Chapter 6
Introduction to Vectors

Review of Prerequisite Skills, p. 273

1. a. $\frac{\sqrt{3}}{2}$ **c.** $\frac{1}{2}$ **e.** $\frac{\sqrt{2}}{2}$

b. $-\sqrt{3}$ **d.** $\frac{\sqrt{3}}{2}$ **f.** 1

2. Find BC using the Pythagorean theorem, $AC^2 = AB^2 + BC^2$.

$$\begin{aligned} BC^2 &= AC^2 - AB^2 \\ &= 10^2 - 6^2 \\ &= 64 \\ BC &= 8 \end{aligned}$$

Next, use the ratio $\tan A = \frac{\text{opposite}}{\text{adjacent}}$.

$$\begin{aligned} \tan A &= \frac{BC}{AB} \\ &= \frac{8}{6} \\ &= \frac{4}{3} \end{aligned}$$

3. a. To solve ΔABC, find measures of the sides and angles whose values are not given: AB, $\angle B$, and $\angle C$. Find AB using the Pythagorean theorem, $BC^2 = AB^2 + AC^2$.

$$\begin{aligned} AB^2 &= BC^2 - AC^2 \\ &= (37.0)^2 - (22.0)^2 \\ &= 885 \\ AB &= \sqrt{885} \\ &\doteq 29.7 \end{aligned}$$

Find $\angle B$ using the ratio $\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$.

$$\begin{aligned} \sin B &= \frac{AC}{BC} \\ &= \frac{22.0}{37.0} \\ \angle B &\doteq 36.5° \\ \angle C &= 90° - \angle B \\ \angle C &= 90° - 36.5° \\ \angle C &\doteq 53.5° \end{aligned}$$

b. Find measures of the angles whose values are not given. Find $\angle A$ using the cosine law, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

$$\begin{aligned} &= \frac{5^2 + 8^2 - 10^2}{2(5)(8)} \\ &= \frac{-11}{80} \\ \angle A &\doteq 97.9° \end{aligned}$$

Find $\angle B$ using the sine law.

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin A}{a} \\ \frac{\sin B}{5} &= \frac{\sin(97.9°)}{10} \\ \sin B &\doteq 0.5 \\ \angle B &\doteq 29.7° \end{aligned}$$

Find $\angle C$ using the sine law.

$$\begin{aligned} \frac{\sin C}{c} &= \frac{\sin A}{a} \\ \frac{\sin C}{8} &= \frac{\sin(97.9°)}{10} \\ \sin C &\doteq 0.8 \\ \angle C &\doteq 52.4° \end{aligned}$$

4. Since the sum of the internal angles of a triangle equals 180°, determine the measure of $\angle Z$ using $\angle X = 60°$ and $\angle Y = 70°$.

$$\begin{aligned} \angle Z &= 180° - (\angle X + \angle Y) \\ &= 180° - (60° + 70°) \\ &= 50° \end{aligned}$$

Find XY and YZ using the sine law.

$$\begin{aligned} \frac{XY}{\sin Y} &= \frac{XY}{\sin Z} \\ \frac{XY}{\sin 70°} &= \frac{6}{\sin 50°} \\ XZ &\doteq 7.36 \\ \frac{YZ}{\sin X} &= \frac{XY}{\sin Z} \\ \frac{YZ}{\sin 60°} &= \frac{6}{\sin 50°} \\ YZ &\doteq 6.78 \end{aligned}$$

5. Find each angle using the cosine law.

$$\begin{aligned} \cos R &= \frac{RS^2 + RT^2 - ST^2}{2(RS)(RT)} \\ &= \frac{4^2 + 7^2 - 5^2}{2(4)(7)} \end{aligned}$$

$$= \frac{5}{7}$$

$$\angle R \doteq 44°$$

$$\cos S = \frac{RS^2 + ST^2 - RT^2}{2(RS)(ST)}$$

$$= \frac{4^2 + 5^2 - 7^2}{2(4)(5)}$$

$$= -\frac{1}{5}$$

$$\angle S \doteq 102°$$

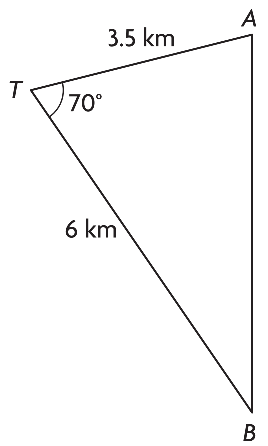
$$\cos T = \frac{RT^2 + ST^2 - RS^2}{2(RT)(ST)}$$

$$= \frac{7^2 + 5^2 - 4^2}{2(7)(5)}$$

$$= \frac{29}{35}$$

$$\angle T \doteq 34°$$

6.



Find AB (the distance between the airplanes) using the cosine law.

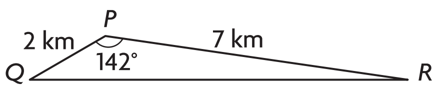
$$AB^2 = AT^2 + BT^2 - 2(AT)(BT)\cos T$$

$$= (3.5\text{ km})^2 + (6\text{ km})^2 - 2(3.5\text{ km})(6\text{ km})\cos 70°$$

$$\doteq 33.89\text{ km}^2$$

$$AB \doteq 5.82\text{ km}$$

7.



Find QR using the cosine law.

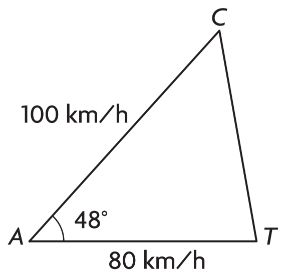
$$QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos P$$

$$= (2\text{ km})^2 + (7\text{ km})^2 - 2(2\text{ km})(7\text{ km})\cos 142°$$

$$\doteq 75.06\text{ km}^2$$

$$QR \doteq 8.66\text{ km}$$

8.



Find AC and AT using the speed of each vehicle and the elapsed time (in hours) until it was located, distance = speed × time.

$$AC = 100\text{ km/h} \times \frac{1}{4}\text{ h}$$

$$= 25\text{ km}$$

$$AT = 80\text{ km/h} \times \frac{1}{3}\text{ h}$$

$$= 26\frac{2}{3}\text{ km}$$

Find CT using the cosine law.

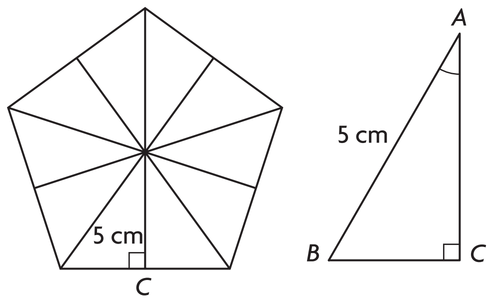
$$CT^2 = AC^2 + AT^2 - 2(AC)(AT)\cos A$$

$$= (25\text{ km})^2 + \left(26\frac{2}{3}\text{ km}\right)^2 - 2(25\text{ km})\left(26\frac{2}{3}\text{ km}\right)\cos 48°$$

$$\doteq 443.94\text{ km}^2$$

$$CT \doteq 21.1\text{ km}$$

9.



The pentagon can be divided into 10 congruent right triangles with height AC and base BC.

$$10 \times \angle A = 360°$$

$$\angle A = 36°$$

Find AC and BC using trigonometric ratios.

$$AC = AB \times \cos A$$

$$= 5\cos 36°$$

$$\doteq 4.0\text{ cm}$$

$$BC = AB \times \sin A$$

$$= 5\sin 36°$$

$$\doteq 2.9\text{ cm}$$

The area of the pentagon is the sum of the areas of the 10 right triangles. Use the area of $\triangle ABC$ to determine the area of the pentagon.

$$\text{Area}_{\text{pentagon}} = 10 \times \frac{1}{2}(BC)(AC)$$
$$= 10 \times \frac{1}{2}(2.9 \text{ cm})(4.0 \text{ cm})$$
$$= 59.4 \text{ cm}^2$$

6.1 An Introduction to Vectors, pp. 279–281

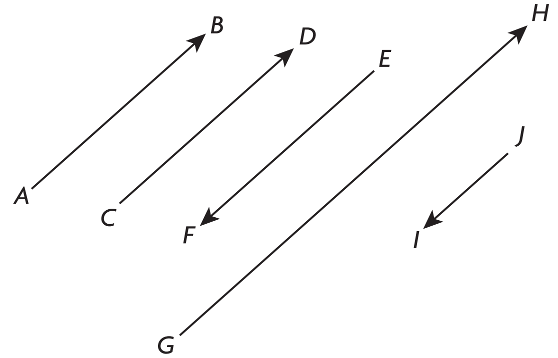
1. a. False. Two vectors with the same magnitude can have different directions, so they are not equal.
b. True. Equal vectors have the same direction and the same magnitude.
c. False. Equal or opposite vectors must be parallel and have the same magnitude. If two parallel vectors have different magnitude, they cannot be equal or opposite.
d. False. Equal or opposite vectors must be parallel and have the same magnitude. Two vectors with the same magnitude can have directions that are not parallel, so they are not equal or opposite.
2. Vectors must have a magnitude and direction. For some scalars, it is clear what is meant by just the number. Other scalars are related to the magnitude of a vector.

