CHAPTER 8: Discrete Functions: Financial Applications

Note: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 474

1. a) Each term in the sequence is equal to the previous term plus 4. So the next two terms are 23 and 27:
$t_4 = 19 + 4 = 23$
$t_5 = 23 + 4 = 27$
The general term, t_n, is $3 + 4n$.
For example,
$t_1 = 3 + (4 \times 1) = 7$
$t_2 = 3 + (4 \times 2) = 11$
The recursive formula is $t_1 = 7$, $t_n = t_{n-1} + 4$, where $n > 1$.
b) Each term in the sequence is equal to the previous term minus 27. So the next two terms are -50 and -77:
$t_4 = -23 - 27 = -50$
$t_5 = -50 - 27 = -77$
The general term, t_n, is $85 - 27n$.
For example,
$t_1 = 85 - (27 \times 1) = 58$
$t_2 = 85 - (27 \times 2) = 31$
The recursive formula is $t_1 = 58$, $t_n = t_{n-1} - 27$, where $n > 1$.
c) Each term in the sequence is equal to the previous term multiplied by 4. So the next two terms are 1280 and 5120:
$t_4 = 320 \times 4 = 1280$
$t_5 = 1280 \times 4 = 5120$
The general term, t_n, is $5 \times 4^{n-1}$.
For example,
$t_1 = 5 \times 4^0 = 5$
$t_2 = 5 \times 4^1 = 20$
The recursive formula is $t_1 = 5$, $t_n = 4t_{n-1}$, where $n > 1$.
d) Each term in the sequence is equal to the previous term multiplied by $-\frac{1}{2}$. So the next two terms in the series are -125 and $62\frac{1}{2}$:
$t_4 = 250 \times -\frac{1}{2} = -125$
$t_5 = -125 \times -\frac{1}{2} = 62\frac{1}{2}$
The general term, t_n, is $1000 \times \left(-\frac{1}{2}\right)^{n-1}$. For example,
$t_1 = 1000 \times \left(-\frac{1}{2}\right)^0$
$t_2 = 1000 \times \left(-\frac{1}{2}\right)^1$
The recursive formula is $t_1 = 1000$,
$t_n = \left(-\frac{1}{2}\right)t_{n-1}$, where $n > 1$.
2. a) In an arithmetic sequence, there is a common difference, d, between terms. If $t_4 = 46$ then $t_5 = 46 + d$ and $t_6 = (46 + d) + d$.
Substituting $t_6 = 248$,
$248 = 46 + 2d$
$2d = 202$
$d = 101$
So the fifth term, t_5, is $46 + 101 = 147$.
b) As determined in **a)**, $d = 101$.
c) Start with t_4 and subtract the common difference to find the first term, t_1:
$t_4 = 46$
$t_3 = 46 - d$
$t_2 = (46 - d) - d = 46 - 2d$
$t_1 = (46 - 2d) - d = 46 - 3d = -257$.
d) The general term, t_n, is $-358 + 101n$. So the 100th term, t_{100}, is $-358 + (100 \times 101) = 9742$.

3. a) There is a constant ratio between the terms, so it is a geometric sequence:

$$9724.05 \div 9261 = 1.05$$
$$10\,210.2525 \div 9724.05 = 1.05$$

b) As determined in **a)**, the common ratio between the terms is 1.05. Start with t_4 and divide by the common ratio to find the first term, t_1:

$t_4 = 9261$
$t_3 = 9261 \div r$
$t_2 = (9261 \div r) \div r = 9261 \div r^2$
$t_1 = (9261 \div r^2) \div r = 9261 \div r^3 = 8000$

The recursive formula is $t_1 = 8000$, $t_n = (1.05)t_{n-1}$, where $n > 1$.

c) The general term, t_n, is $8000 \times (1.05)^{n-1}$: $t_1 = 8000 \times (1.05)^0$; $t_2 = 8000 \times (1.05)^1$; and so on.

d) The 10th term, t_{10}, is $8000 \times (1.05)^9 \doteq 12\,410.6257$.

4. a) $a = 3, d = 2$

$$t_{10} = 3 + (2 \times 9) = 21$$
$$S_n = \frac{n(t_1 + t_n)}{2}$$
$$S_{10} = \frac{10(3 + 21)}{2} = 120$$

b) $a = -27, d = 6$

$$t_{10} = -27 + (6 \times 9) = 27$$
$$S_n = \frac{n(t_1 + t_n)}{2}$$
$$S_{10} = \frac{10(-27 + 27)}{2} = 0$$

c) $a = 48, d = -17$

$$t_{10} = 48 + (-17 \times 9) = -105$$
$$S_n = \frac{n(t_1 + t_n)}{2}$$
$$S_{10} = \frac{10(48 + (-105))}{2} = -285$$

d) $t_1 = 8\,192\,000, r = \dfrac{-1}{2}$

$$S_n = t_1\left(\frac{1 - r^n}{1 - r}\right)$$
$$S_{10} = 8\,192\,000 \times \left(\frac{1 - \left(\frac{-1}{2}\right)^{10}}{1 - \left(\frac{-1}{2}\right)}\right) = 5\,456\,000$$

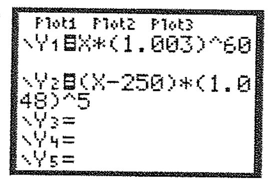
5. a) The city's population can be expressed as a geometric sequence with a common ratio of 1.05:

$p_1 = 200\,000$ (current population)
$p_2 = 1.05 \times 200\,000 = 210\,000$ (after one year)
$p_3 = 1.05 \times 210\,000 = 220\,500$ (after two years)
$p_4 = 1.05 \times 220\,500 = 231\,525$ (after three years)

b) The general term of the sequence is $p_n = 200\,000 \times (1.05)^{n-1}$. So the population in 10 years is $p_{11} = 200\,000 \times (1.05)^{10} \doteq 325\,779$. (Be careful. The population in 10 years is not p^{10}. This year is p_1, next year is p_2, and so on.)

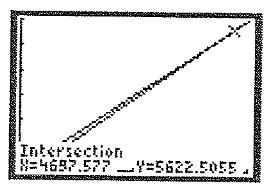
6. Recognizing that $4096 = 64^2$ and that $64 = 2^6$ gives $4096 = (2^6)^2 = 2^{12}$. So $x = 12$.

7. a)



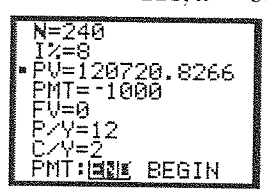
For $2^x = 1\,000\,000$, $x \doteq 19.93$.

b)



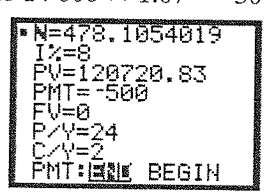
For $5 \times 3^x = 228$, $x \doteq 3.48$.

c)



For $14\,000 \times 1.07^x = 30\,000$, $x \doteq 11.26$.

d)



For $250 \times 1.0045^{12x} = 400$, $x \doteq 8.72$.

8.1 Simple Interest, pp. 481–482

1. a) i) 1st year: $A = 500(1 + 0.064) = \$532$
2nd year: $A = 500(1 + 0.128) = \$564$
3rd year: $A = 500(1 + 0.192) = \$596$
ii) $A = 500(1 + (0.064 \times 15)) = \$980$
b) i) 1st year: $A = 1250(1 + 0.041) = \$1301.25$
2nd year: $A = 1250(1 + 0.082) = \$1352.50$;
3rd year: $A = 1250(1 + 0.123) = \$1403.75$
ii) $A = 1250(1 + (0.041 \times 15)) = \$2018.75$
c) i) 1st year: $A = 25\,000(1 + 0.05) = \$26\,250$
2nd year: $A = 25\,000(1 + 0.10) = \$27\,500$;
3rd year: $A = (\$25\,000)(1 + 0.15) = \$28\,750$
ii) $A = (\$25\,000)(1 + (0.05 \times 15)) = \$43\,750$
d) i) 1st year:
$A = (\$1700)(1 + 0.023) = \$1739.10$
2nd year: $A = (\$1700)(1 + 0.046) = \$1778.20$
3rd year: $A = (\$1700)(1 + 0.069) = \$1817.30$
ii) $A = (\$1700)(1 + (0.023 \times 15))$
$= \$2286.50$

2. a) The principal is represented by the point at which the graph intersects with the vertical axis. That point is (0, \$2000), so the principal is \$2000.
b) From the graph, the interest earned in 5 years appears to be approximately \$600. If you know the interest rate, you can compute the interest exactly. After 4 years, the interest earned is $(\$2500 - \$2000) = \$500$. Use this to compute the interest rate:
$500 = 2000(r \times 4)$
$0.25 = 4r$
$r = 0.0625$
So the interest after 5 years is
$I = 2000(0.0625 \times 5) = \$625$
c) As determined in **b)**, the interest rate is 0.0625, or 6.25%.
d) $A(t) = 2000(1 + 0.025t)$
$= 2000 + 125t$

3. The interest is calculated using $I = Prt$:
$200 = 850(0.07 \times t)$
$200 = 59.5t$
$t \doteq 3.361$ years
$\doteq 3$ years, 132 days

4. Using the formula for simple interest, $I = Prt$,
$$26.19 = 2845 \times r \times \frac{12}{365}$$
$26.19 \doteq 93.53r$
$r \doteq 0.280$, or 28%/a

5. a) $I = 500 \times 0.048 \times 8 = \$192$
$A = 500 + 192 = \$692$
b) $I = 3200 \times 0.098 \times 12 = \$3763.20$
$A = 3200 + 3763.20 = \$6963.20$
c) $I = 5000 \times 0.039 \times \frac{16}{12} = \$260$
$A = 5000 + 260 = \$5260$
d) $I = 128 \times 0.18 \times \frac{5}{12} = \$9.60$
$A = 128 + 9.60 = \$137.60$
e) $I = 50\,000 \times 0.24 \times \frac{17}{52} = \$3923.08$
$A = 50\,000 + 3923.08 = \$53\,923.08$
f) $I = 4500 \times 0.12 \times \frac{100}{365} = \$147.95$
$A = 4500 + 147.95 = \$4647.95$

6. Using the formula for total amount earning simple interest, $A = P + Prt$,
$8000 = 4800 + 4800(r \times 8.5)$
$8000 = 4800 + 40\,800r$
$3200 = 40\,800r$
$r \doteq 0.0784$
$r \doteq 7.84\%/a$

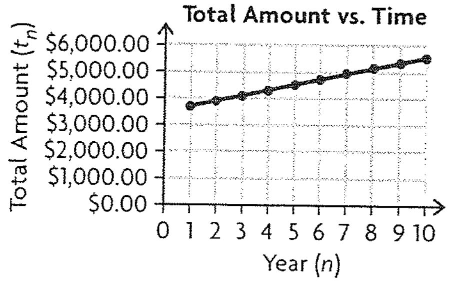
7. Using the formula for simple interest, $I = Prt$,
$$250 = P\left(0.063 \times \frac{1}{12}\right)$$
$250 = P(0.00525)$
$P \doteq 47\,619.05$
You must invest at least \$47 619.05 at 6.3%/a to earn \$250 in interest each month.

8. a) $I = 3500 \times 0.055 \times 1$
$= 192.5$
Using the formula for simple interest, $I = Prt$, Nina's deposit increases by \$192.50 each year.
b) Using the formula for total amount earning simple interest, $A = P + Prt$,
After one year: $A = 3500 + 3500(0.055)$
$= 3500 + 192.5$
$= 3692.50$
After two years: $A = 3500 + 3500(0.055 \times 2)$
$= 3500 + 3500(0.11)$
$= 3885.00$
After three years: $A = 3500 + 3500(0.055 \times 3)$
$= 3500 + 3500(0.165)$
$= 4077.50$
After four years: $A = 3500 + 3500(0.055 \times 4)$
$= 3500 + 3500(0.22)$
$= 4270.00$

After five years: $A = 3500 + 3500(0.055 \times 5)$
$= 3500 + 3500(0.275)$
$= 4462.50$

c) The total amount after n years is
$t_n = 3500 + 3500(0.055n)$
$= 3500 + 192.5n$

d)



9. a) Because Ahmad's account is a simple interest account, it earns the same amount each quarter. So the amount earned in the first quarter is equal to the amount earned in the second quarter: $3994.32 - P = 4248.64 - 3994.32$
$P = 3740$
Ahmad's original investment was \$3740.

b) The interest rate is the same in each quarter. Using the formula for total amount earning simple interest, $A = P + Prt$, and the first quarter figures,
$3994.32 = 3740 + 3740(r \times 1)$
$3994.32 = 3740 + 3740r$
$r = 0.068$
$r = 6.8\%/\text{q}$ or $27.2\%/\text{a}$

10. a) The interest in the first year is equal to the interest in the second year:
$2081.25 - P = 2312.5 - 2081.25$
$P = 1850$
Anita originally borrowed \$1850.

b) The interest Anita pays is the same each year.
$I = 2081.25 - 1850$
$= 231.25$
So the total amount is $t_n = 1850 + 231.25n$.

c) Using the solution for **b)**,
$7500 = 1850 + 231.25n$
$n \doteq 24.43$ years
Expressing 0.43 years as days,
$0.43 \times 365 = 156.95$
Anita will owe \$7500 after 24 years and 157 days.

11. Using the formula for total amount earning simple interest, $A = P + Prt$,
$A_L = 5200 + 5200(0.03t)$
$A_D = 3600 + 3600(0.05t)$
Set $A_L = A_D$ and solve for t to find when the investments are equal:
$5200 + 5200(0.03t) = 3600 + 3600(0.05t)$
$5200 + 156t = 3600 + 180t$
$24t = 1600$
$t = 66\frac{2}{3}$ years
After 66 years and 8 months, Dave's investment will be worth more than Len's.

