta = -\frac{1}{2}.$

You know that cosine is negative in quadrants 2 and 3.

$$\begin{aligned}\theta &= \cos^{-1}(\cos\theta)\\ &= \cos^{-1}\left(-\frac{1}{2}\right)\\ &= 120°\\ &= 180° - 60°\end{aligned}$$

The solution in quadrant 3 is 180° + 60°, or 240°.

3. a) Cosine is defined as $\frac{\text{adjacent}}{\text{hypotenuse}}$, and from the Pythagorean Theorem,

$(\text{adjacent})^2 + (\text{opposite})^2 = (\text{hypotenuse})^2$

Dividing by $(\text{hypotenuse})^2$,

$$1 = \frac{(\text{adjacent})^2}{(\text{hypotenuse})^2} + \frac{(\text{opposite})^2}{(\text{hypotenuse})^2}$$

$$\frac{(\text{opposite})^2}{(\text{hypotenuse})^2} = 1 - \frac{(\text{adjacent})^2}{(\text{hypotenuse})^2}$$

$$\begin{aligned}\frac{\text{opposite}}{\text{hypotenuse}} &= \sqrt{1 - \frac{(\text{adjacent})^2}{(\text{hypotenuse})^2}}\\ &= \sqrt{1 - (\cos\theta)^2}\\ &= \sqrt{1 - \left(-\frac{5}{13}\right)^2}\\ &= \sqrt{\frac{144}{169}}\\ &= \frac{\sqrt{12}}{13}\end{aligned}$$

$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}$, so

$\sin\theta = \frac{\sqrt{12}}{13}$

$$\begin{aligned}\cos\theta\sin\theta &= \left(-\frac{5}{13}\right)\left(\frac{\sqrt{12}}{13}\right)\\ &= -\frac{60}{169}\end{aligned}$$

b) $\cot\theta\tan\theta = \frac{1}{\tan\theta}\tan\theta$

$= \frac{\tan\theta}{\tan\theta}$

$= 1$

4. i. a) $\sin^2\theta + \cos^2\theta = 1$

$$\frac{1}{\cos^2\theta}(\sin^2\theta + \cos^2\theta) = \left(\frac{1}{\cos^2\theta}\right)(1)$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

b) $\sin^2\theta + \cos^2\theta = 1$

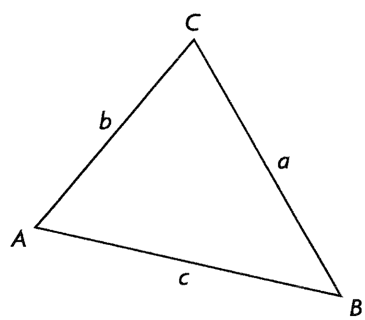
$$\frac{1}{\sin^2\theta}(\sin^2\theta + \cos^2\theta) = \left(\frac{1}{\sin^2\theta}\right)(1)$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

ii. Both identities are derived from the identity $\sin^2\theta + \cos^2\theta = 1$

5. a)

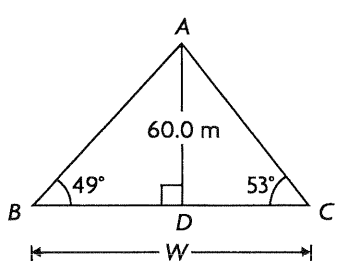


b) $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = c^2 + a^2 - 2ca\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

c) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



6. a) $\tan 49° = \frac{60}{BD}$

$BD = \frac{60}{\tan 49°}$

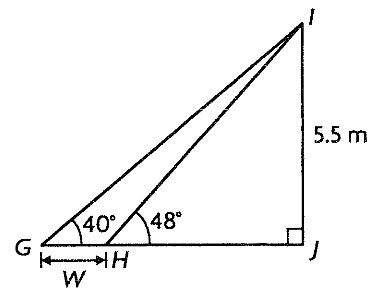
$\tan 53° = \frac{60}{DC}$

$DC = \frac{60}{\tan 53°}$

$w = BD + DC$

$= \frac{60}{\tan 49°} + \frac{60}{\tan 53°}$

$\doteq 97.4$ m



b) $\tan 40° = \frac{5.5}{GJ}$

$GJ = \frac{5.5}{\tan 40°}$

$\tan 48° = \frac{5.5}{HJ}$

$HJ = \frac{5.5}{\tan 48°}$

$w = GH$

$= GJ - HJ$

$= \frac{5.5}{\tan 40°} - \frac{5.5}{\tan 48°}$

$\doteq 1.6$ m

7. a) This is an ambiguous case of the sine law, and h is $2.8(\sin 41°)$, or about 1.8 cm, which is greater than the length of a, so no triangle exists.

b) This is an ambiguous case of the sine law, and h is $6.1(\sin 20°)$, or about 2.09 cm, which is less than a, which is less than c, so there exist two triangles.