- Height is a scalar. Height is the distance (see below) from one end to the other end. No direction is given.
- Temperature is a scalar. Negative temperatures are below freezing, but this is not a direction.
- Weight is a vector. It is the force (see below) of gravity acting on your mass.
- Mass is a scalar. There is no direction given.
- Area is a scalar. It is the amount space inside a two-dimensional object. It does not have direction.
- Volume is a scalar. It is the amount of space inside a three-dimensional object. No direction is given.
- Distance is a scalar. The distance between two points does not have direction.
- Displacement is a vector. Its magnitude is related to the scalar distance, but it gives a direction.
- Speed is a scalar. It is the rate of change of distance (a scalar) with respect to time, but does not give a direction.
- Force is a vector. It is a push or pull in a certain direction.
- Velocity is a vector. It is the rate of change of displacement (a vector) with respect to time. Its magnitude is related to the scalar speed.

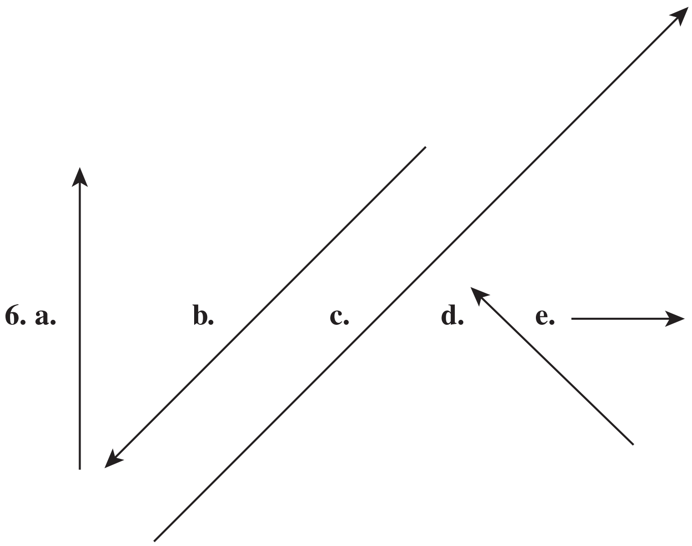
3. Answers may vary. For example: Friction resists the motion between two surfaces in contact by acting in the opposite direction of motion.

- A rolling ball stops due to friction which resists the direction of motion.
- A swinging pendulum stops due to friction resisting the swinging pendulum.

4. Answers may vary. For example:
a. $\overrightarrow{AD} = \overrightarrow{BC}$; $\overrightarrow{AB} = \overrightarrow{DC}$; $\overrightarrow{AE} = \overrightarrow{EC}$; $\overrightarrow{DE} = \overrightarrow{EB}$
b. $\overrightarrow{AD} = -\overrightarrow{CB}$; $\overrightarrow{AB} = -\overrightarrow{CD}$; $\overrightarrow{AE} = -\overrightarrow{CE}$; $\overrightarrow{ED} = -\overrightarrow{EB}$; $\overrightarrow{DA} = -\overrightarrow{BC}$
c. $\overrightarrow{AC}$ & $\overrightarrow{DB}$; $\overrightarrow{AE}$ & $\overrightarrow{EB}$; $\overrightarrow{EC}$ & $\overrightarrow{DE}$; $\overrightarrow{AB}$ & $\overrightarrow{CB}$
5.

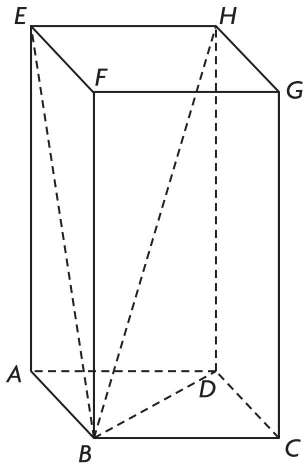


a. $\overrightarrow{AB} = \overrightarrow{CD}$
b. $\overrightarrow{AB} = -\overrightarrow{EF}$
c. $|\overrightarrow{AB}| = |\overrightarrow{EF}|$ but $\overrightarrow{AB} \neq \overrightarrow{EF}$
d. $\overrightarrow{GH} = 2\overrightarrow{AB}$
e. $\overrightarrow{AB} = -2\overrightarrow{JI}$



7. a. 100 km/h, south
b. 50 km/h, west
c. 100 km/h, northeast
d. 25 km/h, northwest
e. 60 km/h, east

8. a. 400 km/h, due south
b. 70 km/h, southwesterly
c. 30 km/h southeasterly
d. 25 km/h, due east
9. a. i. False. They have equal magnitude, but opposite direction.
ii. True. They have equal magnitude.
iii. True. The base has sides of equal length, so the vectors have equal magnitude.
iv. True. They have equal magnitude and direction.
b.



To calculate $|\overrightarrow{BD}|$, $|\overrightarrow{BE}|$ and $|\overrightarrow{BH}|$, find the lengths of their corresponding line segments BD, BE and BH using the Pythagorean theorem.

$$\begin{aligned} BD^2 &= AB^2 + AD^2 \\ &= 3^2 + 3^2 \\ BD &= \sqrt{18} \\ BE^2 &= AB^2 + AE^2 \\ &= 3^2 + 8^2 \\ BE &= \sqrt{73} \\ BH^2 &= BD^2 + DH^2 \\ &= (\sqrt{18})^2 + 8^2 \\ BH &= \sqrt{82} \end{aligned}$$

10. a. The tangent vector describes James's velocity at that moment. At point A his speed is 15 km/h and he is heading north. The tangent vector shows his velocity is 15 km/h, north.
b. The length of the vector represents the magnitude of James's velocity at that point. James's speed is the same as the magnitude of James's velocity.
c. The magnitude of James's velocity (his speed) is constant, but the direction of his velocity changes at every point.
d. Point C
e. This point is halfway between D and A, which is $\frac{7}{8}$ of the way around the circle. Since he is running 15 km/h and the track is 1 km in circumference, he can run around the track 15 times in one hour. That means each lap takes him 4 minutes. $\frac{7}{8}$ of 4 minutes is 3.5 minutes.
f. When he has travelled $\frac{3}{8}$ of a lap, James will be halfway between B and C and will be heading southwest.
11. a. Find the magnitude of $\overrightarrow{AB}$ using the distance formula.

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(x_A - x_B)^2 + (y_B - y_A)^2} \\ &= \sqrt{(-4 + 1)^2 + (3 - 2)^2} \\ &= \sqrt{10} \text{ or } 3.16 \end{aligned}$$

b. $\overrightarrow{CD} = \overrightarrow{AB}$. $\overrightarrow{AB}$ moves from $A(-4, 2)$ to $B(-1, 3)$ or $(x_B, y_B) = (x_A + 3, y_A + 1)$. Use this to find point D.

$$\begin{aligned} (x_D, y_D) &= (x_C + 3, y_C + 1) \\ &= (-6 + 3, 0 + 1) \\ &= (-3, 1) \end{aligned}$$

c. $\overrightarrow{EF} = \overrightarrow{AB}$. Find point E using

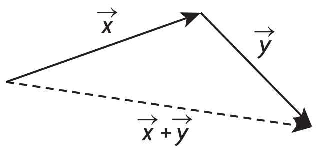
$$\begin{aligned} (x_A, y_A) &= (x_B - 3, y_B - 1). \\ (x_E, y_E) &= (x_F - 3, y_F - 1) \\ &= (3 - 3, -2 - 1) \\ &= (0, -3) \end{aligned}$$

d. $\overrightarrow{GH} = -\overrightarrow{AB}$, and moves in the opposite direction as $\overrightarrow{AB}$.

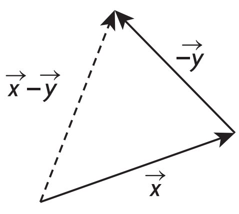
$$\begin{aligned} (x_H, y_H) &= (x_G - 3, y_G - 1). \\ (x_H, y_H) &= (x_G - 3, y_G - 1) \\ &= (3 - 3, 1 - 1) \\ &= (0, 0) \end{aligned}$$

6.2 Vector Addition, pp. 290–292

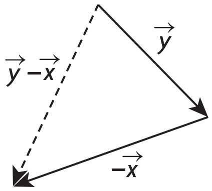
1. a.



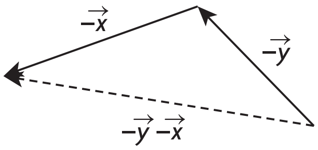
b.



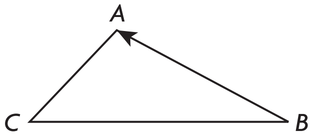
c.



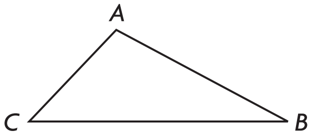
d.



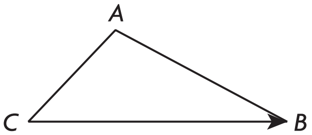
2. a. $\overrightarrow{BA}$



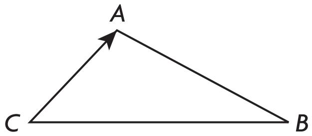
b. $\vec{0}$



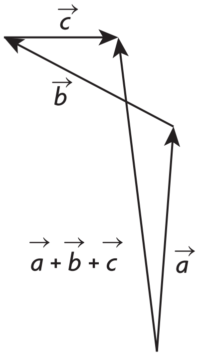
c. $\overrightarrow{CB}$



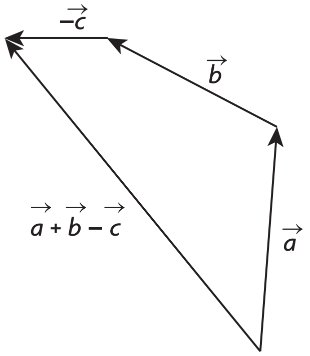
d. $\overrightarrow{CA}$



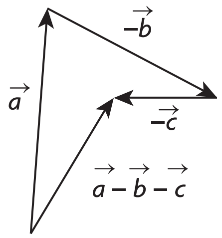
3. a.