12. Lottie's function is equivalent to the formula for total amount earning simple interest, $A = P + Prt$:

$$\begin{array}{ccccccc} A(t) & = & 750 & + & (27.75 & \times & t) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ A & = & P & + & (Pr & \times & t) \end{array}$$

Since P represents the principal, Lottie invested \$750. Compute the interest rate, r, as follows:
$Pr = 27.75$
$750r = 27.75$
$r = 0.037$
$r = 3.7\%/\text{a}$

13. The formula for total amount earning simple interest is $A = P + Prt$. Double the original amount is $2P$. So substitute and solve for D to find a formula for the doubling time:
$2P = P + PrD$
$P = PrD$
$$D = \frac{P}{Pr}$$
$$D = \frac{1}{r}$$

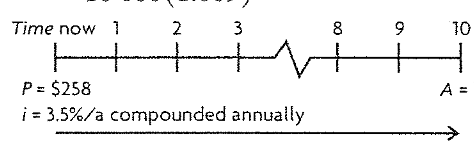
14. Each year Sara's parents increase the principal by \$500. Think of it as a new, \$500 investment each year. The total amount of the first investment, which lasts for 25 years, is
$A_1 = 500 + 500(0.064 \times 25) = 1300$
The total amount of the second investment, which lasts for 24 year, is
$A_2 = 500 + 500(0.064 \times 24) = 1268$
A general formula for the investment made on Sara's nth birthday is
$A_n = 500 + 500(0.064 \times (25 - n))$
Including the investment on the day Sara was born, there are 26 investments. You can use a spreadsheet and the general formula to compute the total of all 26: \$23 400.

8.2 Compound Interest: Future Value, pp. 490–492

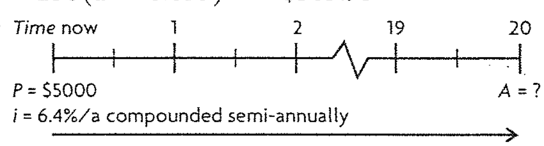
1.

	Interest Rate per Compounding Period, i	Number of Compounding Periods, n
a)	$\frac{1}{2} \times 0.054 = 0.027$	$5 \times 2 = 10$
b)	$\frac{1}{12} \times 0.036 = 0.003$	$3 \times 12 = 36$
c)	$\frac{1}{4} \times 0.029 = 0.007\,25$	$7 \times 4 = 28$
d)	$\frac{1}{52} \times 0.026 = 0.0005$	$\frac{10}{12} \times 52 = 4\frac{1}{3}$

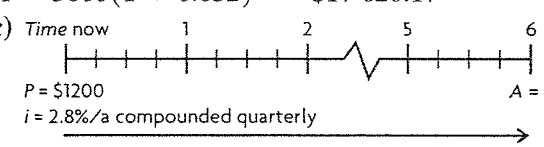
2. a) i) (1st year)
$A = P(1 + rt)$
$= 10\,000(1 + 0.072(1))$
$= \$10\,720$
(2nd year)
$A = 10\,720(1 + 0.072(1))$
$= \$11\,491.84$
(3rd year)
$A = 11\,491.84(1 + 0.072(1)) = \$12\,319.25$
(4th year)
$A = 12\,319.25(1 + 0.072(1)) = \$13\,206.24$
(5th year)
$A = 13\,206.24(1 + 0.072(1)) = \$14\,157.09$
ii) $A(n) = P(1 + i)^n$
$= 10\,000(1.072)^n$
b) i) (1st half-year)
$A = P(1 + rt)$
$= 10\,000(1 + 0.019(1))$
$= \$10\,190$
(2nd half-year)
$A = 10\,190(1 + 0.019(1))$
$= \$10\,383.61$
(3rd half-year)
$A = 10\,383.61(1 + 0.019(1)) = \$10\,580.90$
(4th half-year)
$A = 10\,580.90(1 + 0.019(1)) = \$10\,781.94$
(5th half-year)
$A = 10\,781.94(1 + 0.019(1)) = \$10\,986.80$
ii) $A(n) = P(1 + i)^n$
$= 10\,000(1.019)^n$
c) i) (1st quarter)
$A = P(1 + rt)$
$= 10\,000(1 + 0.017(1))$
$= \$10\,170$
(2nd quarter)
$A = 10\,170(1 + 0.017(1))$
$= \$10\,342.89$
(3rd quarter)
$A = 10\,342.89(1 + 0.017(1)) = \$10\,518.72$
(4th quarter)
$A = 10\,518.72(1 + 0.017(1)) = \$10\,697.54$
(5th quarter)
$A = 10\,697.54(1 + 0.017(1)) = \$10\,879.40$
ii) $A(n) = P(1 + i)^n$
$= 10\,000(1.017)^n$
d) i) (1st month)
$A = P(1 + rt)$
$= 10\,000(1 + 0.009(1))$
$= \$10\,090$
(2nd month)
$A = 10\,090(1 + 0.009(1))$
$= \$10\,180.81$
(3rd month)
$A = 10\,180.81(1 + 0.009(1)) = \$10\,272.44$
(4th month)
$A = 10\,272.44(1 + 0.009(1)) = \$10\,364.89$
(5th month)
$A = 10\,364.89(1 + 0.009(1)) = \$10\,458.17$
ii) $A(n) = P(1 + i)^n$
$= 10\,000(1.009)^n$

3. a)


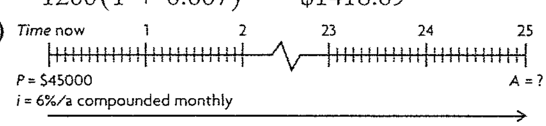
$A = 258(1 + 0.035)^{10} = \$363.93$

b)


$A = 5000(1 + 0.032)^{40} = \$17\,626.17$

c)


$A = 1200(1 + 0.007)^{24} = \$1418.69$

d)


$A = 45\,000(1 + 0.005)^{24} = \$200\,923.64$

4. a) $A = 4000(1.03)^4 = \$4502.04$
$I = 4502.04 - 4000 = \$502.04$
b) $A = 7500(1.005)^{72} = \$10\,740.33$
$I = 10\,740.33 - 7500 = \$3240.33$

c) $A = 15\,000(1.006)^{20} = \$16\,906.39$

$I = 16\,906.39 - 15\,000 = \$1906.39$

d) $A = 28\,200(1.0275)^{20} = \$48\,516.08$

$I = 48\,516.08 - 28\,200 = \$20\,316.08$

e) $A = 850(1.0001)^{365} = \$881.60$

$I = 881.60 - 850 = \$31.60$

f) $A = 2225(1.001)^{47} = \$2332.02$

$I = 2332.02 - 2225 = \$107.02$

5. a) $r = \dfrac{4494.404240}{4240} = 0.06 = 6\%$

b) Using the formula for future value, $A = P(1 + i)^n$, and the first year's data,

$4240 = P(1.06)^1$

$P = 4000$

Sima's original investment was \$4000.

6. Using the formula for future value,

$25\,000 = 10\,000(1 + 0.006)^n$

$2.5 = 1.006^n$

$n \doteq 154$ months $= 12$ years, 10 months

7. Compute the total Serena will pay for Option 1:

$i = \dfrac{0.1}{4}$

$= 0.025$

$A = 15\,000(1 + 0.025)^{40} = \$40\,275.96$

Compute the amount Serena will pay for the first 5 years of Option 2:

$i_1 = \dfrac{0.12}{4}$

$= 0.03$

$A_1 = 15\,000(1 + 0.03)^{20} = \$27\,091.67$

Use that amount to compute the amount after 5 more years of Option 2:

$i_2 = \dfrac{0.06}{4}$

$= 0.015$

$A_2 = 27\,091.67(1 + 0.015)^{20} = \$36\,488.55$

Serena saves \$3787.41 with Option 2:

$40\,275.96 - 36\,488.55 = \$3787.41$

8. Ted's formula is equivalent to the formula for future value:

$$\begin{array}{ccccc} A(n) & = & 5000 & \times & (1.0075)^{12n} \\ \downarrow & & \downarrow & & \downarrow \\ A & = & P & \times & (1 + i)^n \end{array}$$

The principal, P, is \$5000. The interest rate, i, is $12 \times 0.0075 = 0.09$, or $9\%/\text{a}$. Interest is compounded monthly.

9. Compute the total Margaret pays under Plan A:

$A_A = 949.99 + (949.99 \times 0.10 \times 2)$

$= 949.99 + 190$

$= 1339.99$

Compute the total she pays under Plan B:

$A_B = 949.99(1 + 0.0125)^8$

$= 1049.25$

Margaret will pay \$290.74 less under Plan B.

10. $A = 1000(1 + 1.05)^7 = \$1407.10$

11. Compute the value after the first 3 years:

$A_1 = 9000(1 + 0.025)^{12} = \$12\,104.00$

Use that value to compute the value after the next 2 years:

$A_2 = 12\,104(1 + 0.045)^4 = \$14\,434.24$

12. Suppose Cliff invests P dollars for n years. The future value for the first option is

$A_1 = P + (P \times 0.1 \times n)$

The future value for the second options is

$A_2 = P(1.05)^n$

Use a spreadsheet or a calculator to find the year, n, in which the future values are equal:

$P + (P \times 0.1 \times n) = P(1.05)^n$

$1 + 0.1n = 1.05^n$

$n \doteq 26$

For the first 26 years, the first option is better. After that, the exponential growth of the second option makes it a better choice.

13. For example, how long will it take the following two investments to be worth the same amount?

a) \$5000 at 5%/a compounded annually

b) \$3000 at 7%/a compounded annually

$A_a = 5000(1.05)^n$

$A_b = 3000(1.07)^n$

The two investments are the same after approximately 27 years:

◇	A	B	C	D
1	Year	\$3000 Investment	\$5000 Investment	Diff
2	1	3210.00	5250.00	2040.00
3	2	3434.70	5512.50	2077.80
4	3	3675.13	5788.13	2113.00
5	4	3932.39	6077.53	2145.14
6	5	4207.66	6381.41	2173.75
7	6	4502.19	6700.48	2198.29
8	7	4817.34	7035.50	2218.16
9	8	5154.56	7387.28	2232.72
10	9	5515.38	7756.64	2241.26
11	10	5901.45	8144.47	2243.02
12	11	6314.56	8551.70	2237.14
13	12	6756.57	8979.28	2222.71
14	13	7229.54	9428.25	2198.71
15	14	7735.60	9899.66	2164.06
16	15	8277.09	10394.64	2117.55
17	16	8856.49	10914.37	2057.88
18	17	9476.45	11460.09	1983.65
19	18	10139.80	12033.10	1893.30
20	19	10849.58	12634.75	1785.17
21	20	11609.05	13266.49	1657.44
22	21	12421.69	13929.81	1508.13
23	22	13291.21	14626.30	1335.10
24	23	14221.59	15357.62	1136.03
25	24	15217.10	16125.50	908.40
26	25	16282.30	16931.77	649.48
27	26	17422.06	17778.36	356.30
28	27	18641.60	18667.28	25.68
29	28	19946.52	19600.65	−345.87

14. Compare the future value of a one-dollar investment after one year to rank the rates:

Rate	Compounding period	Future value
6.5%	quarterly	$1.0165^4 = 1.067651543$
6.55%	semi-annually	$1.03275^2 = 1.066572563$
6.45%	monthly	$1.005375^{12} = 1.066441361$
6.6%	annually	1.066

15. Compute Anna's balance on July 1, 2001:

$A_1 = 2000(1 + 0.005)^{60} = \$2697.70$

Now use that value to compute the balance on January 1, 2008:

$A_2 = 2697.70(1 + 0.02)^{26} = \$4514.38$

16. Compute the balance after the first year:

$$i_1 = \frac{0.04}{4} = 0.01$$

$A_1 = 4000(1 + 0.01)^4 = \$4162.42$

Compute the balance after the second year:

$$i_2 = \frac{0.042}{4} = 0.0105$$

$A_2 = 4162.42(1 + 0.0105)^4 = \$4340.01$

Compute the balance after the third year:

$$i_3 = \frac{0.044}{4} = 0.011$$

$A_3 = 4340.01(1 + 0.011)^4 = \$4534.14$

17. Compute the balance on Rachel's 5th birthday:

$A_1 = 500(1 + 0.004)^{60} = \$635.32$

Add \$500 to the balance and compute the balance on Rachel's 10th birthday:

$$A_2 = (635.32 + 500)(1 + 0.004)^{60} = \$1442.58$$

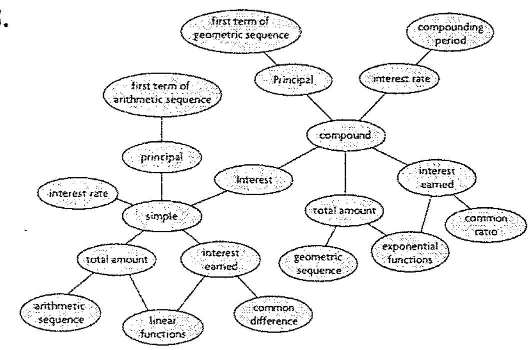
Add \$500 to the balance and compute the balance on Rachel's 15th birthday:

$$A_3 = (1442.58 + 500)(1 + 0.004)^{60} = \$2468.32$$

Add \$500 to the balance and compute the balance on Rachel's 18th birthday:

$$A_4 = (2468.32 + 500)(1 + 0.004)^{36} = \$3427.08$$

18.



19. There are 20 investment periods in Anita's savings plan. Each period, she adds \$500 to the balance. The first \$500 is invested for all 20 periods, so the future value of that \$500 is

$A_1 = 500(1.034)^{20} = \$975.85$

The second \$500 is invested for 19 periods, so the future value of that \$500 is

$A_2 = 500(1.034)^{19} = \$943.76$

A general formula for the future value of the nth \$500 investment is

$A_n = 500(1.034)^{21-n}$

You can use a spreadsheet and the general formula to compute the total future value:

◇	A	B	C
1	Period	Investment	Future Value
2	1	500	943.76
3	2	500	912.72
4	3	500	882.71
5	4	500	853.69
6	5	500	825.62
7	6	500	798.47
8	7	500	772.21
9	8	500	746.82
10	9	500	722.26
11	10	500	698.51
12	11	500	675.55
13	12	500	653.33
14	13	500	631.85
15	14	500	611.07
16	15	500	590.98
17	16	500	571.55
18	17	500	552.75
19	18	500	534.58
20	19	500	517.00
21	20	500	500.00
22			13995.44

Anita will have \$13 995.44 after 10 years.