$\frac{\sin C}{6.1} = \frac{\sin 20°}{2.1}$

$\sin C = 6.1\left(\frac{\sin 20°}{2.1}\right)$

$\doteq 0.993$

$C = \sin^{-1}(0.993)$

$\doteq 83°$

$\angle B \doteq 180° - (83° + 20°)$

$\doteq 180° - 103°$

$\doteq 77°$

$$\frac{b}{\sin 77°} = \frac{2.1}{\sin 20°}$$

$$b \doteq (\sin 77°)\left(\frac{2.1}{\sin 20°}\right)$$

$$\doteq 6.0 \text{ cm}$$

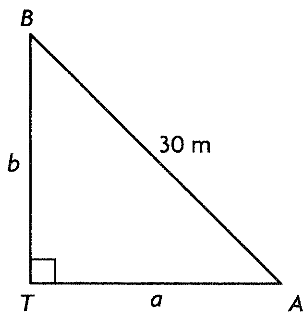
So for one triangle, $\angle C$ is about 83°, $\angle B$ is about 77°, and b is about 6.0 cm.
For the other triangle, $\angle C$ is about 180° − 83°, or 97°, $\angle B$ is about 180° − (97° + 20°), or 63°, and b is given by the formula

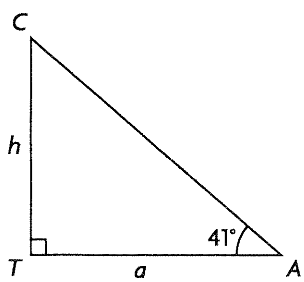
$$\frac{b}{\sin 63°} = \frac{2.1}{\sin 20°}$$

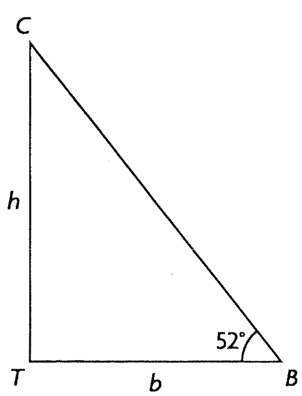
$$b \doteq (\sin 63°)\left(\frac{2.1}{\sin 20°}\right)$$

$$\doteq 5.5 \text{ cm}$$

8.







$$\tan 41° = \frac{h}{a}$$

$$a = \frac{h}{\tan 41°}$$

$$\tan 52° = \frac{h}{b}$$

$$b = \frac{h}{\tan 52°}$$

$$a^2 + b^2 = 30^2$$

$$a = \sqrt{30^2 - b^2}$$

$$\frac{h}{\tan 41°} = \sqrt{30^2 - b^2}$$

$$= \sqrt{30^2 - \left(\frac{h}{\tan 52°}\right)^2}$$

$$\frac{h^2}{(\tan 41°)^2} = 30^2 - \left(\frac{h}{\tan 52°}\right)^2$$

$$h^2 = (\tan 41°)^2\left(30^2 - \frac{h^2}{(\tan 52°)^2}\right)$$

$$= 30^2(\tan 41°)^2 - \frac{(\tan 41°)^2}{(\tan 52°)^2}h^2$$

$$\left(1 + \frac{(\tan 41°)^2}{(\tan 52°)^2}\right)h^2 = 30^2(\tan 41°)^2$$

$$h^2 = \frac{30^2(\tan 41°)^2}{1 + \frac{(\tan 41°)^2}{(\tan 52°)^2}}$$

$$h = \sqrt{\frac{30^2(\tan 41°)^2}{1 + \frac{(\tan 41°)^2}{(\tan 52°)^2}}}$$

$$\doteq 22 \text{ m}$$