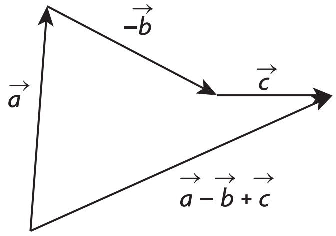
b.



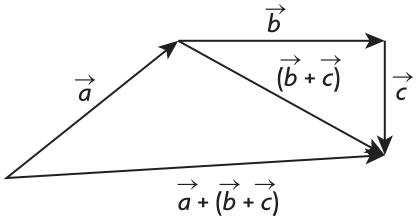
c.



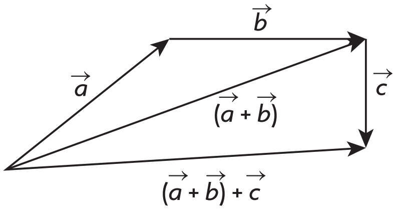
d.



4. a.

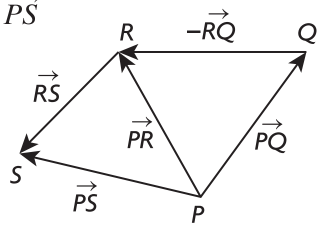


b.

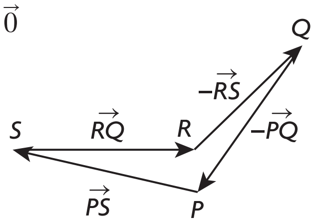


c. The resultant vectors are the same. The order in which you add vectors does not matter.
$\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)$

5. a. $\overrightarrow{PS}$



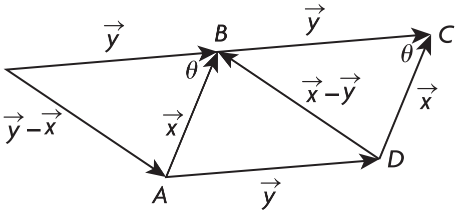
b. $\vec{0}$



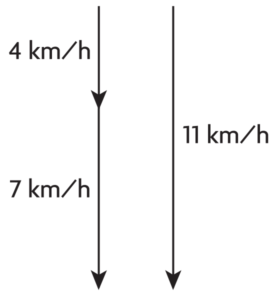
6. $\vec{x}+\vec{y}=\overrightarrow{MR}+\overrightarrow{RS}$
$=\overrightarrow{MS}$
$\vec{z}+\vec{t}=\overrightarrow{ST}+\overrightarrow{TQ}$
$=\overrightarrow{SQ}$

so
$(\vec{x}+\vec{y})+(\vec{z}+\vec{t})=\overrightarrow{MS}+\overrightarrow{SQ}$
$=\overrightarrow{MQ}$

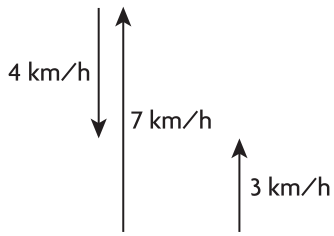
7. a. $-\vec{x}$
b. $\vec{y}$
c. $\vec{x} + \vec{y}$
d. $-\vec{x} + \vec{y}$
e. $\vec{x} + \vec{y} + \vec{z}$
f. $-\vec{x} - \vec{y}$
g. $-\vec{x} + \vec{y} + \vec{z}$
h. $-\vec{x} - \vec{z}$
8. a.



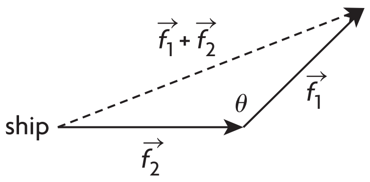
b. See the figure in part a. for the drawn vectors.
$|\vec{y} - \vec{x}|^2 = |\vec{y}|^2 + |\vec{x}|^2 - 2|\vec{y}||-\vec{x}| \cos(\theta)$ and $|-\vec{x}| = |\vec{x}|$, so $|\vec{y} - \vec{x}|^2 = |\vec{x} - \vec{y}|^2$
9. a. Maria's velocity is 11 km/h downstream.
b.



c.



Maria's speed is 3 km/h.
10. a.

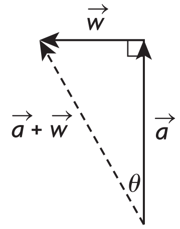


b. The vectors form a triangle with side lengths $|\vec{f}_1|$, $|\vec{f}_2|$ and $|\vec{f}_1 + \vec{f}_2|$. Find $|\vec{f}_1 + \vec{f}_2|$ using the cosine law.

$$|\vec{f}_1 + \vec{f}_2|^2 = |\vec{f}_1|^2 + |\vec{f}_2|^2 - 2|\vec{f}_1||\vec{f}_2| \cos(\theta)$$

$$|\vec{f}_1 + \vec{f}_2| = \sqrt{|\vec{f}_1|^2 + |\vec{f}_2|^2 - 2|\vec{f}_1||\vec{f}_2| \cos(\theta)}$$

11.



Find $|\vec{a} + \vec{w}|$ using the Pythagorean theorem.

$$\begin{aligned}|\vec{a} + \vec{w}|^2 &= |\vec{a}|^2 + |\vec{w}|^2 \\ &= (150\text{ km/h})^2 + (80\text{ km/h})^2 \\ &= 28900 \\ |\vec{a} + \vec{w}|^2 &= 170\end{aligned}$$

Find the direction of $\vec{a} + \vec{w}$ using the ratio

$$\tan(\theta) = \frac{|\vec{w}|}{|\vec{a}|}$$

$$\begin{aligned}\theta &= \tan^{-1} \frac{80\text{ km/h}}{150\text{ km/h}} \\ &\doteq \text{N } 28.1° \text{ W}\end{aligned}$$

$\vec{a} + \vec{w} = 170$ km/h, N 28.1° W
12. $\vec{x}$, $\vec{y}$, and $\vec{x} + \vec{y}$ form a right triangle. Find $|\vec{x} + \vec{y}|$ using the Pythagorean theorem.

$$\begin{aligned}|\vec{x} + \vec{y}|^2 &= |\vec{x}|^2 + |\vec{y}|^2 \\ &= 7^2 + 24^2 \\ &= 625 \\ |\vec{x} + \vec{y}| &= 25\end{aligned}$$

Find the angle between $\vec{x}$ and $\vec{x} + \vec{y}$ using the ratio

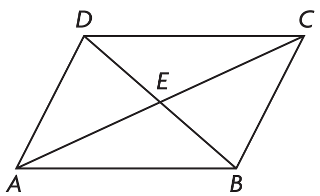
$$\tan(\theta) = \frac{|\vec{y}|}{|\vec{x}|}$$

$$\begin{aligned}\theta &= \tan^{-1} \frac{24}{7} \\ &\doteq 73.7°\end{aligned}$$

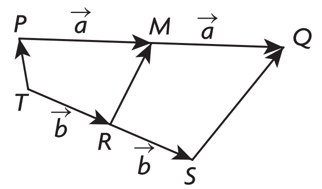
13. Find $|\overrightarrow{AB} + \overrightarrow{AC}|$ using the cosine law and the supplement to the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$$\begin{aligned}&|\overrightarrow{AB} + \overrightarrow{AC}|^2 \\ &= |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{AC}| \cos(30°) \\ &= 1^2 + 1^2 - 2(1)(1)\frac{\sqrt{3}}{2} \\ &= 2 - \sqrt{3} \\ &|\overrightarrow{AB} + \overrightarrow{AC}| \doteq 0.52\end{aligned}$$

14.



The diagonals of a parallelogram bisect each other.
So $\overrightarrow{EA} = -\overrightarrow{EC}$ and $\overrightarrow{ED} = -\overrightarrow{EB}$.
Therefore, $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \vec{0}$.

15.


Multiple applications of the Triangle Law for adding vectors show that

$\overrightarrow{RM} + \vec{b} = \vec{a} + \overrightarrow{TP}$ (since both are equal to the undrawn vector $\overrightarrow{TM}$), and that

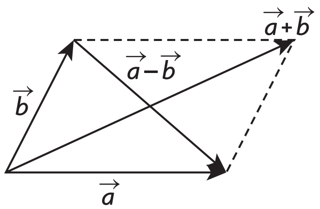
$\overrightarrow{RM} + \vec{a} = \vec{b} + \overrightarrow{SQ}$ (since both are equal to the undrawn vector $\overrightarrow{RQ}$)

Adding these two equations gives

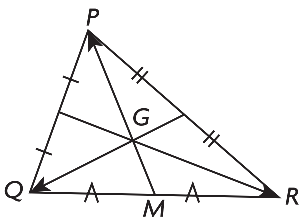
$2\,\overrightarrow{RM} + \vec{a} + \vec{b} = \vec{a} + \vec{b} + \overrightarrow{TP} + \overrightarrow{SQ}$

$2\,\overrightarrow{RM} = \overrightarrow{TP} + \overrightarrow{SQ}$

16. $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ represent the diagonals of a parallelogram with sides $\vec{a}$ and $\vec{b}$.



Since $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ and the only parallelogram with equal diagonals is a rectangle, the parallelogram must also be a rectangle.

17.