20. a) The future value of a one-year investment at 6.3%/a compounded semi-annually is $P(1.0315)^2$. Set this equal to the same investment at a rate, I, that is compounded annually, and solve for i:

$P(1.0315)^2 = P(1 + i)^1$

$1.0315^2 = 1 + i$

$1.0640 \doteq 1 + i$

$i \doteq 0.0640 = 6.40\%$

b) The future value of a one-year investment at 4.2%/a compounded monthly is $P(1.0035)^{12}$. Set this equal to the same investment at a rate, I, that is compounded annually, and solve for i:

$P(1.0035)^{12} = P(1 + i)^1$

$1.0035^{12} = 1 + i$

$1.0428 \doteq 1 + i$

$i \doteq 0.0428 = 4.28\%$

c) The future value of a one-year investment at 3.2%/a compounded quarterly is $P(1.008)^4$. Set this equal to the same investment at a rate, I, that is compounded annually, and solve for i:

$P(1.008)^4 = P(1 + i)^1$

$1.008^4 = 1 + i$

$1.0324 \doteq 1 + i$

$i \doteq 0.0324 = 3.24\%$

8.3 Compound Interest: Present Value, pp. 498–499

1. a) $PV = \dfrac{10\,000}{(1.04)^{10}} = \$6755.64$

b) $PV = \dfrac{100\,000}{(1.031)^{10}} = \$73\,690.81$

c) $PV = \dfrac{23\,000}{(1.013)^{60}} = \$10\,506.47$

d) $PV = \dfrac{2500}{(1.0055)^{1200}} = \$3.46$

2. Compare the present value of Kevin's investment to the present value of Lui's investment:

$PV_K = \dfrac{10\,000}{(1.05)^{20}} = \$3768.89$

$PV_L = \dfrac{10\,000}{(1.004)^{240}} = \$3836.27$

Lui would have to invest more money than Kevin to reach his goal.

3. a) $PV = \dfrac{10\,000}{(1.06)^4} = \$7920.94$

$I = 10\,000 - 7920.94 = \$2079.06$

b) $PV = \dfrac{6200}{(1.041)^6} = \$4871.78$

$I = 6200 - 4871.78 = \$1328.22$

c) $PV = \dfrac{20\,000}{(1.014)^{60}} = \$8684.66$

$I = 20\,000 - 8684.66 = \$11\,315.34$

d) $PV = \dfrac{12\,800}{(1.0035)^{108}} = \$8776.74$

$I = 12\,800 - 8776.74 = \$4023.26$

4. $i = 0.072$

$n = 5$

$PV = \dfrac{A}{(1 + i)^n}$

$PV = \dfrac{12\,033.52}{(1.072)^5} = \$8500.00$

5. Compute the present value of Nazir's loan:

$PV = \dfrac{1429.50}{(1.015)^{24}} = \$1000.00$

Add the \$900 that he saved to the present value of the loan to find the cost of the TV: \$1900.00.

6. $PV = \dfrac{15\,000}{(1.025)^{40}} = \$5586.46$

7. Working backwards from Colin's third and final payment,

$PV_3 = \dfrac{5000}{(1.0179)^{20}} = \$3506.45$

$$PV_2 = \frac{(5000 + 3506.45)}{(1.0179)^8} = \$7380.87$$

$$PV_1 = \frac{(5000 + 7380.87)}{(1.0179)^{12}} = \$10\ 006.67$$

8. $$2500 = \frac{6000}{(1 + i)^{40}}$$

$$2500(1 + i)^{40} = 6000$$

$$(1 + i)^{40} = 2.40$$

$$1 + i \doteq 1.02212$$

$$i \doteq 0.02212$$

The annual rate is $4i$,

$$4i \doteq 0.08848,$$

which rounds to 8.85%.

9. $i_1 = 0.069$

$n_1 = 30$

$$PV = \frac{A}{(1 + i)^n}$$

$$PV_F = \frac{25\ 000}{(1.069)^{30}} = \$3377.60$$

$i_2 = 0.00575$

$n_2 = 360$

$$PV = \frac{A}{(1 + i)^n}$$

$$PV_D = \frac{25\ 000}{(1.00575)^{360}} = \$3173.40$$

Franco invested $204.20 more than David.

10. Working backwards from Sally's second investment,

$$PV_2 = \frac{14\ 784.56}{(1.018)^{24}} = \$9635.22$$

$$PV_1 = \frac{9635.22}{(1.06)^5} = \$7200.00$$

11. a) First find the present value of the investment with the 5-year guarantee. Working backwards from the last 20 years of the investment,

$$PV_2 = \frac{25\ 000}{(1.01)^{80}} = \$11\ 277.95$$

$$PV_1 = \frac{11\ 277.95}{(1.008)^{20}} = \$9616.56$$

Now find the present value of the investment with the 8-year guarantee. Working backwards from the last 17 years of the investment,

$$PV_2 = \frac{25\ 000}{(1.0125)^{68}} = \$10\ 741.82$$

$$PV_1 = \frac{10\ 741.82}{(1.008)^{32}} = \$8324.17$$

Steve should choose the investment option with the 8-year guarantee because it requires a smaller initial investment.

b) As shown in **a)**, Steve needs to invest $8324.17.

12. Present value is an exponential function with ratio $(1 + i)^{-1}$, so the amount decreases the farther you go into the past, just like the amount of radioactive material descreases as time goes on.

13. $$5000 = \frac{12\ 000}{1.0272^n}$$

$$5000 \times 1.027^{2n} = 12\ 000$$

$$1.027^{2n} = 2.4$$

$$2n \doteq 33$$

$$n \doteq 16\frac{1}{2} \text{ years}$$

14. $$P = \frac{3P}{(1 + i)^{40}}$$

$$P(1 + i)^{40} = 3P$$

$$(1 + i)^{40} = 3$$

$$1 + i \doteq 1.02785$$

$$i \doteq 0.02785$$

The annual rate is $4i$,

$$4i \doteq 0.1114,$$

which rounds to 11.14%.

15. Compute the total payment amount and substitute it into the present value formula to find the amount originally borrowed:

$$A = 30(268.17) = \$8045.10$$

$$PV = \frac{8045.1}{(1.016)^{30}} = \$4997.12$$

The original amount borrowed is $4997.12. The total of the payments is $8045.10. So the total interest paid is the difference, $3047.98.

16. Start with the formula for future value of an investment earning simple interest, $A = P + Prt$. Write PV for P, n for t, i for r. Solve for PV:

$$A = PV + (PV \times in)$$

$$A = PV(1 + in)$$

$$PV = \frac{A}{1 + in}$$

Mid-Chapter Review, p. 503

1. a) $I = 5400 \times 0.067 \times 15 = \$5427.00$

$A = 5400 + 5427 = \$10\ 827.00$

b) $I = 400 \times 0.096 \times \frac{16}{12} = \$51.20$

$A = 400 + 51.20 = \$451.20$

c) $I = 15\ 000 \times 0.143 \times \frac{80}{52} = \$3300.00$

$A = 15\ 000 + 3300 = \$18\ 300.00$

d) $I = 2500 \times 0.271 \times \frac{150}{365} = \$278.42$

$A = 2500 + 278.42 = \$2778.42$

2. $1200 = 5300 \times 0.072 \times t$

$1200 = 381.6t$

$t \doteq 3.145$ years $\doteq$ 3 years, 53 days

3. a) $1079.20 - 1014.60 = \$64.60$

b) $1014.60 - 64.60 = \$950.00$

c) $r = \frac{64.6}{950} = 0.068 = 6.8\%/\text{m}$ or $81.6\%/\text{a}$

4. a) $A = 6300(1 + 0.049)^7 = \$8805.80$

$I = 8805.80 - 6300 = \$2505.80$

b) $A = 14\,000(1 + 0.044)^{21} = \$34\,581.08$

$I = 34\,581.08 - 14\,000 = 20\,581.08$

c) $A = 120\,000(1 + 0.011)^{176} = \$822\,971.19$

$I = 822\,971.19 - 120\,000 = \$702\,971.19$

d) $A = 298(1 + 0.057)^6 = \$415.59$

$I = 415.59 - 298 = \$117.59$

5. $34\,000 = 15\,000(1.006)^n$

$1.006^n \doteq 2.2667$

$n \doteq 137$ months $= 11$ years, 5 months

6. Sara finances \$1612.00. Using the formula for future value,

$$2112 = 1612\left(1 + \frac{i}{2}\right)^3$$

$$\left(1 + \frac{i}{2}\right)^3 \doteq 1.31017$$

$$1 + \frac{i}{2} \doteq 1.09423$$

$$\frac{i}{2} \doteq 0.09423$$

$$i \doteq 0.18845 = 18.85\%/\text{a}$$

7. $i = 0.023$

$n = 100$

$$PV = \frac{A}{(1 + i)^n}$$

$$PV = \frac{25\,000}{1.023^{100}} = \$2572.63$$

8. $i = 0.037$

$n = 130$

$$PV = \frac{A}{(1 + i)^n}$$

$$PV = \frac{39\,382.78}{1.037^{130}} = \$350.00$$

9. a) The interest rate remains constant. So use the first two statements to compute the rate:

$I = 9125.56 - 8715.91 = \$409.65$

$$\frac{i}{2} = \frac{409.65}{8715.91} = 0.047$$

$i = 0.094 = 9.4\%/a$

b) $PV = \frac{8715.91}{1.047^1} = \$8324.65$

8.4 Annuities: Future Value, pp. 511–512

1. a) (1st invest)

$FV = 2500(1.082)^{24} = \$16\,572.74$

(2nd invest) $FV = 2500(1.082)^{23} = \$15\,316.76$

(3rd invest) $FV = 2500(1.082)^{22} = \$14\,155.97$

(4th invest) $FV = 2500(1.082)^{21} = \$13\,083.15$

b) The values form a geometric sequence with a common ratio of 1.082.

c) $FV = 2500 \times \frac{1.082^{25} - 1}{0.082} = \$188\,191.50$

2. a) $FV = 100 \times \frac{1.003^{600} - 1}{0.003} = \$167\,778.93$

b) $FV = 1500 \times \frac{1.0155^{60} - 1}{0.0155} = \$146\,757.35$

c) $FV = 500 \times \frac{1.028^{16} - 1}{0.028} = \$9920.91$

d) $FV = 4000 \times \frac{1.045^{10} - 1}{0.045} = \$49\,152.84$

3. The future value of Lois's annuity is \$59 837.37:

$$FV = 650 \times \frac{1.023^{50} - 1}{0.023} = \$59\,837.37$$

The total of her investments is \$32 500:

$50 \times 650 = \$32\,500$

The interest earned is the difference:

$59\,837.37 - 32\,500 = \$27\,337.37$

4. $i = \frac{0.054}{12}$

$= 0.0045$

$n = 3 \times 12 = 36$

$$FV = R \times \left(\frac{((1 + i)^n - 1)}{i}\right)$$

$$FV = 125.43 \times \frac{1.0045^{36} - 1}{0.0045} = \$4889.90$$

5. a) $FV = 1500 \times \frac{1.063^{10} - 1}{0.063} = \$20\,051.96$

b) $FV = 250 \times \frac{1.018^6 - 1}{0.018} = \$1569.14$

c) $FV = 2400 \times \dfrac{1.012^{28} - 1}{0.012} = \$79\ 308.62$

d) $FV = 25 \times \dfrac{\left(1\dfrac{2}{300}\right)^{420} - 1}{\dfrac{2}{300}} = \$57\ 347.07$

6. a) $1\ 000\ 000 = R \times \dfrac{1.0085^{480} - 1}{0.0085}$

$$8500 = R \times (1.0085^{480} - 1)$$
$$R = \$148.77$$

b) $1\ 000\ 000 = R \times \dfrac{1.00425^{480} - 1}{0.00425}$

$$4250 = R \times (1.000425)^{480} - 1$$
$$R = \$638.38$$

7. The total of Kiki's regular payments is the same for all four options. The rate of compound interest is the same for all four options. The only difference is the compounding period. In this case, the first option is the best because it compounds monthly, the most frequent of the options.

8. $i = \dfrac{0.052}{4}$

$$= 0.013$$
$$R = 250$$
$$FV = 6500$$
$$FV = R \times \left(\frac{((1 + i)^n - 1)}{i}\right)$$
$$6500 = 250 \times \frac{1.013^{4n} - 1}{0.013}$$
$$84.50 = 250(1.013^{4n} - 1)$$
$$0.338 = 1.013^{4n} - 1$$
$$1.338 = 1.013^{4n}$$
$$4n \doteq 22.54$$
$$n \doteq 5.64 \text{ years} \doteq 5 \text{ years, } 7 \text{ months}$$

9. a) Compute Sonja's regular payment:

$$500\ 000 = R_S \times \frac{1.0055^{420} - 1}{0.0055}$$
$$R_S = 305.19$$

Compute Anita's regular payment:

$$500\ 000 = R_A \times \frac{1.009^{420} - 1}{0.009}$$
$$R_A = 106.94$$

Compare the regular payments:

$305.19 - 106.94 = 198.25$

Sonja must invest \$198.25 more per month.

b) Compute the future value of Anita's investment using Sonja's monthly payment:

$$FV = 305.19 \times \frac{1.009^{420} - 1}{0.009}$$
$$= 1\ 426\ 980.31$$

Compare that future value to the original future value:

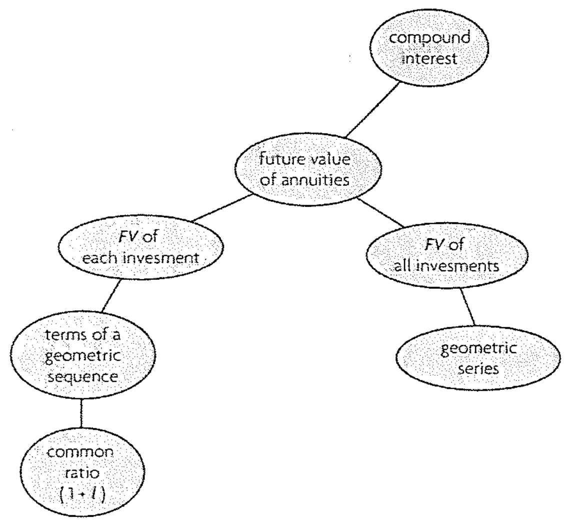
$1\ 426\ 980.31 - 500\ 000 = 926\ 980.31$

Anita will have \$926 980.31 more.

10. $25\ 000 = 150 \times \dfrac{(1 + i)^{120} - 1}{i}$

$$166\frac{2}{3} = \frac{(1 + i)^{120} - 1}{i}$$
$$166\frac{2}{3}i = (1 + i)^{120} - 1$$

11.



12. a) Set up a spreadsheet to compute the balance after every payment:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2					10000.00
3	1	250	40.00	210.00	9790.00
4	2	250	39.16	210.84	9579.16
5	3	250	38.32	211.68	9367.48
6	4	250	37.47	212.53	9154.95
7	5	250	36.62	213.38	8941.57
8	6	250	35.77	214.23	8727.33
9	7	250	34.91	215.09	8512.24
10	8	250	34.05	215.95	8296.29
11	9	250	33.19	216.81	8079.48
12	10	250	32.32	217.68	7861.79
13	11	250	31.45	218.55	7643.24
14	12	250	30.57	219.43	7423.81
15	13	250	29.70	220.30	7203.51
16	14	250	28.81	221.19	6982.32
17	15	250	27.93	222.07	6760.25
18	16	250	27.04	222.96	6537.29
19	17	250	26.15	223.85	6313.44
20	18	250	25.25	224.75	6088.70
21	19	250	24.35	225.65	5863.05
22	20	250	23.45	226.55	5636.50
23	21	250	22.55	227.45	5409.05
24	22	250	21.64	228.36	5180.69
25	23	250	20.72	229.28	4951.41
26	24	250	19.81	230.19	4721.21
27	25	250	18.88	231.12	4490.10
28	26	250	17.96	232.04	4258.06
29	27	250	17.03	232.97	4025.09
30	28	250	16.10	233.90	3791.19
31	29	250	15.16	234.84	3556.36
32	30	250	14.23	235.77	3320.58
33	31	250	13.28	236.72	3083.86
34	32	250	12.34	237.66	2846.20
35	33	250	11.38	238.62	2607.58
36	34	250	10.43	239.57	2368.02
37	35	250	9.47	240.53	2127.49
38	36	250	8.51	241.49	1886.00
39	37	250	7.54	242.46	1643.54
40	38	250	6.57	243.43	1400.12
41	39	250	5.60	244.40	1155.72
42	40	250	4.62	245.38	910.34
43	41	250	3.64	246.36	663.98
44	42	250	2.66	247.34	416.64
45	43	250	1.67	248.33	168.30
46	44	250	0.67	249.33	-81.02

The balance is close to zero after 44 payments, which is 3 years and 8 months.

b) Compute Carmen's total payments, keeping in mind that her last payment will be less than $250:

$(43 \times 250) + (168.30 + 0.67) = \$10\,918.97$

The total interest is the difference between the total payments and the original loan amount:

$10\,918.97 - 10\,000 = \$918.97$

13. Set up a spreadsheet to compute the balance after every payment. Try different payment amounts. After 20 years of monthly payments of $924.32, the balance is near zero:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2					123000.00
3	1	924.32	676.50	247.82	122752.10
4	2	924.32	675.14	249.18	122503.00
5	3	924.32	673.77	250.55	122252.44
6	4	924.32	672.39	251.93	122000.51
7	5	924.32	671.00	253.32	121747.19
8	6	924.32	669.61	254.71	121492.48
9	7	924.32	668.21	256.11	121236.37
10	8	924.32	666.80	257.52	120978.85
11	9	924.32	665.38	258.94	120719.92
12	10	924.32	663.96	260.36	120459.56
13	11	924.32	662.53	261.79	120197.76
14	12	924.32	661.09	263.23	119934.53
15	13	924.32	659.64	264.68	119669..85
16	14	924.32	658.18	266.14	119403.72
17	15	924.32	656.72	267.60	119136.12
18	16	924.32	655.25	269.07	118867.04
19	17	924.32	653.77	270.55	119596.49
20	18	924.32	652.28	272.04	118324.45
21	19	924.32	650.78	273.54	118050.92
22	20	924.32	649.28	275.04	117775.88
23	21	924.32	647.77	276.55	117499.33
24	22	924.32	646.25	278.07	117221.25
25	23	924.32	644.72	279.60	116941.65
26	24	924.32	643.18	281.14	116660.51

...

231	229	924.32	58.86	865.46	9835.47
232	230	924.32	54.10	870.22	8965.25
233	231	924.32	49.31	875.01	8090.23
234	232	924.32	44.50	879.82	7210.41
235	233	924.32	39.66	884.66	6325.75
236	234	924.32	34.79	889.53	5436.22
237	235	924.32	29.90	894.42	4541.80
238	236	924.32	24.98	899.34	3642.46
239	237	924.32	20.03	904.29	2738.17
240	238	924.32	15.06	909.26	1828.91
241	239	924.32	10.06	914.26	914.65
242	240	924.32	5.03	919.29	-4.64

14. $100R = R \times \dfrac{1.007^n - 1}{0.007}$

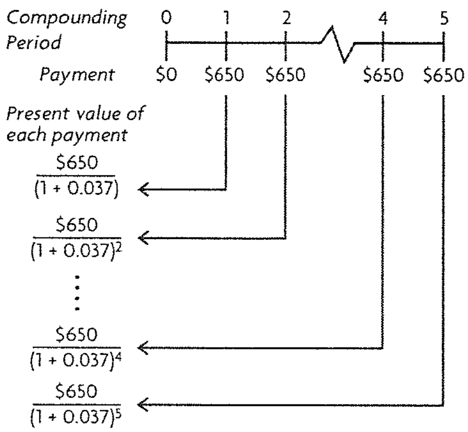
$$100 = \frac{1.007^n - 1}{0.007}$$

$$1.7 = 1.007^n$$

$$n \doteq 76 \text{ payments}$$

8.5 Annuities: Present Value, pp. 520–522

1. a) i) There are 5 payments: $i = 3.7\%/a$ compounded annually

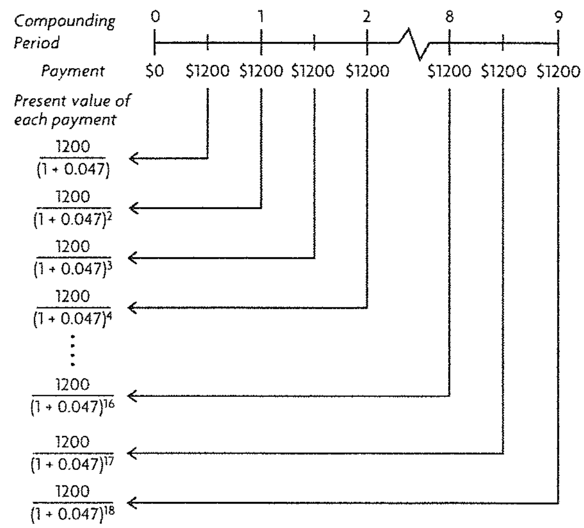


ii) $PV = 650(1.037)^{-1} + 650(1.037)^{-2} + 650(1.037)^{-3} + \cdots + 650(1.037)^{-5}$

iii) $PV = 650 \times \dfrac{1 - 1.037^{-5}}{0.037} = \$2918.24$

iv) Compute the total paid: $650 \times 5 = \$3250$
Subtract the present value from **iii)** to compute the total interest paid:
$3250 - 2918.24 = \$331.77$

b) i) There are 18 payments (9×2):
$i = 9.4\%$/a compounded semi-annually

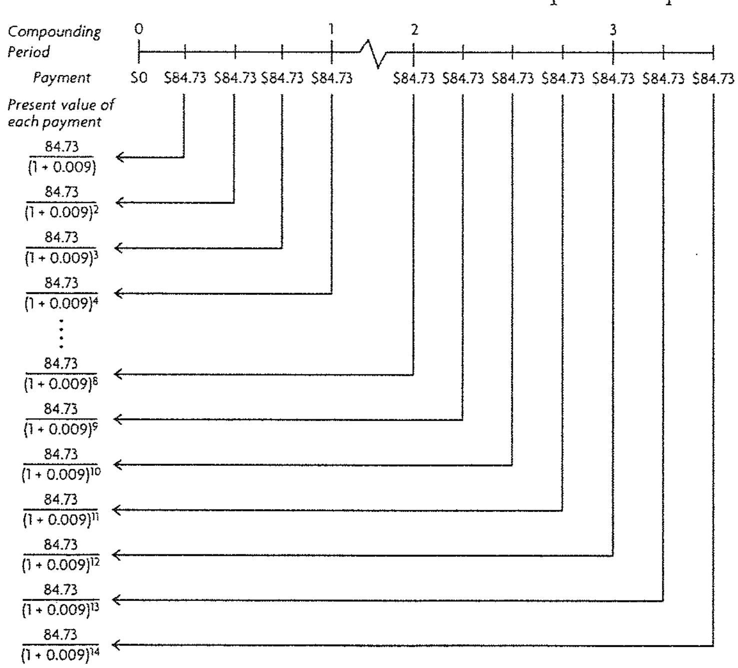


ii) $PV = 1200(1.047)^{-1} + 1200(1.047)^{-2} + 1200(1.047)^{-3} + \cdots + 1200(1.047)^{-18}$

iii) $PV = 1200 \times \dfrac{1 - 1.047^{-18}}{0.047} = \$14\,362.17$

iv) Compute the total paid:
$1200 \times 18 = \$21\,600$
Subtract the present value from **iii)** to compute the total interest paid:
$21\,600 - 14\,362.17 = \$7237.83$

c) i) There are 14 payments $\left(3\frac{1}{2} \times 4\right)$:
$i = 3.6\%$/a compounded quarterly



ii) $PV = 84.73(1.009)^{-1} + 84.73(1.009)^{-2} + 84.73(1.009)^{-3} + \cdots + 84.73(1.009)^{-14}$

iii) $PV = 84.73 \times \frac{1 - 1.009^{-14}}{0.009} = \$1109.85$

iv) Compute the total paid:

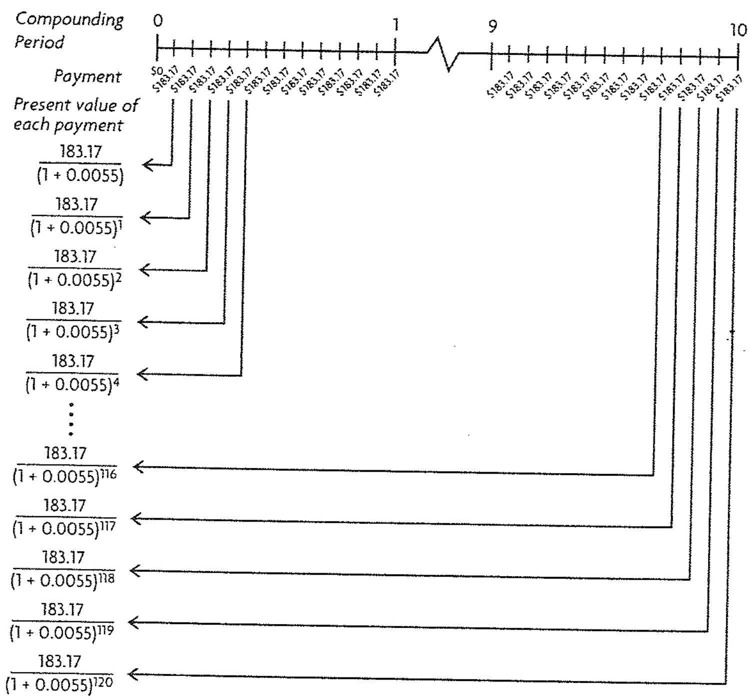
$84.73 \times 14 = \$1186.22$

Subtract the present value from **iii)** to compute the total interest paid:

$1186.22 - 1109.85 = \$76.37$

d) i) There are 120 payments (10×12):

$i = 6.6\%$/a compounded monthly



ii) $PV = 183.17(1.0055)^{-1} + 183.17(1.0055)^{-2} + 183.17(1.0055)^{-3} + \cdots + 183.17(1.0055)^{-120}$

iii)

$PV = 183.17 \times \frac{1 - 1.0055^{-120}}{0.0055} = \$16\,059.45$

iv) Compute the total paid:

$183.17 \times 120 = \$21\,980.40$

Subtract the present value from **iii)** to compute the total interest paid:

$21\,980.40 - 16\,059.45 = \$5920.95$

2. a) i) $PV_1 = 8000(1.09)^{-1} = \$7339.45$

$PV_2 = 8000(1.09)^{-2} = \$6733.44$

$PV_3 = 8000(1.09)^{-3} = \$6177.47$

$PV_4 = 8000(1.09)^{-4} = \$5667.40$

$PV_5 = 8000(1.09)^{-5} = \$5199.45$

$PV_6 = 8000(1.09)^{-6} = \$4770.14$

$PV_7 = 8000(1.09)^{-7} = \$4376.27$

ii) $PV = 8000(1.09)^{-1} + 8000(1.09)^{-2} + 8000(1.09)^{-3} + \cdots + 8000(1.09)^{-7}$

iii) $PV = 8000 \times \frac{1 - 1.09^{-7}}{0.09} = \$40\,263.62$

b) i) $PV_1 = 300(1.04)^{-1} = \$288.46$

$PV_2 = 300(1.04)^{-2} = \$277.37$

$PV_3 = 300(1.04)^{-3} = \$266.70$

$PV_4 = 300(1.04)^{-4} = \$256.44$

$PV_5 = 300(1.04)^{-5} = \$246.58$

$PV_6 = 300(1.04)^{-6} = \$237.09$

$PV_7 = 300(1.04)^{-7} = \$227.98$

ii) $PV = 300(1.04)^{-1} + 300(1.04)^{-2} + 300(1.04)^{-3} + \cdots + 300(1.04)^{-7}$

iii) $PV = 300 \times \frac{1 - 1.04^{-7}}{0.04} = \$1800.62$

c) i) $PV_1 = 750(1.02)^{-1} = \$735.29$
$PV_2 = 750(1.02)^{-2} = \$720.88$
$PV_3 = 750(1.02)^{-3} = \$706.74$
$PV_4 = 750(1.02)^{-4} = \$692.88$
$PV_5 = 750(1.02)^{-5} = \$679.30$
$PV_6 = 750(1.02)^{-6} = \$665.98$
$PV_7 = 750(1.02)^{-7} = \$652.92$
$PV_8 = 750(1.02)^{-8} = \$640.12$

ii) $PV = 750(1.02)^{-1} + 750(1.02)^{-2} + 750(1.02)^{-3} + \cdots + 750(1.02)^{-8}$

iii) $PV = 750 \times \dfrac{1 - 1.02^{-8}}{0.02} = \$5494.11$

3. a) $R = 5000$
$i = 0.072$
$n = 5$

$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$PV = 5000 \times \frac{1 - 1.072^{-5}}{0.072} = \$20\,391.67$$

b) $R = 250$
$i = 0.024$
$n = 24$

$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$PV = 250 \times \frac{1 - 1.024^{-24}}{0.024} = \$4521.04$$

c) $R = 2550$
$i = 0.001$
$n = 100$

$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$PV = 25.50 \times \frac{1 - 1.001^{-100}}{0.001} = \$2425.49$$

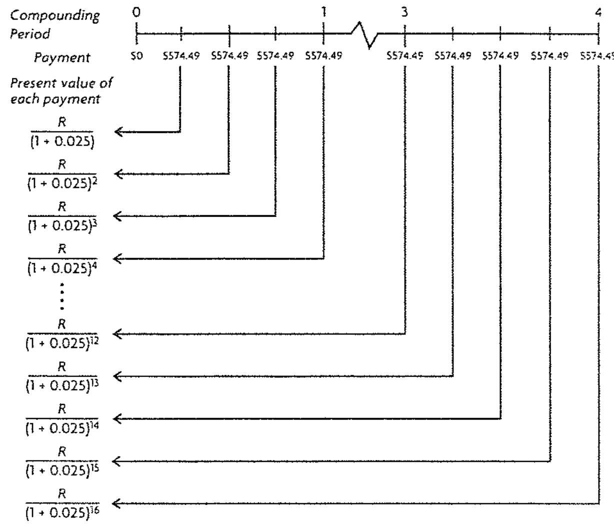
d) $R = 48.50$
$i = 0.0195$
$n = 30$

$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$PV = 48.50 \times \frac{1 - 1.0195^{-30}}{0.0195} = \$1093.73$$

4. $1300 = R \times \dfrac{1 - 1.015^{-24}}{0.015}$
$R = \$64.90$

5. a) Lily makes 16 payments: $i = 10\%/a$ compounded quarterly



b) $7500 = R(1.025)^{-1} + R(1.025)^{-2} + R(1.025)^{-3} + \cdots + R(1.025)^{-16}$

c) $7500 = R \times \dfrac{1 - 1.025^{-16}}{0.025}$
$R = \$574.49$

6. a) Calculate the present value of the loaned amount:

$$PV = 40 \times \frac{1 - 1.015^{-10}}{0.015} = \$368.89$$

Add the \$50 down payment that Rocco made:
$368.89 + 50 = \$418.89$

b) Over 10 months, Rocco pays \$400, excluding the down payment. He borrowed \$368.89. The difference, \$31.11, is the amount of interest Rocco paid.