Let point M be defined as shown. Two applications of the Triangle Law for adding vectors show that

$\overrightarrow{GQ} + \overrightarrow{QM} + \overrightarrow{MG} = \vec{0}$

$\overrightarrow{GR} + \overrightarrow{RM} + \overrightarrow{MG} = \vec{0}$

Adding these two equations gives

$\overrightarrow{GQ} + \overrightarrow{QM} + 2\,\overrightarrow{MG} + \overrightarrow{GR} + \overrightarrow{RM} = \vec{0}$

From the given information,

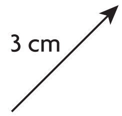
$2\,\overrightarrow{MG} = \overrightarrow{GP}$ and

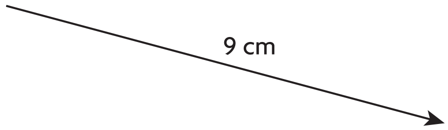
$\overrightarrow{QM} + \overrightarrow{RM} = \vec{0}$ (since they are opposing vectors of equal length), so

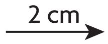
$\overrightarrow{GQ} + \overrightarrow{GP} + \overrightarrow{GR} = \vec{0}$, as desired.

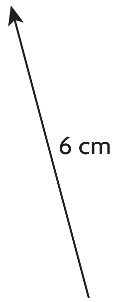
6.3 Multiplication of a Vector by a Scalar, pp. 298–301

1. A vector cannot equal a scalar.

2. a.


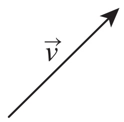
b.


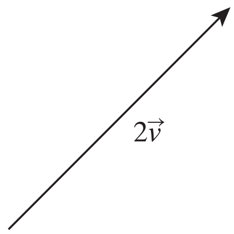
c.


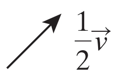
d.


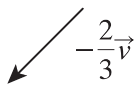
3. E25°N describes a direction that is 25° toward the north of due east (90° east of north), in other words $90° - 25° = 65°$ toward the east of due north. N65°E and “a bearing of 65°” both describe a direction that is 65° toward the east of due north. So, each is describing the same direction in a different way.

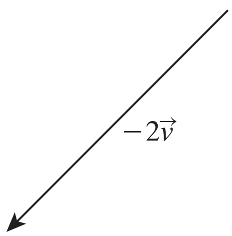
4. Answers may vary. For example:

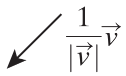


a.


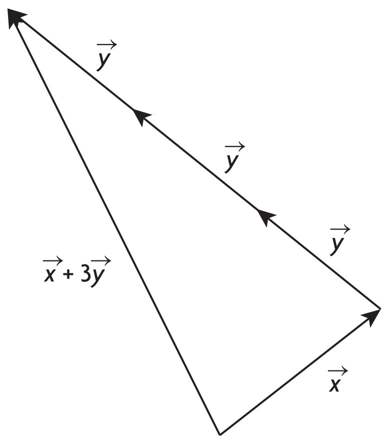
b.


c.


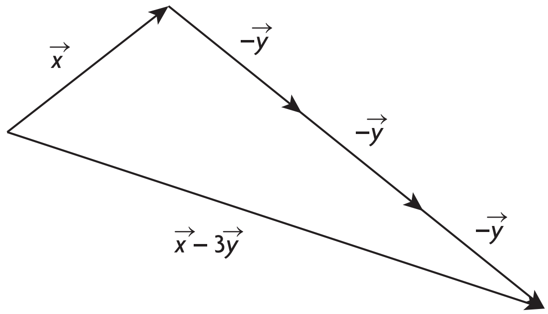
d.


e.


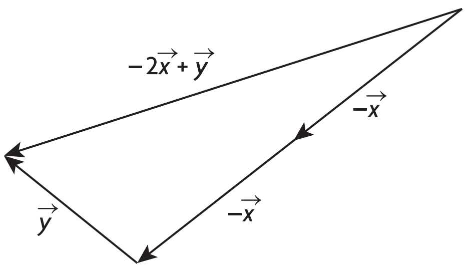
5. a.



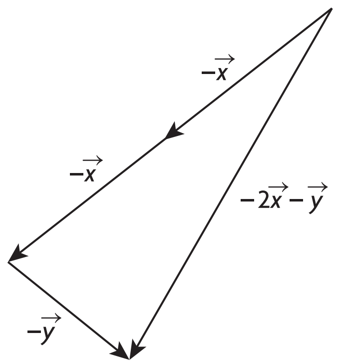
b.



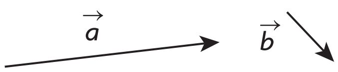
c.



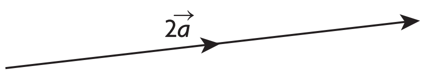
d.



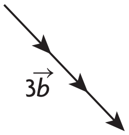
6. Answers may vary. For example:



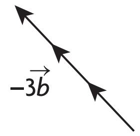
a.



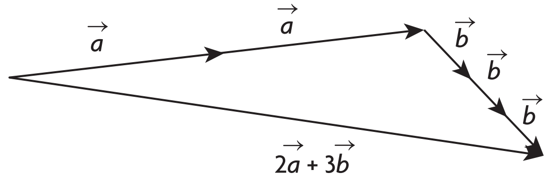
b.



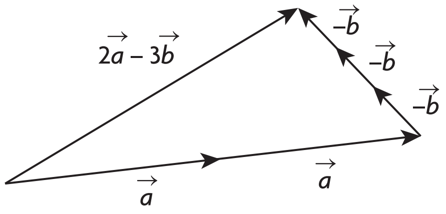
c.



d.



e.



7. a.

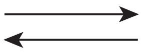
$$\vec{c} = 2\vec{a}, \vec{b} = \frac{3}{2}\vec{a}$$
$$m\vec{c} + n\vec{b} = \vec{0}$$
$$m(2\vec{a}) + n\left(\frac{3}{2}\vec{a}\right) = \vec{0}$$
$$m(4\vec{a}) + n\,(3\vec{a}) = \vec{0}$$

$m = 3$ and $n = -4$ satisfy the equation, as does any multiple of the pair $(3, -4)$. There are infinitely many values possible.

b.

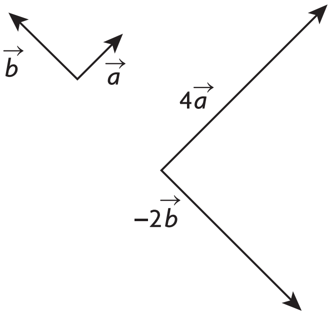
$$\vec{c} = 2\vec{a}, \vec{b} = \frac{3}{2}\vec{a}$$
$$d\vec{a} + e\vec{b} + f\vec{c} = \vec{0}$$
$$d\vec{a} + e\left(\frac{3}{2}\vec{a}\right) + f(2\vec{a}) = \vec{0}$$
$$2d\vec{a} + 3e\vec{a} + 4f\vec{a} = \vec{0}$$

$d = 2$, $e = 0$, and $f = -1$ satisfy the equation, as does any multiple of the triple $(2, 0, -1)$. There are infinitely many values possible.

8.  or 

$\vec{a}$ and $\vec{b}$ are collinear, so $\vec{a} = k\vec{b}$, where k is a nonzero scalar. Since $|\vec{a}| = |\vec{b}|$, k can only be -1 or 1.

9.



Yes

10. Two vectors are collinear if and only if they can be related by a scalar multiple. In this case $\vec{a} \neq k\vec{b}$

a. collinear

b. not collinear

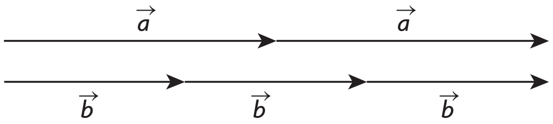
c. not collinear

d. collinear

11. a. $\frac{1}{|\vec{x}|}\vec{x}$ is a vector with length 1 unit in the same direction as $\vec{x}$.

b. $-\frac{1}{|\vec{x}|}\vec{x}$ is a vector with length 1 unit in the opposite direction of $\vec{x}$.

12.



$$m = \frac{2}{3}$$

13. a. $-\frac{2}{3}\vec{a}$

b. $\frac{1}{3}\vec{a}$

c. $\frac{1}{3}|\vec{a}|$

d. $\frac{2}{3}|\vec{a}|$

e. $\frac{4}{3}\vec{a}$

14. $\vec{x}$ and $\vec{y}$ make an angle of 90°, so you may find $|2\vec{x} + \vec{y}|$ using the Pythagorean theorem.

$$|2\vec{x} + \vec{y}|^2 = |2\vec{x}|^2 + |\vec{y}|^2$$
$$= 2^2 + 1^2$$
$$|2\vec{x} + \vec{y}| = \sqrt{5} \text{ or } 2.24$$

Find the direction of $2\vec{x} + \vec{y}$ using the ratio

$$\tan(\theta) = \frac{|\vec{y}|}{|2\vec{x}|}$$
$$\theta = \tan^{-1}\frac{1}{2}$$
$$\doteq 26.6° \text{ from } \vec{x} \text{ towards } 2\vec{x} + \vec{y}$$

15. Find $|2\vec{x} + \vec{y}|$ using the cosine law, and the supplement to the angle between $\vec{x}$ and $\vec{y}$.

$$|2\vec{x} + \vec{y}|^2 = |2\vec{x}|^2 + |\vec{y}|^2 - 2|2\vec{x}||\vec{y}|\cos(150°)$$
$$= 2^2 + 1^2 - 2(2)(1)\frac{-\sqrt{3}}{2}$$
$$|2\vec{x} + \vec{y}| \doteq 2.91$$

Find the direction of $2\vec{x} + \vec{y}$ using the sine law.