7. $128\,000 = R \times \dfrac{1 - 1.0065^{-300}}{0.0065}$
$R = \$971.03$

8. a) The Pecas are financing \$64 000. Compute the monthly payment for the 7-year loan. There are 84 payments:
$i = 0.01$
$n = 84$

$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$64\,000 = R_7 \times \frac{1 - 1.01^{-84}}{0.01}$$

$R_7 = \$1029.70$

Compute the monthly payment for the 10-year loan. There are 120 payments:

$i = 0.01$

$n = 120$

$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$64\,000 = R_{10} \times \frac{1 - 1.01^{-120}}{0.01}$$

$R_{10} = \$810.72$

b) Compute the amount of interest they pay for the 7-year loan:

$A_7 = 1029.70 \times 84$

$= 86\,494.80$

$I_7 = \$\,86\,494.80 - 64\,000$

$= \$\,22\,494.80$

Compute the amount of interest they pay for the 10-year loan:

$A_{10} = 810.72 \times 120$

$= 97\,286.40$

$I_{10} = 97\,286.40 - 64\,000 = \$\,33\,286.40$

The shorter term loan saves the Pecas \$10 791.60 in interest.

9. Charles will pay \$552.60 per month for 60 months if he borrows \$29 000 from the bank to pay for the car in cash:

$$29\,000 = R \times \frac{1 - 1.0045^{-60}}{0.0045}$$

$R = \$552.60$

If Charles finances the full \$32 000 at the dealership, he will pay \$566.51 per month for 60 months:

$$32\,000 = R \times \frac{1 - 1.002^{-60}}{0.002}$$

$R = \$566.51$

Since both deals are for the same number of months, the bank-financed deal, which is less per month, will cost Charles less.

10. a) $35\,000 = R_5 \times \dfrac{1 - 1.007^{-60}}{0.007}$

$R_5 = 716.39$

$$35\,000 = R_{10} \times \frac{1 - 1.007^{-120}}{0.007}$$

$R_{10} = 432.08$

$$35\,000 = R_{15} \times \frac{1 - 1.007^{-180}}{0.007}$$

$R_{15} = 342.61$

b) $I_5 = 60(716.39) - 35\,000 = \$7983.40$

$I_{10} = 120(432.08) - 35\,000 = \$16\,849.60$

$I_{15} = 180(342.61) - 35\,000 = \$26\,669.80$

11. a) Calculate the present value of the loaned amount:

$$PV = 25 \times \frac{1 - 1.0155^{-12}}{0.0155} = \$271.84$$

Add the \$45 down payment to get the price of the stereo:

$271.84 + 45 = \$316.84$

b) Pedro financed \$271.84. Compute his total payments:

$12 \times 25 = \$300$

The total interest is the difference:

$300 - 271.84 = \$28.16$

12. You can use a spreadsheet to find an interest rate for which monthly payments of \$75.84 leave a near-zero balance after 30 months.

◇	A	B	C	D	E	F
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance	Interest Rate
2					1800.00	0.19
3	1	75.84	28.50	47.34	1752.66	
4	2	75.84	27.75	48.09	1704.57	
5	3	75.84	26.99	48.85	1655.72	
6	4	75.84	26.22	49.62	1606.10	
7	5	75.84	25.43	50.41	1555.68	
8	6	75.84	24.63	51.21	1504.48	
9	7	75.84	23.82	52.02	1452.46	
10	8	75.84	23.00	52.84	1399.61	
11	9	75.84	22.16	53.68	1345.94	
12	10	75.84	21.31	54.53	1291.41	
13	11	75.84	20.45	55.39	1236.01	
14	12	75.84	19.57	56.27	1179.74	
15	13	75.84	18.68	57.16	1122.58	
16	14	75.84	17.77	58.07	1064.52	
17	15	75.84	16.85	58.99	1005.53	
18	16	75.84	15.92	59.92	945.61	
19	17	75.84	14.97	60.87	884.74	
20	18	75.84	14.01	61.83	822.91	
21	19	75.84	13.03	62.81	760.10	
22	20	75.84	12.03	63.81	696.30	
23	21	75.84	11.02	64.82	631.48	
24	22	75.84	10.00	65.84	565.64	
25	23	75.84	8.96	66.88	498.76	
26	24	75.84	7.90	97.94	430.81	
27	25	75.84	6.82	69.02	361.80	
28	26	75.84	5.73	70.11	291.68	
29	27	75.84	4.62	71.22	220.46	
30	28	75.84	3.49	72.35	148.11	
31	29	75.84	2.35	73.49	74.62	
32	30	75.84	1.18	74.66	−0.04	

An interest rate of 19.00%/a results in a near-zero balance after 30 months.

13. Calculate how much Leo will have in his retirement account after 20 years:

$A = 50\,000(1.028)^{80} = \$455\,427.42$

Now calculate the quarterly withdrawals that that amount will support for 10 years. Assume the interest rate remains the same.

$$i = 0.028$$
$$n = 40$$
$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$
$$455\,427.42 = R \times \frac{1 - 1.028^{-40}}{0.028}$$
$$R = \$19\,070.96$$

14. Calculate how much Charmaine needs 25 years from now to support her \$2500 per month withdrawal from her retirement account. She plans to make 180 withdrawals after retirement. Use the present value formula:

$$R = 2500$$
$$i = 0.0075$$
$$n = 180$$
$$PV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$
$$PV = 2500 \times \frac{1 - 1.0075^{-180}}{0.0075} = \$246\,483.52$$

Calculate how much Charmaine needs to save each month over the next 25 years to reach that goal. Use the future value formula:

$$246\,483.52 = R \times \frac{1.0075^{300} - 1}{0.0075}$$
$$R = \$219.85$$

15. a) Calculate the weekly payment that Option B will support for 25 years:

$$660\,000 = R \times \frac{1 - 1.00127^{-1300}}{0.00127}$$
$$R = \$1037.45$$

Option B is a better choice for a person who expects to live 25 more years.

b) Calculate how long Option B can support a \$1000 per week payment.

$$660\,000 = 1000 \times \frac{1 - 1.00127^{-n}}{0.00127}$$
$$660 = \frac{1 - 1.00127^{-n}}{0.00127}$$
$$0.8382 = 1 - 1.00127^{-n}$$
$$0.1618 = 1.00127^{-n}$$
$$n \doteq 1435 \text{ weeks}$$

After about $27\frac{1}{2}$ years there will be no money left in Option B. So Option A may be a better choice for someone who expects to live more than $27\frac{1}{2}$ years.

16. a) For example, a lump sum is a one-time payment; an annuity has multiple payments over time. If a contest prize can be collected either as a lump sum or an annuity, the annuity earns interest until the last payment is made. The lump sum earns interest only if the contest winner invests the lump sum payment into an account that earns interest.

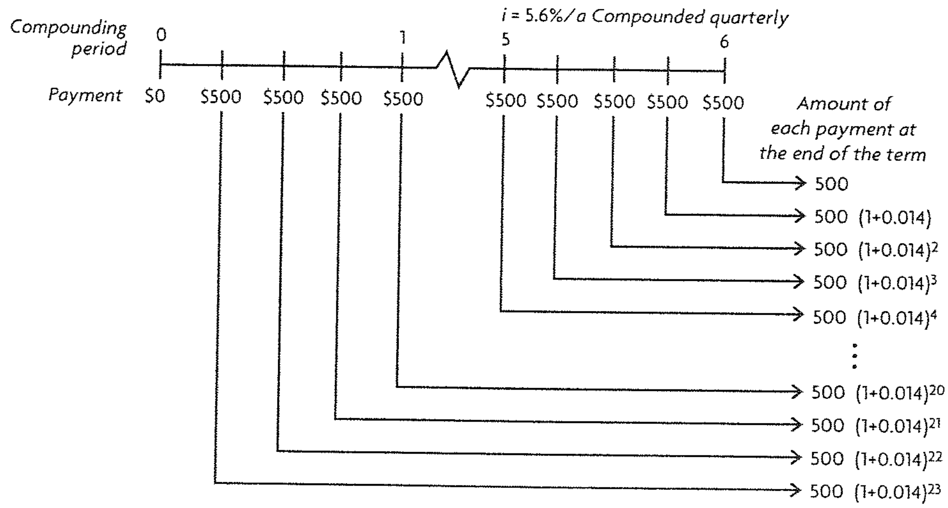
b) For example, future value is the value of an investment some time in the future; present value is the value of the investment now. If the parents of a child are planning for their child's education, they could deposit an amount (present value) into an account that earns interest. When the child is ready for college, the balance in the account (future value) can be used for tuition and other fees.

17. a) You can use a spreadsheet to compute the future values and find their sum:

◇	A	B	C
1	Year	Investment	Future Value of Investment
2			
3	1	500	1024.77
4	2	500	946.23
5	3	500	873.71
6	4	500	806.75
7	5	500	744.92
8	6	500	687.83
9	7	500	635.12
10	8	500	586.44
11	9	500	541.50
12	10	500	500.00
13			**7347.29**

$$PV = \frac{7347.29}{1.0083^{10}} = \$3310.11$$

b) For example, how much would you need to invest now at 5.6%/a compounded quarterly to provide \$500 quarterly for the next 6 years?



◇	A	B	C
1	Quarter	Investment	Future Value of Investment
2			
3	1	500	688.40
4	2	500	678.90
5	3	500	669.53
6	4	500	660.28
7	5	500	651.17
8	6	500	642.17
9	7	500	633.31
10	8	500	624.56
11	9	500	615.94
12	10	500	607.44
13	11	500	599.05
14	12	500	590.78
15	13	500	582.62
16	14	500	574.58
17	15	500	566.65
18	16	500	558.82
19	17	500	551.11
20	18	500	543.50
21	19	500	535.99
22	20	500	528.59
23	21	500	521.30
24	22	500	514.10
25	23	500	507.00
26	24	500	500.00
27			**14145.78**

$$PV = \frac{14\,145.78}{1.014^{24}} = \$10\,132.49$$

c) Use the present value formula and substitute the future value, FV, for the amount, A:

$$PV = \frac{A}{(1+i)^n}$$
$$= \frac{FV}{(1+i)^n}$$

18. You can use a spreadsheet to see when Kyla's loan balance is near zero:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2					17000.00
3	1	325	102.00	223.00	16777.00
4	2	325	100.66	224.34	16552.66
5	3	325	99.32	225.68	16326.98
6	4	325	97.96	227.04	16099.94
7	5	325	96.60	228.40	15871.54
8	6	325	95.23	229.77	15641.77
9	7	325	93.85	231.15	15410.62
10	8	325	92.46	232.54	15178.08
11	9	325	91.07	233.93	14944.15
12	10	325	89.66	235.34	14708.82
13	11	325	88.25	236.75	14472.07
14	12	325	86.83	238.17	14233.90
15	13	325	85.40	239.60	13994.31

...

55	53	325	20.63	304.37	3134.20
56	54	325	18.81	306.19	2828.00
57	55	325	16.97	308.03	2519.97
58	56	325	15.12	309.88	2210.09
59	57	325	13.26	311.74	1898.35
60	58	325	11.39	313.61	1584.74
61	59	325	9.51	315.49	1269.25
62	60	325	7.62	317.38	951.86
63	61	325	5.71	319.29	632.58
64	62	325	3.80	321.20	311.37
65	63	325	1.87	323.13	−11.76

The balance is near zero after 63 payments, which is 5 years and 3 months.

19. $$R \times \frac{(1+i)^m - 1}{i} = W \times \frac{1-(1+i)^{-n}}{i}$$
$$R = W \times \frac{1-(1+i)^{-n}}{i} \times \frac{i}{(1+i)^m - 1}$$
$$R = W \times \frac{1-(1+i)^{-n}}{(1+i)^m - 1}$$

8.6 Using Technology to Investigate Financial Problems, pp. 530–531

1. a) About 12 years

◇	A	B	C
1	Compounding Period	Interest Earned	Balance
2			5000.00
3	1	415.00	5415.00
4	2	449.45	5864.45
5	3	486.75	6351.19
6	4	527.15	6878.34
7	5	570.90	7449.25
8	6	618.29	8067.53
9	7	669.61	8737.14
10	8	725.18	9462.32
11	9	785.37	10247.69
12	10	850.56	11098.25
13	11	921.15	12019.41
14	12	997.61	13017.02

b) About 7 years

◇	A	B	C
1	Compounding Period	Interest Earned	Balance
2			2500.00
3	1	85.00	2585.00
4	2	87.89	2672.89
5	3	90.88	2763.77
6	4	93.97	2857.74
7	5	97.16	2954.90
8	6	100.47	3055.37
9	7	103.88	3159.25
10	8	107.41	3266.66
11	9	111.07	3377.73
12	10	114.84	3492.57
13	11	118.75	3611.32
14	12	122.78	3734.10
15	13	126.96	3861.06
16	14	131.28	3992.34
17	15	135.74	4128.08

c) About 19 years

◇	A	B	C
1	Compounding Period	Interest Earned	Balance
2			450.00
3	1	13.95	463.95
4	2	14.38	478.33
5	3	14.83	493.16
6	4	15.29	508.45
7	5	15.76	524.21
8	6	16.25	540.46
9	7	16.75	557.22
10	8	17.27	574.49

...