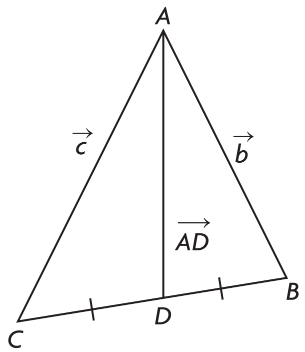
$$\frac{\sin\theta}{|\vec{y}|} = \frac{\sin(150°)}{|2\vec{x} + \vec{y}|}$$
$$\sin\theta \doteq (1)\frac{\frac{1}{2}}{2.91}$$
$$\theta \doteq 9.9° \text{ from } \vec{x} \text{ towards } \vec{y}$$

16. $\vec{b} = \frac{1}{|\vec{a}|}\vec{a}$

$$|\vec{b}| = \left|\frac{1}{|\vec{a}|}\vec{a}\right|$$
$$|\vec{b}| = \frac{1}{|\vec{a}|}|\vec{a}|$$
$$|\vec{b}| = 1$$

$\vec{b}$ is a positive multiple of $\vec{a}$, so it points in the same direction as $\vec{a}$ and has magnitude 1. It is a unit vector in the same direction as $\vec{a}$.

17.



$$\overrightarrow{AD} = \vec{c} + \overrightarrow{CD}$$
$$\overrightarrow{AD} = \vec{b} + \overrightarrow{BD}$$
$$2\overrightarrow{AD} = \vec{c} + \vec{b} + \overrightarrow{CD} + \overrightarrow{BD}$$

But $\overrightarrow{CD} + \overrightarrow{BD} = 0$.

So $2\overrightarrow{AD} = \vec{c} + \vec{b}$, or $\overrightarrow{AD} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{b}$.

18. $\overrightarrow{PM} = \vec{a}$ and $\overrightarrow{PN} = \vec{b}$

so $\overrightarrow{MN} = \overrightarrow{PN} - \overrightarrow{PM}$

$= \vec{b} - \vec{a}$

$\overrightarrow{PQ} = 2\vec{a}$ and $\overrightarrow{PR} = 2\vec{b}$

so $\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$

$= 2\vec{b} - 2\vec{a}$

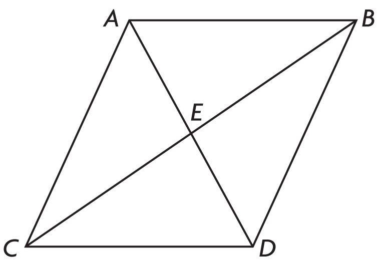
Notice that

$2\overrightarrow{MN} = 2\vec{b} - 2\vec{a}$

$= \overrightarrow{QR}$

We can conclude that $\overrightarrow{QR}$ is parallel to $\overrightarrow{MN}$ and $|\overrightarrow{QR}| = 2|\overrightarrow{MN}|$.

19.



Answers may vary. For example:

a. $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{CD}$

b. $\vec{u} = \overrightarrow{AD}$ and $\vec{v} = \overrightarrow{AE}$

c. $\vec{u} = \overrightarrow{AC}$ and $\vec{v} = \overrightarrow{DB}$

d. $\vec{u} = \overrightarrow{ED}$ and $\vec{v} = \overrightarrow{AD}$

20. a. Since the magnitude of $\vec{x}$ is three times the magnitude of $\vec{y}$ and because the given sum is 0, $m\vec{x}$ must be in the opposite direction of $n\vec{y}$ and $|n| = 3|m|$.

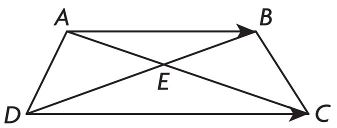
b. Whether $\vec{x}$ and $\vec{y}$ are collinear or not, $m = 0$ and $n = 0$ will make the given equation true.

21. a. $\overrightarrow{CD} = \vec{b} - \vec{a}$

b.
$$\begin{aligned}\overrightarrow{BE} &= 2\vec{b} - 2\vec{a}\\ &= 2(\vec{b} - \vec{a})\\ &= 2\overrightarrow{CD}\end{aligned}$$

The two are therefore parallel (collinear) and $\left|\overrightarrow{BE}\right| = 2\left|\overrightarrow{CD}\right|$

22.



Applying the triangle law for adding vectors shows that

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

The given information states that

$$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{DC}$$

$$\frac{3}{2}\overrightarrow{AB} = \overrightarrow{DC}$$

By the properties of trapezoids, this gives

$\frac{3}{2}\overrightarrow{AE} = \overrightarrow{EC}$, and since

$\overrightarrow{AC} = \overrightarrow{AE} + \overrightarrow{EC}$, the original equation gives

$$\begin{aligned}\overrightarrow{AE} + \frac{3}{2}\overrightarrow{AE} &= \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}\\ \frac{5}{2}\overrightarrow{AE} &= \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}\\ \overrightarrow{AE} &= \frac{2}{5}\overrightarrow{AD} + \frac{3}{5}\overrightarrow{AB}\end{aligned}$$

6.4 Properties of Vectors, pp. 306–307

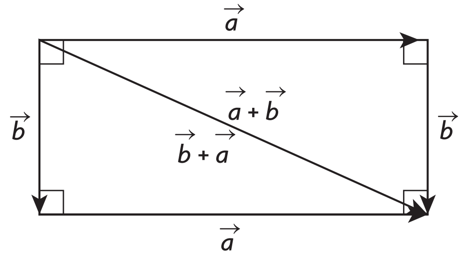
1. a. 0

b. 1

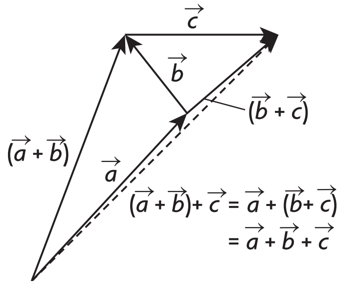
c. $\vec{0}$

d. 1

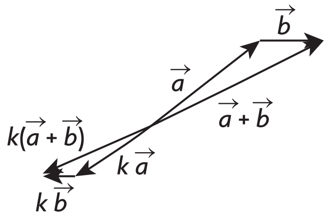
2. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



3.



4. Answers may vary. For example:



5.
$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{RQ} + \overrightarrow{SR} + \overrightarrow{TS} + \overrightarrow{PT}\\ &= (\overrightarrow{RQ} + \overrightarrow{SR}) + (\overrightarrow{TS} + \overrightarrow{PT})\\ &= (\overrightarrow{SR} + \overrightarrow{RQ}) + (\overrightarrow{PT} + \overrightarrow{TS})\\ &= \overrightarrow{SQ} + \overrightarrow{PS}\\ &= \overrightarrow{PS} + \overrightarrow{SQ}\\ &= \overrightarrow{PQ}\end{aligned}$$

6. a. $\overrightarrow{EC}$

b. $\vec{0}$

c. Yes, the diagonals of a rectangular prism are of equal length

7.
$$\begin{aligned}&= 3\vec{a} - 6\vec{b} - 15\vec{c} - 6\vec{a} + 12\vec{b} - 6\vec{c} - \vec{a}\\ &\quad + 3\vec{b} - 3\vec{c}\\ &= -4\vec{a} + 9\vec{b} - 24\vec{c}\end{aligned}$$

8. a.
$$\begin{aligned}&= 6\vec{i} - 8\vec{j} + 2\vec{k} + 6\vec{i} - 9\vec{j} + 3\vec{k}\\ &= 12\vec{i} - 17\vec{j} + 5\vec{k}\end{aligned}$$

b.
$$\begin{aligned}&= 3\vec{i} - 4\vec{j} + \vec{k} - 10\vec{i} + 15\vec{j} - 5\vec{k}\\ &= -7\vec{i} + 11\vec{j} - 4\vec{k}\end{aligned}$$

c.
$$\begin{aligned}&= 2(3\vec{i} - 4\vec{j} + \vec{k} + 6\vec{i} - 9\vec{j} + 3\vec{k})\\ &\quad -3(-6\vec{i} + 8\vec{j} - 2\vec{k} + 14\vec{i} - 21\vec{j} + 7\vec{k})\\ &= -6\vec{i} + 13\vec{j} - 7\vec{k}\end{aligned}$$

9. Solve the first equation for $\vec{x}$.

$$\vec{x} = \frac{1}{2}\vec{a} - \frac{3}{2}\vec{y}$$

Substitute into the second equation.

$$6\vec{b} = -\left(\frac{1}{2}\vec{a} - \frac{3}{2}\vec{y}\right) + 5\vec{y}$$

$$\vec{y} = \frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}$$

Lastly, find $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.

$$\vec{x} = \frac{1}{2}\vec{a} - \frac{3}{2}\left(\frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}\right)$$

$$= \frac{5}{13}\vec{a} - \frac{18}{13}\vec{b}$$

10. $\vec{a} = \vec{x} - \vec{y}$

$$= \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z} - (\vec{b} + \vec{z})$$

$$= \frac{2}{3}\vec{y} - \frac{2}{3}\vec{z} - \vec{b}$$

$$= \frac{2}{3}(\vec{y} - \vec{z}) - \vec{b}$$

$$= \frac{2}{3}\vec{b} - \vec{b}$$

$$= -\frac{1}{3}\vec{b}$$

11. a. $\overrightarrow{AG} = \vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{BH} = -\vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{CE} = -\vec{a} - \vec{b} + \vec{c}$

$\overrightarrow{DF} = \vec{a} - \vec{b} + \vec{c}$

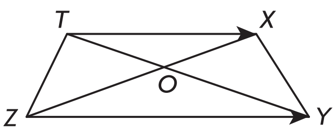
b. $|\overrightarrow{AG}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= |-\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= |\overrightarrow{BH}|^2$

Therefore, $|\overrightarrow{AG}| = |\overrightarrow{BH}|$

12.