70	68	107.87	3587.85
71	69	111.22	3698.80
72	70	114.66	3813.46
73	71	118.22	3931.68
74	72	121.88	4053.56
75	73	125.66	4179.22
76	74	129.56	4308.77
77	75	133.57	4442.35
78	76	137.71	4580.06

d) About 8 years

◇	A	B	C
1	Compounding Period	Interest Earned	Balance
2			15000.00
3	1	45.00	15045.00
4	2	45.14	15090.14
5	3	45.27	15135.41
6	4	45.41	15180.81
7	5	45.54	15226.35
8	6	45.68	15272.03
9	7	45.82	15317.85
10	8	45.95	15363.80

...

91	89	58.57	19582.78
92	90	58.75	19641.53
93	91	58.92	19700.45
94	92	59.10	19759.55
95	93	59.28	19818.83
96	94	59.46	19878.29
97	95	59.63	19937.92
98	96	59.81	19997.74
99	97	59.99	20057.73

2. a) 7.10%

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0710		2500.00
3	1	357.69	177.50	180.19	2319.81
4	2	357.69	164.71	192.98	2126.83
5	3	357.69	151.00	206.69	1920.14
6	4	357.69	136.33	221.36	1698.78
7	5	357.69	120.61	237.08	1461.70
8	6	357.69	103.78	253.91	1207.80
9	7	357.69	85.75	271.94	935.86
10	8	357.69	66.45	291.24	644.62
11	9	357.69	45.77	311.92	332.69
12	10	357.69	23.62	334.07	−1.38

b) 5.80%

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0580		15000.00
3	1	1497.95	435.00	1062.95	13937.05
4	2	1497.95	404.17	1093.78	12843.27
5	3	1497.95	372.45	1125.50	11717.78
6	4	1497.95	339.82	1158.13	10559.65
7	5	1497.95	306.23	1191.72	9367.92
8	6	1497.95	271.67	1226.28	8141.64
9	7	1497.95	236.11	1261.84	6879.80
10	8	1497.95	199.51	1298.44	5581.37
11	9	1497.95	161.86	1336.09	4245.28
12	10	1497.95	123.11	1374.84	2870.44
13	11	1497.95	83.24	1414.71	1455.73
14	12	1497.95	42.22	1455.73	0.00

c) 16.30%

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.1630		3500.00
3	1	374.56	142.63	231.94	3268.07
4	2	374.56	133.17	241.39	3026.68
5	3	374.56	123.34	251.22	2775.46
6	4	374.56	113.10	261.46	2514.00
7	5	374.56	102.45	272.11	2241.88
8	6	374.56	91.36	283.20	1958.68
9	7	374.56	79.82	294.74	1663.93
10	8	374.56	67.81	306.75	1357.18
11	9	374.56	55.31	319.25	1037.92
12	10	374.56	42.30	332.26	705.66
13	11	374.56	28.76	345.80	359.86
14	12	374.56	14.66	359.90	−0.04

d) 22.19%

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.2219		450.00
3	1	29.62	8.32	21.30	428.70
4	2	29.62	7.93	21.69	407.01
5	3	29.62	7.53	22.09	384.91
6	4	29.62	7.12	22.50	362.41
7	5	29.62	6.70	22.92	339.49
8	6	29.62	6.28	23.34	316.15
9	7	29.62	5.85	23.77	292.38
10	8	29.62	5.41	24.21	268.16
11	9	29.62	4.96	24.66	243.50
12	10	29.62	4.50	25.12	218.39
13	11	29.62	4.04	25.58	192.80
14	12	29.62	3.57	26.05	166.75
15	13	29.62	3.08	26.54	140.21
16	14	29.62	2.59	27.03	113.19
17	15	29.62	2.09	27.53	85.66
18	16	29.62	1.58	28.04	57.62
19	17	29.62	1.07	28.55	29.07
20	18	29.62	0.54	29.08	−0.01

3. $99.86

◇	A	B	C
1	Investment Period	Investment	Future Value
2	1	99.86	135.19
3	2	99.86	133.79
4	3	99.86	132.40
5	4	99.86	131.02
6	5	99.86	129.66
7	6	99.86	128.31
8	7	99.86	126.98
9	8	99.86	125.66
10	9	99.86	124.35
11	10	99.86	123.06
12	11	99.86	121.78
13	12	99.86	120.52
14	13	99.86	119.26
15	14	99.86	118.02
16	15	99.86	116.80
17	16	99.86	115.58
18	17	99.86	114.38
19	18	99.86	113.20
20	19	99.86	112.02
21	20	99.86	110.85
22	21	99.86	109.70
23	22	99.86	108.56
24	23	99.86	107.43
25	24	99.86	106.32
26	25	99.86	105.21
27	26	99.86	104.12
28	27	99.86	103.04
29	28	99.86	101.97
30	29	99.86	100.91
31	30	99.86	99.86
32			**3499.96**

4. a) $817.16

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0660		120000.00
3	1	817.76	660.00	157.76	119842.24
4	2	817.76	659.13	158.63	119683.61
5	3	817.76	658.26	159.50	119524.11
6	4	817.76	657.38	160.38	119363.73
7	5	817.76	656.50	161.26	119202.48
8	6	817.76	655.61	162.15	119040.33
9	7	817.76	654.72	163.04	118877.29
10	8	817.76	653.83	163.93	118713.36

...

298	296	817.76	22.13	795.63	3228.73
299	297	817.76	17.76	800.00	2428.72
300	298	817.76	13.36	804.40	1624.32
301	299	817.76	8.93	808.83	815.50
302	300	817.76	4.49	813.27	2.22

b) About 5 years and 2 months:
300 months − 238 months = 62 months

...

38	36	817.76	626.61	191.15	113738.33
39	37	15000	625.56	14374.44	99363.89
40	38	817.76	546.50	270.66	99093.23
41	39	817.76	545.01	272.15	98821.08

...

236	234	817.76	24.10	793.06	3587.92
237	235	817.76	19.73	797.43	2790.49
238	236	817.76	15.35	801.81	1988.68
239	237	817.76	10.94	806.22	1182.46
240	238	817.76	6.50	810.66	371.80
241	239	817.76	2.04	815.12	−443.31

c) The total interest without the $15 000 payment is $125 330.22. The total interest with the payment is $88 961.48. So Nadia saves $36 368.74 in interest with the $15 000 payment.

5.

...

38	36	817.76	626.61	191.15	113738.33
39	37	15000	625.56	14374.44	99363.89
40	38	817.76	546.50	270.66	99093.23
41	39	817.76	545.01	272.15	98821.08

...

236	234	817.76	24.10	793.06	3587.92
237	235	817.76	19.73	797.43	2790.49
238	236	817.76	15.35	801.81	1988.68
239	237	817.76	10.94	806.22	1182.46
240	238	817.76	6.50	810.66	371.80
241	239	817.76	2.04	815.12	−443.31

The future value of the Bank A investment is $79 805.09. The future value of the Bank B investment is $83 456.12. With Bank B, they will earn $3651.03 more.

6. The loan will be paid off in 8 years:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0500		25000.00
3	1	250	104.17	145.83	24854.17
4	2	250	103.56	146.44	24707.73
5	3	250	102.95	147.05	24560.67
6	4	250	102.34	147.66	24413.01
7	5	250	101.72	148.28	24264.73

...

24	22	250	90.86	159.14	21647.32
25	23	250	90.20	159.80	21487.52
26	24	250	89.53	160.47	21327.05
27	25	**300**	88.86	211.14	21115.92
28	26	300	87.98	212.02	20903.90
29	27	300	87.10	212.90	20691.00

...

48	46	300	69.60	230.40	16473.06
49	47	300	68.64	231.36	16241.69
50	48	300	67.67	232.33	16009.37
51	49	**350**	66.71	283.29	15726.07
52	50	350	65.53	284.47	15441.60
53	51	350	64.34	285.66	15155.94

...

72	70	350	40.86	309.14	9496.49
73	71	350	39.57	310.43	9186.06
74	72	350	38.28	311.72	8874.34
75	73	**400**	36.98	363.02	8511.32
76	74	400	35.46	364.54	8146.78
77	75	400	33.94	366.06	7780.72

...

95	93	400	5.50	394.50	924.66
96	94	400	3.85	396.15	528.51
97	95	400	2.20	397.80	130.71
98	96	400	0.54	399.46	−268.74

7. a) After 5 years, Natalie's outstanding loan balance is $123 354.95:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0420		150000.00
3	1	924.86	525.00	399.86	149600.14
4	2	924.86	523.60	401.26	149198.88
5	3	924.86	522.20	402.66	148796.22
6	4	924.86	520.79	404.07	148392.14

...

57	55	924.86	441.97	482.89	125794.86
58	56	924.86	440.28	484.58	125310.28
59	57	924.86	438.59	486.27	124824.01
60	58	924.86	436.88	487.98	124336.03
61	59	924.86	435.18	489.68	123846.35
62	60	924.86	433.46	491.40	123354.95

Use that as the amount for a new loan.
A monthly payment of $1143.52 will bring the balance near zero in 15 years:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0750		123354.95
3	1	1143.52	770.97	372.55	122982.40
4	2	1143.52	768.64	374.88	122607.52
5	3	1143.52	766.30	377.22	122230.29
6	4	1143.52	763.94	379.58	121850.71
7	5	1143.52	761.57	381.95	121468.76

...

178	176	1143.52	35.07	1108.45	4502.08
179	177	1143.52	28.14	1115.38	3386.70
180	178	1143.52	21.17	1122.35	2264.35
181	179	1143.52	14.15	1129.37	1134.98
182	180	1143.52	7.09	1136.43	−1.45

b) If Natalie leaves her monthly payment at $924.86, it will take an additional 9 years to repay the loan:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0750		123354.95
3	1	924.86	770.97	153.89	123201.06
4	2	924.86	770.01	154.85	123046.20
5	3	924.86	769.04	155.82	122890.38
6	4	924.86	768.06	156.80	122733.59
7	5	924.86	767.08	157.78	122575.81

...

285	283	924.86	33.04	891.82	4394.73
286	284	924.86	27.47	897.39	3497.34
287	285	924.86	21.86	903.00	2594.34
288	286	924.86	16.21	908.65	1685.69
289	287	924.86	10.54	914.32	771.37
290	288	924.86	4.82	920.04	−148.67

$$\begin{aligned} 288 \text{ months} - 180 \text{ months} &= 108 \text{ months} \\ &= 9 \text{ years} \end{aligned}$$

8. Peter is paying 12.36%/a interest:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.1236		2404.00
3	1	147	24.76	122.24	2281.76
4	2	147	23.50	123.50	2158.26
5	3	147	22.23	124.77	2033.49
6	4	147	20.94	126.06	1907.44
7	5	147	19.65	127.35	1780.09
8	6	147	18.33	128.67	1651.42
9	7	147	17.01	129.99	1521.43
10	8	147	15.67	131.33	1390.10
11	9	147	14.32	132.68	1257.42
12	10	147	12.95	134.05	1123.37
13	11	147	11.57	135.43	987.94
14	12	147	10.18	136.82	851.12
15	13	147	8.77	138.23	712.88
16	14	147	7.34	139.66	573.23
17	15	147	5.90	141.10	432.13
18	16	147	4.45	142.55	289.58
19	17	147	2.98	144.02	145.56
20	18	147	1.50	145.50	0.06

9. a) Doubling the payment does not necessarily ensure that the amortization period stays the same. For example, suppose you borrow $3500 at 6.6%/a compounded monthly amortized over 2 years. The monthly payment is $156.07. Suppose the interest rate doubles. To keep the amortization period the same, the monthly payment needs to be $166.73, which is less than twice the original payment. Here is the 6.6% amortization table:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0660		3500.00
3	1	156.07	19.25	136.82	3363.18
4	2	156.07	18.50	137.57	3225.61
5	3	156.07	17.74	138.33	3087.28
6	4	156.07	16.98	139.09	2948.19
7	5	156.07	16.22	139.85	2808.33
8	6	156.07	15.45	140.62	2667.71
9	7	156.07	14.67	141.40	2526.31
10	8	156.07	13.89	142.18	2384.14
11	9	156.07	13.11	142.96	2241.18
12	10	156.07	12.33	143.74	2097.44
13	11	156.07	11.54	144.53	1952.90
14	12	156.07	10.74	145.33	1807.57
15	13	156.07	9.94	146.13	1661.44
16	14	156.07	9.14	146.93	1514.51
17	15	156.07	8.33	147.74	1366.77
18	16	156.07	7.52	148.55	1218.22
19	17	156.07	6.70	149.37	1068.85
20	18	156.07	5.88	150.19	918.66
21	19	156.07	5.05	151.02	767.64
22	20	156.07	4.22	151.85	615.79
23	21	156.07	3.39	152.68	463.11
24	22	156.07	2.55	153.52	309.59
25	23	156.07	1.70	154.37	155.22
26	24	156.07	0.85	155.22	0.00