Applying the triangle law for adding vectors shows that

$\overrightarrow{TY} = \overrightarrow{TZ} + \overrightarrow{ZY}$

The given information states that

$\overrightarrow{TX} = 2\,\overrightarrow{ZY}$

$\frac{1}{2}\overrightarrow{TX} = \overrightarrow{ZY}$

By the properties of trapezoids, this gives $\frac{1}{2}\overrightarrow{TO} = \overrightarrow{OY}$, and since $\overrightarrow{TY} = \overrightarrow{TO} + \overrightarrow{OY}$, the original equation gives

$$\overrightarrow{TO} + \frac{1}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$$

$$\frac{3}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$$

$$\overrightarrow{TO} = \frac{2}{3}\overrightarrow{TZ} + \frac{1}{3}\overrightarrow{TX}$$

Mid-Chapter Review, pp. 308–309

1. a. $\overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{BA} = \overrightarrow{CD}$

$\overrightarrow{AD} = \overrightarrow{BC}$

$\overrightarrow{CB} = \overrightarrow{DA}$

There is not enough information to determine if there is a vector equal to $\overrightarrow{AP}$.

b. $|\overrightarrow{PD}| = |\overrightarrow{DA}|$

$= |\overrightarrow{BC}|$ (parallelogram)

2. a. $\overrightarrow{RV}$

b. $\overrightarrow{RV}$

c. $\overrightarrow{PS}$

d. $\overrightarrow{RU}$

e. $\overrightarrow{PS}$

f. $\overrightarrow{PQ}$

3. a. Find $|\vec{a} + \vec{b}|$ using the cosine law, and the supplement to the angle between the vectors.

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos 60°$

$= 3^2 + 4^2 - 2(3)(4)\frac{1}{2}$

$= 3$

$|\vec{a} + \vec{b}| = \sqrt{3}$

b. Find θ using the ratio

$$\tan\theta = \frac{|\vec{b}|}{|\vec{a}|}$$

$$= \frac{4}{3}$$

$$\theta = \tan^{-1}\frac{4}{3}$$

$$\doteq 53°$$

4. $t = 4$ or $t = -4$

5. In quadrilateral $PQRS$, look at $\triangle PQR$. Joining the midpoints B and C creates a vector $\overrightarrow{BC}$ that is parallel to $\overrightarrow{PR}$ and half the length of $\overrightarrow{PR}$. Look at $\triangle SPR$. Joining the midpoints A and D creates a vector $\overrightarrow{AD}$ that is parallel to $\overrightarrow{PR}$ and half the length of $\overrightarrow{PR}$. $\overrightarrow{BC}$ is parallel to $\overrightarrow{AD}$ and equal in length to $\overrightarrow{AD}$. Therefore, $ABCD$ is a parallelogram.

6. a. Find $|\vec{u} - \vec{v}|$ using the cosine law. Note $|-\vec{v}| = |\vec{v}|$ and the angle between $\vec{u}$ and $-\vec{v}$ is 120°.

$|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |-\vec{v}|^2 - 2|\vec{u}||-\vec{v}| \cos 60°$

$$= 8^2 + 10^2 - 2(8)(10)\left(\frac{1}{2}\right)$$

$$|\vec{u} - \vec{v}| = 2\sqrt{21}$$

b. Find the direction of $\vec{u} - \vec{v}$ using the sine law.

$$\frac{\sin\theta}{|-\vec{v}|} = \frac{\sin 60°}{|\vec{u} - \vec{v}|}$$

$$\sin\theta = \frac{5}{\sqrt{21}}\sin 60°$$

$$\theta = \sin^{-1}\frac{5}{\sqrt{28}}$$

$$\doteq 71°$$

c. $\frac{1}{|\vec{u} + \vec{v}|}(\vec{u} + \vec{v}) = \frac{1}{2\sqrt{21}}(\vec{u} + \vec{v})$

d. Find $|5\vec{u} + 2\vec{v}|$ using the cosine law.

$$|5\vec{u} + 2\vec{v}|^2 = |5\vec{u}|^2 + |2\vec{v}|^2 - 2|5\vec{u}||2\vec{v}|\cos 120°$$

$$= 40^2 + 20^2 - 2(40)(20)\left(-\frac{1}{2}\right)$$

$$|5\vec{u} + 2\vec{v}| = 20\sqrt{7}$$

7. Find $|2\vec{p} - \vec{q}|$ using the cosine law.

$$|2\vec{p} - \vec{q}|^2 = |2\vec{p}|^2 + |-\vec{q}|^2 - 2|2\vec{p}||-\vec{q}|\cos 60°$$

$$= 2^2 + 1^2 - 2(2)(1)\left(\frac{1}{2}\right) = 3$$

8. $|\vec{m} + \vec{n}| = |\vec{m}| - |\vec{n}|$

9. $\overrightarrow{BC} = -\vec{y}$

$\overrightarrow{DC} = \vec{x}$

$\overrightarrow{BD} = -\vec{x} - \vec{y}$

$\overrightarrow{AC} = \vec{x} - \vec{y}$

10. Construct a parallelogram with sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$. Since the diagonals bisect each other, $2\overrightarrow{OB}$ is the diagonal equal to $\overrightarrow{OA} + \overrightarrow{OC}$. Or $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ and $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$. So, $\overrightarrow{OB} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$. And $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$. Now $\overrightarrow{OB} = \overrightarrow{OA} + \frac{1}{2}\left(\overrightarrow{OC} - \overrightarrow{OA}\right)$ Multiplying by 2 gives $2\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$.

11. $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$

$3\vec{x} - \vec{y} + 2\vec{y} = \overrightarrow{AD}$

$3\vec{x} + \vec{y} = \overrightarrow{AD}$

$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\vec{x} + \overrightarrow{BD} = 3\vec{x} + \vec{y}$

$\overrightarrow{BD} = 2\vec{x} + \vec{y}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\vec{x} + \overrightarrow{BC} = 3\vec{x} - \vec{y}$

$\overrightarrow{BC} = 2\vec{x} - \vec{y}$

12. The air velocity of the airplane ($\vec{V}_{\text{air}}$) and the wind velocity ($\vec{W}$) have opposite directions.

$\vec{V}_{\text{ground}} = \vec{V}_{\text{air}} - \vec{W}$

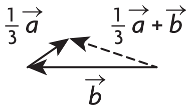
$= 460$ km/h due south

13. a. $\overrightarrow{PT}$

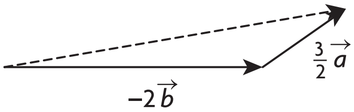
b. $\overrightarrow{PT}$

c. $\overrightarrow{SR}$

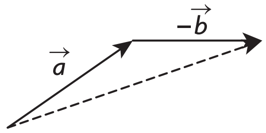
14. a.



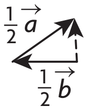
b.



c.



d.



15. $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$

$= 3\vec{b} - \vec{a}$

$\overrightarrow{RS} = \overrightarrow{QS} - \overrightarrow{QR}$

$= -3\vec{a}$

6.5 Vectors in R^2 and R^3, pp. 316–318

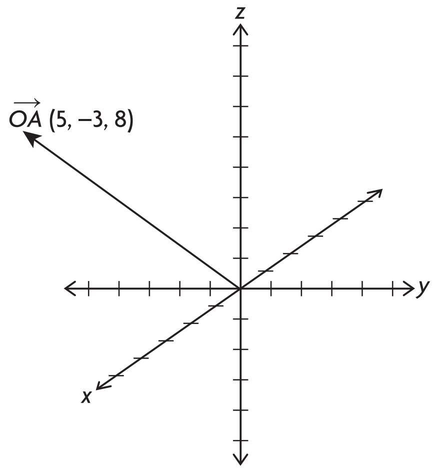
1. No, as the y-coordinate is not a real number.

2. a. We first arrange the x-, y-, and z-axes (each a copy of the real line) in a way so that each pair of axes are perpendicular to each other (i.e., the x- and y-axes are arranged in their usual way to form the xy-plane, and the z-axis passes through the origin of the xy-plane and is perpendicular to this plane). This is easiest viewed as a "right-handed system," where, from the viewer's perspective, the positive z-axis points upward, the positive x-axis points out of the page, and the positive y-axis points rightward in the plane of the page. Then, given point $P(a, b, c)$, we locate this point's unique position by moving a units along the x-axis, then from there b units parallel to the y-axis, and finally c units parallel to the z-axis. It's associated unique position vector is determined by drawing a vector with tail at the origin $O(0, 0, 0)$ and head at P.

b. Since this position vector is unique, its coordinates are unique. Therefore $a = -4$, $b = -3$, and $c = -8$.

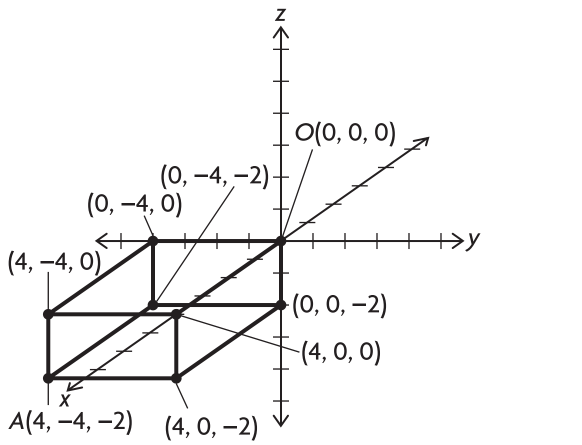
3. a. Since A and B are really the same point, we can equate their coordinates. Therefore $a = 5$, $b = -3$, and $c = 8$.

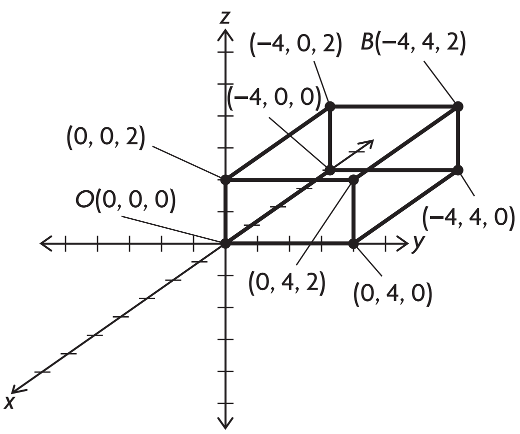
b. From part **a.**, $A(5, -3, 8)$, so $\overrightarrow{OA} = (5, -3, 8)$. Here is a depiction of this vector.

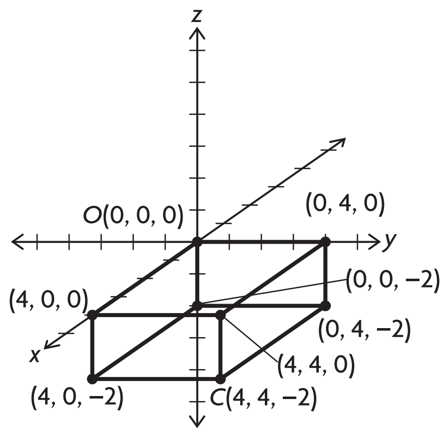


4. This is not an acceptable vector in I^3 as the z-coordinate is not an integer. However, since all of the coordinates are real numbers, this is acceptable as a vector in R^3.

5.







6. a. $A(0, -1, 0)$ is located on the y-axis. $B(0, -2, 0)$, $C(0, 2, 0)$, and $D(0, 10, 0)$ are three other points on this axis.

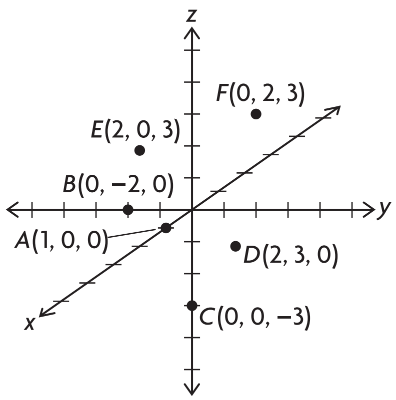
b. $\overrightarrow{OA} = (0, -1, 0)$, the vector with tail at the origin $O(0, 0, 0)$ and head at A.

7. a. Answers may vary. For example: $\overrightarrow{OA} = (0, 0, 1)$, $\overrightarrow{OB} = (0, 0, -1)$, $\overrightarrow{OC} = (0, 0, -5)$

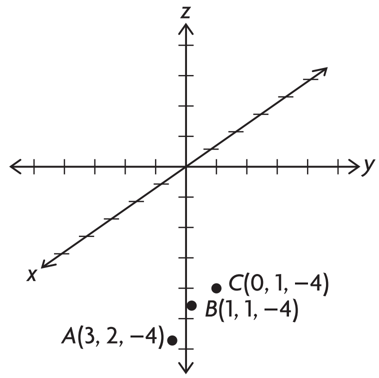
b. Yes, these vectors are collinear (parallel), as they all lie on the same line, in this case the z-axis.

c. A general vector lying on the z-axis would be of the form $\overrightarrow{OA} = (0, 0, a)$ for any real number a. Therefore, this vector would be represented by placing the tail at O, and the head at the point $(0, 0, a)$ on the z-axis.

8.

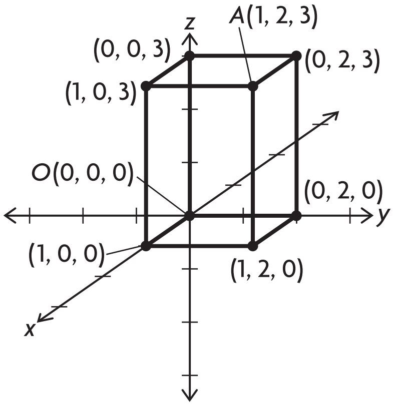


9. a.

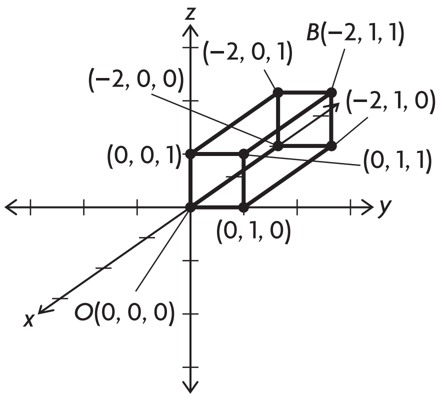


b. Every point on the plane containing points A, B, and C has z-coordinate equal to -4. Therefore, the equation of the plane containing these points is $z = -4$ (a plane parallel to the xy-plane through the point $z = -4$).

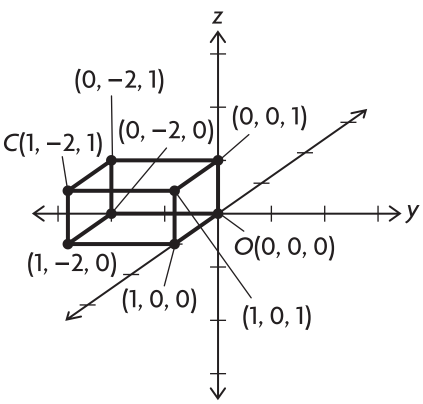
10. a.



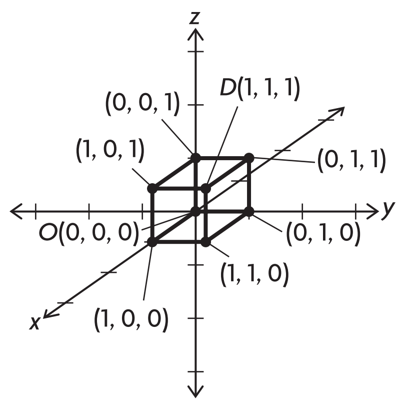
b.



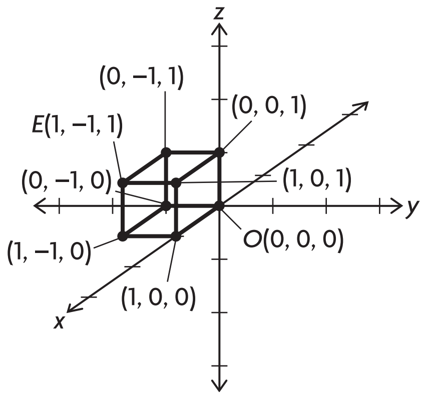
c.



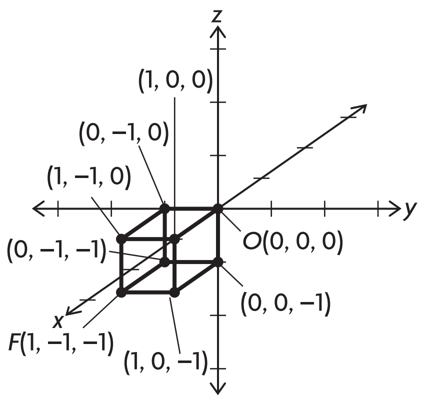
d.



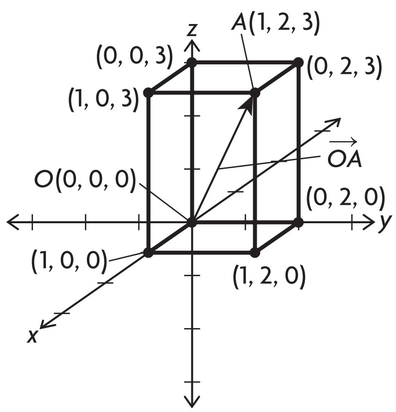
e.



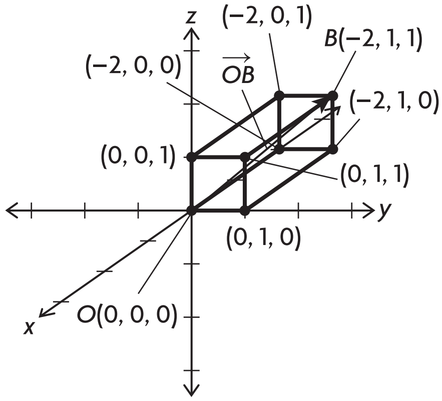
f.