Here is the 13.2% amortization table:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.1320		3500.00
3	1	166.73	38.50	128.23	3371.77
4	2	166.73	37.09	129.64	3242.13
5	3	166.73	35.66	131.07	3111.06
6	4	166.73	34.22	132.51	2978.55
7	5	166.73	32.76	133.97	2844.59
8	6	166.73	31.29	135.44	2709.15
9	7	166.73	29.80	136.93	2572.22
10	8	166.73	28.29	138.44	2433.78
11	9	166.73	26.77	139.96	2293.83
12	10	166.73	25.23	141.50	2152.33
13	11	166.73	23.68	143.05	2009.27
14	12	166.73	22.10	144.63	1864.65
15	13	166.73	20.51	146.22	1718.43
16	14	166.73	18.90	147.83	1570.60
17	15	166.73	17.28	149.45	1421.15
18	16	166.73	15.63	151.10	1270.05
19	17	166.73	13.97	152.76	1117.29
20	18	166.73	12.29	154.44	962.85
21	19	166.73	10.59	156.14	806.71
22	20	166.73	8.87	157.86	648.85
23	21	166.73	7.14	159.59	489.26
24	22	166.73	5.38	161.35	327.91
25	23	166.73	3.61	163.12	164.79
26	24	166.73	1.81	164.92	−0.13

b) If the loan amount doubles, doubling the payment amount ensures the amortization period stays the same. For example, suppose you borrow \$3500 at 6.6%/*a* compounded monthly amorized over 2 years. The monthly payment is \$156.07. Suppose the loan amount doubles. To keep the amortization period the same, the monthly payment needs to be \$312.14, which is twice the original payment. The \$3500 amortization table is shown in **a**). Here is the \$7000 amortization table:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0660		7000.00
3	1	312.14	38.50	273.64	6726.36
4	2	312.14	36.99	275.15	6451.21
5	3	312.14	35.48	276.66	6174.56
6	4	312.14	33.96	278.18	5896.38
7	5	312.14	32.43	279.71	5616.67
8	6	312.14	30.89	281.25	5335.42
9	7	312.14	29.34	282.80	5052.62
10	8	312.14	27.79	284.35	4768.27
11	9	312.14	26.23	285.91	4482.36
12	10	312.14	24.65	287.49	4194.87
13	11	312.14	23.07	289.07	3905.80
14	12	312.14	21.48	290.66	3615.14
15	13	312.14	19.88	292.26	3322.89
16	14	312.14	18.28	293.86	3029.02
17	15	312.14	16.66	295.48	2733.54
18	16	312.14	15.03	297.11	2436.44
19	17	312.14	13.40	298.74	2137.70
20	18	312.14	11.76	300.38	1837.32
21	19	312.14	10.11	302.03	1535.28
22	20	312.14	8.44	303.70	1231.59
23	21	312.14	6.77	305.37	926.22
24	22	312.14	5.09	307.05	619.17
25	23	312.14	3.41	308.73	310.44
26	24	312.14	1.71	310.43	0.01

10. Calculate Laurie's original payment:

$$50\,000 = R \times \frac{1 - (1.0055)^{-120}}{0.0055}$$

$$R = \$570.29$$

Set up a spreadsheet in which the payment changes to $1140.58 after 4 years:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0660		50000.00
3	1	570.29	275.00	295.29	49704.71
4	2	570.29	273.38	295.91	49407.80
5	3	570.29	271.74	298.55	49109.25
6	4	570.29	270.10	300.19	48809.06
...					
49	47	570.29	190.25	380.04	34211.69
50	48	570.29	188.16	382.13	33829.56
51	49	1140.58	186.06	954.52	32875.05
52	50	1140.58	180.81	959.77	31915.28
...					
80	78	1140.58	21.49	1119.09	2788.65
81	79	1140.58	15.34	1125.24	1663.41
82	80	1140.58	9.15	1131.43	531.98
83	81	1140.58	2.93	1137.65	−605.68

The loan will be paid off in 81 months instead of 120 months. That is, it will be paid off 3 years and 3 months sooner.

11. For example,

Technology	Advantages	Disadvantages
Spreadsheet	Can set up the spreasheet so that you only need to type equations once and just input the values in a question Can see how changing one value affects the rest of the calculation	Need a computer
Graphing calculator	May be more easily available	Have to make many keystrokes Could mistype a number or equation Cannot see how changing one value affects the others

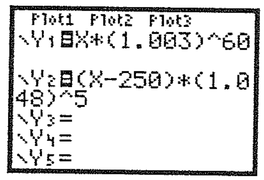
12. Let x represent the price of the guitar. Then the amount that would be paid back to the music store if the store financed the purchase would be:

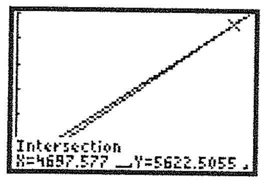
$A = x(1.003)^{60}$

Since the music store gives a discount of $250 if the customer pays cash, the amount that would have to be paid back to the bank if the customer borrowed money from the bank and paid the music store would be:

$A = (x - 250)(1.048)^5$

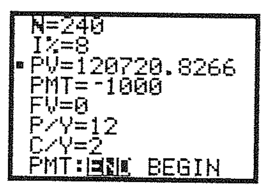
Use a graphing calculator to find when the amount owed for each scenario would be the same. (Also, verify by tracing or inspection that the bank option is cheaper for all prices up to this point).





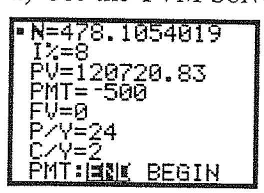
The two graphs intersect when $x \doteq 4697.577$, so for all prices less than or equal to approximately $4697.58, it is worthwhile to take a loan from the bank to purchase the rare guitar.

13. First, use the TVM Solver on a graphing calculator to determine how much money is borrowed, based on the information given.



The amount of the mortgage is $120 720.83.

a) Use the TVM Solver on a grEr year).



Approximately 478 payments will be required. Since there are 24 payments per year, it will take about 19.92 years to repay the loan.

b) Use the TVM Solver on a graphing calculator to determine the number of payments that will be required if payments of $500 are made every two weeks (26 payments per year).

```
▪N=433.4046377
 I%=8
 PV=120720.83
 PMT=-500
 FV=0
 P/Y=26
 C/Y=2
 PMT:END BEGIN
```

Approximately 433 payments will be required. Since there are 26 payments per year, it will take about 16.65 years to repay the loan.

c) Use the TVM Solver on a graphing calculator to determine the number of payments that will be required if payments of $250 are made every week (52 payments per year).

```
▪N=865.4620521
 I%=8
 PV=120720.83
 PMT=-250
 FV=0
 P/Y=52
 C/Y=2
 PMT:END BEGIN
```

Approximately 865 payments will be required. Since there are 52 payments per year, it will take about 16.63 years to repay the loan.

14. It will take 5 years and 2 months to pay off the loan:

◇	A	B	C	D	E	F
1	Payment Number	Payment	Interest Rate/a	Interest Paid	Principal Paid	Balance
2						6800.00
3	1	150	0.13	73.67	76.33	6723.67
4	2	150	0.13	72.84	77.16	6646.51
5	3	150	0.13	72.00	78.00	6568.51
6	4	150	0.13	71.16	78.84	6489.67
7	5	150	0.13	70.30	79.70	6409.97
8	6	150	0.13	69.44	80.56	6329.42
9	7	150	0.13	68.57	81.43	6247.98
10	8	150	0.13	67.69	82.31	6165.67
11	9	150	0.13	66.79	83.21	6082.47
12	10	150	0.13	65.89	84.11	5998.36
13	11	150	0.13	64.98	85.02	5913.34
14	12	150	0.13	64.06	85.94	5827.40
15	13	150	0.125	60.70	89.30	5738.10
16	14	150	0.125	59.77	90.23	5647.88
17	15	150	0.125	58.83	91.17	5556.71
18	16	150	0.125	57.88	92.12	5464.59
19	17	150	0.125	56.92	93.08	5371.51
20	18	150	0.125	55.95	94.05	5277.47
21	19	150	0.125	54.97	95.03	5182.44
22	20	150	0.125	53.98	96.02	5086.42
23	21	150	0.125	52.98	97.02	4989.41
24	22	150	0.125	51.97	98.03	4891.38
25	23	150	0.125	50.95	99.05	4792.33
26	24	150	0.125	49.92	100.08	4692.25
27	25	150	0.12	46.92	103.08	4589.17
28	26	150	0.12	45.89	104.11	4485.07
29	27	150	0.12	44.85	105.15	4379.92
30	28	150	0.12	43.80	106.20	4273.72
31	29	150	0.12	42.74	107.26	4166.45
32	30	150	0.12	41.66	108.34	4058.12
33	31	150	0.12	40.58	109.42	3948.70
34	32	150	0.12	39.49	110.51	3818.19
35	33	150	0.12	38.38	111.62	3726.57
36	34	150	0.12	37.27	112.73	3613.83
37	35	150	0.12	36.14	113.86	3499.97
38	36	150	0.12	35.00	115.00	3384.97
39	37	150	0.115	32.44	117.56	3267.41
40	38	150	0.115	31.31	118.69	3148.72
41	39	150	0.115	30.18	119.82	3028.90
42	40	150	0.115	29.03	120.97	2907.93
43	41	150	0.115	27.87	122.13	2785.79
44	42	150	0.115	26.70	123.30	2662.49
45	43	150	0.115	25.52	124.48	2538.01
46	44	150	0.115	24.32	125.68	2412.33
47	45	150	0.115	23.12	126.88	2285.45
48	46	150	0.115	21.90	128.10	2157.35
49	47	150	0.115	20.67	129.33	2028.02
50	48	150	0.115	19.44	130.56	1897.46
51	49	150	0.11	17.39	132.61	1764.05
52	50	150	0.11	16.18	133.82	1631.03
53	51	150	0.11	14.95	135.05	1495.98
54	52	150	0.11	13.71	136.29	1359.69
55	53	150	0.11	12.46	137.54	1222.16
56	54	150	0.11	11.20	138.80	1083.36
57	55	150	0.11	9.93	140.07	943.29
58	56	150	0.11	8.65	141.35	801.94
59	57	150	0.11	7.35	142.65	659.29
60	58	150	0.11	6.04	143.96	515.33
61	59	150	0.11	4.72	145.28	370.06
62	60	150	0.11	3.39	146.61	223.45
63	61	150	0.105	1.96	148.04	75.41
64	62	150	0.105	0.66	149.34	−73.93

Chapter Review, pp. 534–535

1. a) $I = 3500(0.06)(10) = \$2100.00$
$A = 3500 + 2100 = \$5600.00$

b) $I = 15\,000(0.11)(3) = \$4950.00$
$A = 15\,000 + 4950 = \$19\,950$

c) $I = 280(0.032)\left(\frac{34}{12}\right) = \$25.39$

$A = 25.39 + 280 = \$305.39$

d) $I = 850(0.29)\left(\frac{100}{52}\right) = \$474.04$

$A = 474.04 + 850 = \$1324.04$

e) $I = 21\,000(0.073)\left(\frac{42}{365}\right) = \$176.40$

$A = 176.40 + 21\,000 = \$21\,176.40$

2. a) $11.25 = 2500(i)\left(\frac{1}{12}\right)$

$i = 0.054 = 5.4\%$

b) $A = 2500 + 84(11.25) = \$3445.00$

c) $5000 = 2500 + 2500(0.054)t$
$t \doteq 18.5$ years $= 18$ years and 6 months

3. a) Karl borrowed \$5000.

b) After 2 years the total is \$6000, so the amount of interest is \$6000 − \$5000 or \$1000. The amount of interest per year is $\frac{\$1000}{2}$ or \$500.

$i = \frac{500}{5000}$
$= 0.1$
$= 10\%$

c) \$20 000 − \$5000 = \$15 000
Since the amount of interest per year is \$500, it will take $\frac{15\,000}{5000}$ or 30 years before Karl owes \$20 000.

4. $10\,000 = 4350(1.019)^n$

$\frac{10\,000}{4350} = 1.019^n$

$n \doteq 44.2$

It will take about 11 years.

5. a) $A = 4300(1.091)^8 = \$8631.11$
$I = 8631.11 - 4300 = \$4331.11$

b) $A = 500(1.052)^{23} = \$1604.47$
$I = 1604.47 - 500 = \$1104.47$

c) $A = 25\,000(1.016)^{12} = \$30\,245.76$
$I = 30\,245.76 - 25\,000 = \$5245.76$

d) $A = 307(1.023)^{30} = \$607.31$
$I = 607.31 - 307 = \$300.31$

6. a) At the end of the first year, Deana earns \$400 in interest. So if i is the interest rate and P is the principal she invested,
$400 = Pi$
At the end of the second year, she earns \$432 in interest.
$432 = (P + 400)i$
$432 = Pi + 400i$
$Pi = 432 - 400i$
Substituting \$400 for Pi,
$400 = 432 - 400i$
$i = 0.08 = 8\%$

b) Substituting 0.08 for i,
$400 = 0.08P$
$P = \$5000$

7. Calculate how much Vlad financed:
$2942.37 - 850 = \$2092.37$
Substituting into the formula for future value,
$2147.48 = 2092.37(1 + i)^{18}$

$(1 + i)^{18} = \frac{2147.48}{2092.37}$

$1 + i \doteq 1.001445$
$i \doteq 0.001445$ per compounding period
Multiply by 12 to find the annual rate because the compounding period is monthly:
$i \doteq 1.73\%/\text{a}$

8. a) $PV = \frac{8000}{1.067^5} = \$5784.53$

b) $PV = \frac{1280}{1.044^5} = \$1032.07$

c) $PV = \frac{100\,000}{1.014^{32}} = \$64\,089.29$

d) $PV = \frac{850}{1.0205^{18}} = \$589.91$

9. $PV = \frac{847.53}{1.008^{30}} = \$667.33$

10. $1650 = \frac{2262.70}{(1 + i)^3}$

$(1 + i)^3 = \frac{2262.70}{1650}$

$1 + i \doteq 1.1110$
$i \doteq 0.1110 = 11.10\%/\text{a}$

11. a) $FV = 2500 \times \frac{1.076^{12} - 1}{0.076} = \$46\,332.35$

$I = 46\,332.35 - (2500)(12)$
$= \$16\,332.35$

b) $FV = 500 \times \dfrac{1.036^{19} - 1}{0.036} = \$13\,306.97$

$I = 13\,306.97 - (500)(19) = \$3806.97$

c) $FV = 2500 \times \dfrac{1.01075^{12} - 1}{0.01075} = \$31\,838.87$

$I = 31\,838.87 - (2500)(12) = \$1838.87$

12. $100\,000 = 1500 \times \dfrac{1.011^{n} - 1}{0.011}$

$1.011^{n} = 1.7333$

$n \doteq 50.28$ compounding periods

It will take about 12 years and 7 months for Naomi's account to reach \$100 000.

13. $i = 0.0075$

$n = 72$

$FV = 25\,000$

$$FV = R \times \left(\frac{(1 - (1 + i)^{-n})}{i}\right)$$

$$25\,000 = R \times \frac{1.0075^{72} - 1}{0.0075}$$

$R = \$263.14$

14. a) $PV = 450 \times \dfrac{1 - 1.051^{-12}}{0.051} = \$2276.78$

$I = (450)(12) - 2276.78 = \$423.22$

b) $PV = 2375 \times \dfrac{1 - 1.046^{-9}}{0.046} = \$17\,185.88$

$I = (2375)(9) - 17\,185.88 = \$4189.12$

c) $PV = 185.73 \times \dfrac{1 - 1.032^{-14}}{0.032} = \$2069.70$

$I = (185.73)(14) - 2069.70 = \$530.52$

d) $PV = 105.27 \times \dfrac{1 - 1.016^{-18}}{0.016} = \$1635.15$

$I = (105.27)(18) - 1635.15 = \$259.71$

15. a) $136\,000 = R \times \dfrac{1 - 1.0055^{-240}}{0.0055}$

$R = \$1022.00$

b) Calculate the total of Paul's payments:

$1022 \times 240 = \$245\,280$

The total interest is the difference between the total payments and the present value:

$I = 245\,280 - 136\,000 = \$109\,280$

16. Eden's balance is \$0.00 after 30 months when she pays 20.4%/a interest:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.2040		611.03
3	1	26.17	10.39	15.78	595.25
4	2	26.17	10.12	16.05	579.20
5	3	26.17	9.85	16.32	562.87
6	4	26.17	9.57	16.60	546.27

...

29	27	26.17	1.71	24.46	75.91
30	28	26.17	1.29	24.88	51.03
31	29	26.17	0.87	25.30	25.73
32	30	26.17	0.44	25.73	0.00

17. Chantal finances \$1225.47 over 48 months:

$$1225.47 = R \times \frac{1 - 1.0055^{-48}}{0.0055}$$

$R = \$29.12$

18. Ken saves for 35 years, which is 420 months. Calculate the future value of Ken's investments:

$$FV = 100 \times \frac{1.0045^{420} - 1}{0.0045} =$$

When Ken is 55, his account will be valued at \$124 252.52.

Adam saves for 18 years, which is 216 months. Calculate the monthly payment Adam must make in order for his account to be worth \$124 252.52 at age 55:

$$124\,252.52 = R \times \frac{1.006^{216} - 1}{0.006}$$

$R = \$282.34$

Adam must save \$182.34 more per month than Ken saves.

19. Use a spreadsheet to compare the amortization periods. If Jenny pays \$1000 each month, the balance will be near zero after 124 months, or 10 years, 4 months:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0420		100000.00
3	1	1000	350.00	650.00	99350.00
4	2	1000	347.73	652.28	98697.73
5	3	1000	345.44	654.56	98043.17
6	4	1000	343.15	656.85	97386.32
7	5	1000	340.85	659.15	96727.17

...

122	120	1000	14.90	985.10	3271.47
123	121	1000	11.45	988.55	2282.92
124	122	1000	7.99	992.01	1290.91
125	123	1000	4.52	995.48	295.43
126	124	1000	1.03	998.97	−703.54

If Jenny pays \$1500 each month, the balance will be near zero after 76 months, or 6 years, 4 months:

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0420		100000.00
3	1	1500	350.00	1150.00	98850.00
4	2	1500	345.98	1154.03	97695.98
5	3	1500	341.94	1158.06	96537.91
6	4	1500	337.88	1162.12	95375.79
7	5	1500	333.82	1166.18	94209.61

...

75	73	1500	21.07	1478.93	4539.75
76	74	1500	15.89	1484.11	3055.64
77	75	1500	10.69	1489.31	1566.33
78	76	1500	5.48	1494.52	71.82
79	77	1500	0.25	1499.75	−1427.93

The higher payment will shorten the amortization schedule by 4 years.

20. $PV = 17.85 \times \dfrac{1 - 1.0025^{-130}}{0.0025} = \$1979.06$

Chapter Self-Test, p. 536

1. a) $I = 850(0.09)(6) = \$459.00$

$A = 850 + 459 = \$1309.00$

b) $A = 5460(1.042)^{26} = \$15\,913.05$

$I = 15\,913.05 - 5460 = \$10\,453.05$

c) $A = 230 \times \dfrac{1.004^{78} - 1}{0.004} = \$21\,005.02$

$I = (230)(78) - 21\,005.02 = \$3065.02$

2. a) For Loan 1, simple interest is being charged; there is a common difference in the amount owed from month to month. For Loan 2, compound interest is being charged; there is a common ratio in the amount owed from month to month.

b) (Loan 1) Calculate the amount of interest charged each month:

$3942 - 3796 = \$146$

So the original amount borrowed was \$3650:

$3796 - 146 = \$3650$

Calculate the interest rate:

$i = \dfrac{146}{3650} = 0.04 = 4\%$

(Loan 2) Write the present value of the loan using the numbers for the second year:

$PV = \dfrac{977.53}{(1 + i)^2}$

Write the present value of the loan using the numbers for the third year:

$PV = \dfrac{1036.18}{(1 + i)^3}$

Set them equal to each other and solve for i:

$\dfrac{977.53}{(1 + i)^2} = \dfrac{1036.18}{(1 + i)^3}$

$977.53 = \dfrac{1036.18}{1 + i}$

$1 + i = \dfrac{1036.18}{977.53}$

$i = \dfrac{1036.18}{977.53} - 1$

$i \doteq 0.0600 = 6.00\%$

c) As calculated in **b)**, Loan 1 was \$3650.00. Calculate the present value of Loan 2:

$PV = \dfrac{977.53}{1.06^2} = \$870.00$

d) (Loan 1) $A = 3650(0.04)(10) + 3650$

$= \$5110.00$

(Loan 2) $A = 870(1.06)^{10} = \$1558.04$

3. $25\,000 = P(1.023)^{32}$

$P = \$12\,075.91$

4. $i = 0.004$

$n = 78$

$R = 250$

$FV = R \times \left(\dfrac{(1 - (1 + i)^{-n})}{i}\right)$

$FV = 250 \times \dfrac{1.004^{78} - 1}{0.004} = \$22\,831.55$

5. Compare the one-year future values of the two options. The 5.88% option has 12 compounding periods in a year:

$FV_1 = R \times \dfrac{1.0049^{12} - 1}{0.0049}$

$\doteq 12.329R$

The 6.00% option has one compounding period in a year:

$FV_2 = R \times \dfrac{1.06 - 1}{0.06}$

$\doteq 12.245R$

Simone should choose the option with 5.88%/a compounded monthly.

6. Set up a spreadsheet to find an interest rate for which the sum of the future values is

$450 000 after 360 months. The rate 5.98%/a compounded monthly yields about $450 000:

◇	A	B	C
1	Investment Period	Investment	Future Value
2			0.0598
3	1	450	2680.67
4	2	450	2667.38
5	3	450	2654.15
6	4	450	2640.99
7	5	450	2627.89

...

357	355	450	461.32
358	356	450	459.04
359	357	450	456.76
360	358	450	454.50
361	359	450	452.24
362	360	450	450.00
363			450306.39

7. (First scenario) Calculate how much Yvette needs to support her $5000 withdrawals after 17 years:

$$PV = 5000 \times \frac{1 - 1.006^{-120}}{0.006} = \$426\,832.85$$

Now calculate how much she needs to invest per quarter for 17 years in order to accumulate that amount:

$$426\,832.85 = R \times \frac{1.021^{68} - 1}{0.021}$$
$$R = \$2882.95$$

(Second scenario) The second scenario is similar, except the interest rates are swapped. Calculate how much Yvette needs to support her $5000 withdrawals after 17 years:

$$PV = 5000 \times \frac{1 - 1.007^{-120}}{0.007} = \$405\,017.35$$

Now calculate how much she needs to invest per quarter for 17 years in order to accumulate that amount:

$$405\,017.35 = R \times \frac{1.018^{68} - 1}{0.018}$$
$$R = \$3083.96$$

Yvette needs to invest $201.01 more per quarter if she chooses the second scenario.

Chapters 7 and 8 Cumulative review, pp. 538–539

1. A. The series is an arithmetic series starting at 2.8 with the common difference 0.4. So the sum of the first 21 terms, S_{21}, is

$$S_{21} = \frac{21[5.6 + 20(0.4)]}{2} = 142.8$$

2. C. The sequence 2, 6, 7, 21, 22, ... is not a geometric sequence. There is no common ratio between terms.

3. C. The sequence is a geometric sequence starting at 1 with common ratio $-\frac{2}{3}$, so the 8th term, t_8, is

$$t_8 = 1\left(-\frac{2}{3}\right)^7 = -\frac{128}{2187}$$

4. A and C. Check which values of a and b satisfy both criteria:

(a, b)	Resulting sequence	First 3 terms, arithmetic sequence?	Last 3 terms, geometric sequence?
a) $(1, -6)$	$8, 1, -6, 36$	Yes; $d = -7$	Yes; $r = -6$
b) $(-1, 6)$	$8, -1, 6, 36$	No	No
c) $(16, 24)$	$8, 16, 24, 36$	Yes; $d = 8$	Yes; $r = 1.5$
d) $(12, 24)$	$8, 12, 24, 36$	No	No

5. C. The common ratio is 3:

$1215 = 405r$
$r = 3$

The first term, a, is 5:

$405 = a \times 3^4$
$a = 5$

So $S_9 = 49\,205$:

$$S_9 = \frac{5(3^9 - 1)}{2} = 49\,205$$

6. A. The first 6 terms are

Term	Value
1	-5
2	$-3(-5) + 8 = 23$
3	$-3(23) + 8 = -61$
4	$-3(-61) + 8 = 191$
5	$-3(191) + 8 = -565$
6	$-3(-565) + 8 = 1703$

7. A. Use the fifth row of – triangle to determine the coefficients:

$$\begin{aligned}(x - 3)^5 &= 1(x)^5 + 5(x)^4(-3)^1 \\ &\quad + 10(x)^3(-3)^2 + 10(x)^2(-3)^3 \\ &\quad + 5(x)^1(-3)^4 + 1(-3)^5 \\ &= x^5 - 15x4 + 90x^3 - 270x^2 \\ &\quad + 405x - 243\end{aligned}$$

8. B. Calculate the half-life, h:

$$0.1 = \left(\frac{1}{2}\right)^{\frac{15}{h}}$$

$$\frac{15}{h} \doteq 3.322$$

$$h \doteq 4.52 \text{ days}$$

9. A. If the original investment is P,

$$2P = P(1 + i)^7$$
$$2 = (1 + i)^7$$
$$1 + i \doteq 1.104$$
$$i \doteq 0.104 = 10.4\%/a$$

10. A. Using the formula for the future value of an investment earning compound interest,

$$6546.42 = 5000(1.005)^n$$
$$n \doteq 54 \text{ months} = 4.5 \text{ years}$$

11. B. Calculate how much Marisa will have at the end of 10 years if she invests her \$800 quarterly payments at 8%/a compounded quarterly.

$$FV = 800 \times \frac{1.02^{40} - 1}{0.02}$$
$$\doteq \$48\,321.59$$

50 000 − 48 321.59 = 1678.41, so Marisa would have \$1678.41 more if she took the \$50 000 lump-sum payment.

12. D. $12 \times 0.005 = 0.06 = 6\%$

13. C. Lee finances \$ 1894−150 = \$1744 with monthly payments of \$113. With an interest rate of 20.06%/a, the loan balance is near zero after 18 months:

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.2006		1744.00
3	1	113	29.15	83.85	1660.15
4	2	113	27.75	85.25	1574.91
5	3	113	26.33	86.67	1488.23
6	4	113	24.88	88.12	1400.11
7	5	113	23.41	89.59	1310.52
8	6	113	21.91	91.09	1219.42
9	7	113	20.38	92.62	1126.81
10	8	113	18.84	94.16	1032.65
11	9	113	17.26	95.74	936.91
12	10	113	15.66	97.34	839.57
13	11	113	14.03	98.97	740.60
14	12	113	12.38	100.52	639.99
15	13	113	10.70	102.30	537.68
16	14	113	8.99	104.01	433.67
17	15	113	7.25	105.75	327.92
18	16	113	5.48	107.52	220.40
19	17	113	3.68	109.32	111.09
20	18	113	1.86	111.14	−0.06

14. D. After the first four years, Mr. Los has \$16 229.35:

$$FV = 300 \times \frac{1.005^{48} - 1}{0.005} = \$16\,229.35$$

He borrows \$39 770.65:

56 000 − 16 229.35 = \$39 770.65

If he pays \$525 per month at 8%/a, the balance of the loan will be near zero after 106 months (8 years, 10 months):

◇	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Balance
2			0.0800		39770.65
3	1	525	265.14	259.86	39510.79
4	2	525	263.41	261.59	39249.19
5	3	525	261.66	263.34	38985.85
6	4	525	259.91	265.09	38720.76

. . .

104	102	525	16.61	508.39	1982.96
105	103	525	13.22	511.78	1471.18
106	104	525	9.81	515.78	955.99
107	105	525	6.37	518.63	437.36
108	106	525	2.92	522.08	−84.72

15. B. Increasing the periodic payment and decreasing the interest rate allows you to repay a loan in less time.

16. B. The regular payment on an amortized loan is the fixed periodic payment, which is made up of interest and principal.

17. a) $t_1 = 350$, $t_n = 0.32t_{n-1} + 350$

b) The medication levels off to 514.7 mg:

◇	A	B
1	Dose	mg
2		
3	1	0
4	2	350
5	3	462
6	4	497.84
7	5	509.3088
8	6	512.978816
9	7	514.1532211
10	8	514.5290308
11	9	514.6492898
12	10	514.6877727
13	11	514.7000873
14	12	514.7040279
15	13	514.7056925
16	14	514.7058216
17	15	514.7058629

c) The medication reaches close to 514.7 mg after 10 doses, which is 54 hours after the first dose.

18. a) Each month Mr. Cowan deposits \$25. The sequence 25, 25, 25, … is a geometric sequence with common ratio equal to 1. After n months, the value of the first deposit, which is \$25, is $t_n = 25(1.005)^{n-1}$.

b) There are 168 payments until Bart's 18th birthday:

$18 - 4 = 14$ and $14 \times 12 = 168$

After 168 payments, there will be 557.626 in the account:

$$FV = 25 \times \frac{1.005^{168} - 1}{0.005} = \$6557.62$$

c)

◇	A	B	C
1	Payment Number	Payment	Future Value of Each Payment
2	1	25	57.50
3	2	25	57.21
4	3	25	56.93
5	4	25	56.65
6	5	25	56.36
7	6	25	56.08

…

162	161	25	25.89
163	162	25	25.76
164	163	25	25.63
165	164	25	25.50
166	165	25	25.38
167	166	25	25.25
168	167	25	25.13
169	168	25	25.00
170			**6557.62**

d) Changing the monthly payment to \$50 increases the account value to \$13 115.24:

◇	A	B	C
1	Payment Number	Payment	Future Value of Each Payment
2	1	50	115.00
3	2	50	114.43
4	3	50	113.86
5	4	50	113.29
6	5	50	112.73
7	6	50	112.17

…

162	161	50	51.78
163	162	50	51.52
164	163	50	51.26
165	164	50	51.01
166	165	50	50.75
167	166	50	50.50
168	167	50	50.25
169	168	50	50.00
170			**13115.24**