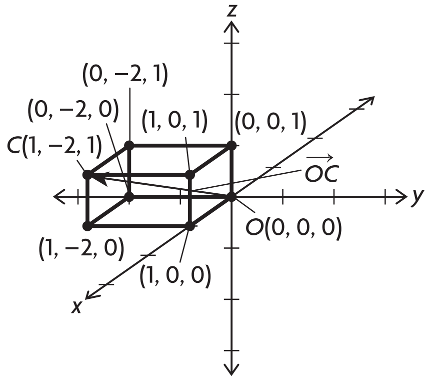
11. a. $\overrightarrow{OA} = (1, 2, 3)$



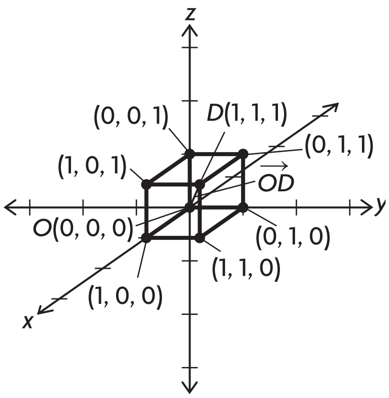
b. $\overrightarrow{OB} = (-2, 1, 1)$



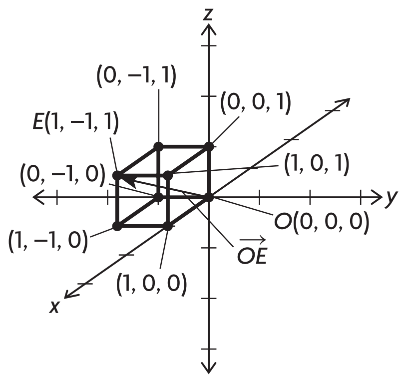
c. $\overrightarrow{OC} = (1, -2, 1)$



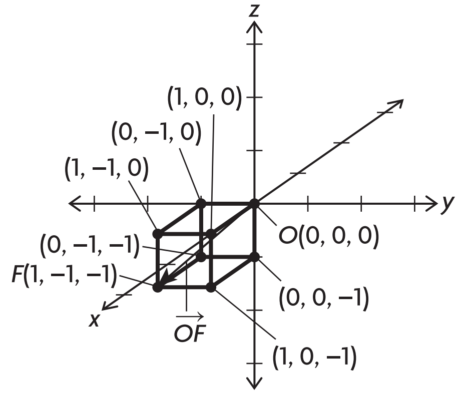
d. $\overrightarrow{OD} = (1, 1, 1)$



e. $\overrightarrow{OE} = (1, -1, 1)$



f. $\overrightarrow{OF} = (1, -1, -1)$



12. a. Since P and Q represent the same point, we can equate their y- and z-coordinates to get the system of equations

$$a - c = 6$$
$$a = 11$$

Substituting this second equation into the first gives

$$11 - c = 6$$
$$c = 5$$

So $a = 11$ and $c = 5$.

b. Since P and Q represent the same point in R^3, they will have the same associated position vector, i.e. $\overrightarrow{OP} = \overrightarrow{OQ}$. So, since these vectors are equal, they will certainly have equal magnitudes, i.e. $|\overrightarrow{OP}| = |\overrightarrow{OQ}|$.

13. $P(x, y, 0)$ represents a general point on the xy-plane, since the z-coordinate is 0. Similarly, $Q(x, 0, z)$ represents a general point in the xz-plane, and $R(0, y, z)$ represents a general point in the yz-plane.

14. a. Every point on the plane containing points M, N, and P has y-coordinate equal to 0. Therefore, the equation of the plane containing these points is $y = 0$ (this is just the xz-plane).

b. The plane $y = 0$ contains the origin $O(0, 0, 0)$, and so since it also contains the points M, N, and P as well, it will contain the position vectors associated with these points joining O (tail) to the given point (head). That is, the plane $y = 0$ contains the vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$, and $\overrightarrow{OP}$.

15. a. $A(-2, 0, 0)$, $B(-2, 4, 0)$, $C(0, 4, 0)$, $D(0, 0, -7)$, $E(0, 4, -7)$, $F(-2, 0, -7)$

b. $\overrightarrow{OA} = (-2, 0, 0)$, $\overrightarrow{OB} = (-2, 4, 0)$,

$\overrightarrow{OC} = (0, 4, 0)$, $\overrightarrow{OD} = (0, 0, -7)$,

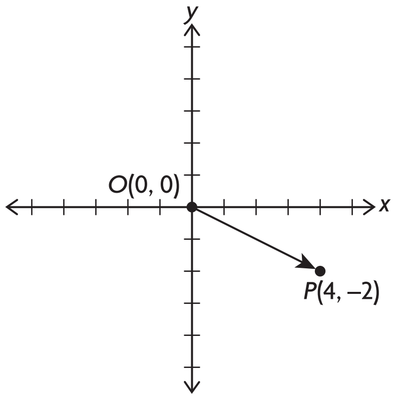
$\overrightarrow{OE} = (0, 4, -7)$, $\overrightarrow{OF} = (-2, 0, -7)$

c. Rectangle $DEPF$ is 7 units below the xy-plane.

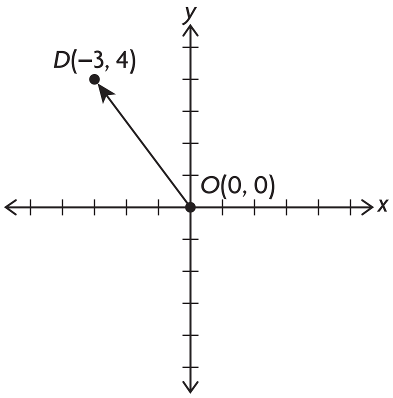
d. Every point on the plane containing points B, C, E, and P has y-coordinate equal to 4. Therefore, the equation of the plane containing these points is $y = 4$ (a plane parallel to the xz-plane through the point $y = 4$).

e. Every point contained in rectangle $BCEP$ has y-coordinate equal to 4, and so is of the form $(x, 4, z)$ where x and z are real numbers such that $-2 \le x \le 0$ and $-7 \le z \le 0$.

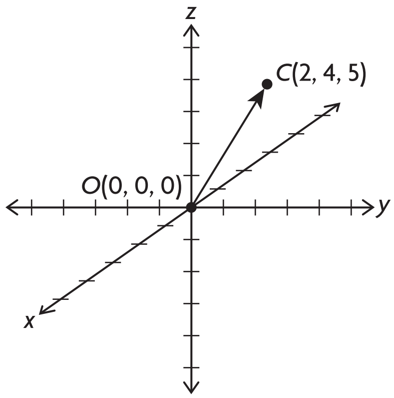
16. a.



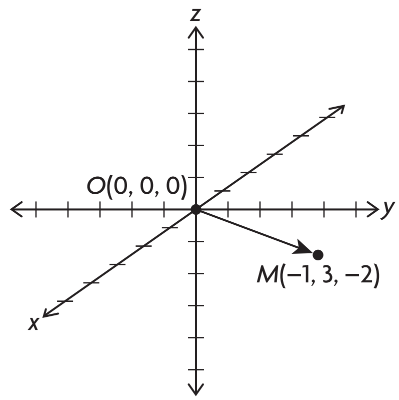
b.



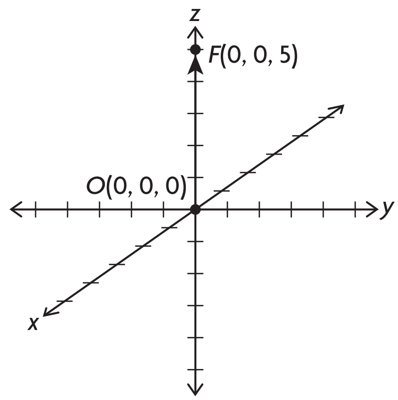
c.



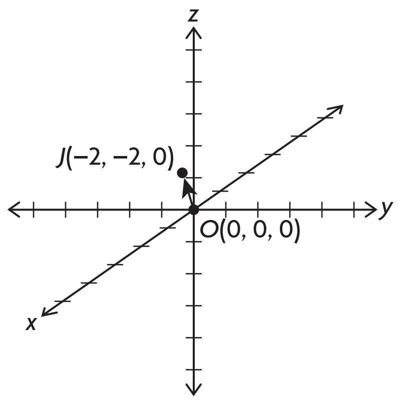
d.



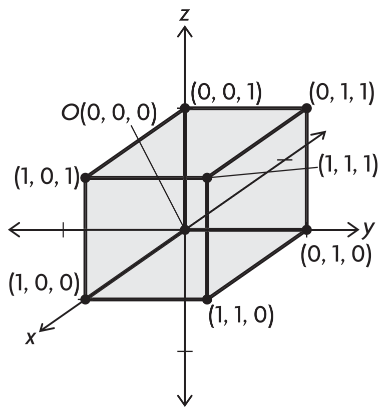
e.



f.



17. The following box illustrates the three dimensional solid consisting of the set of all points (x, y, z) such that $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$.



18. First, $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$ by the triangle law of vector addition, where $\overrightarrow{OA} = (5, -10, 0)$, $\overrightarrow{OB} = (0, 0, -10)$, $\overrightarrow{OP}$ and $\overrightarrow{OA}$ are drawn in standard position (starting from the origin $O(0, 0, 0)$), and $\overrightarrow{OB}$ is drawn starting from the head of $\overrightarrow{OA}$. Notice that $\overrightarrow{OA}$ lies in the *xy*-plane, and $\overrightarrow{OB}$ is perpendicular to the *xy*-plane (so is perpendicular to $\overrightarrow{OA}$). So, $\overrightarrow{OP}$, $\overrightarrow{OA}$, and $\overrightarrow{OB}$ form a right triangle and, by the Pythagorean theorem, $|\overrightarrow{OP}|^2 = |\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2$
Similarly, $\overrightarrow{OA} = \vec{a} + \vec{b}$ by the triangle law of vector addition, where $\vec{a} = (5, 0, 0)$ and $\vec{b} = (0, -10, 0)$, and these three vectors form a right triangle as well. So,

$$\begin{aligned} |\overrightarrow{OA}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 \\ &= 25 + 100 \\ &= 125 \end{aligned}$$

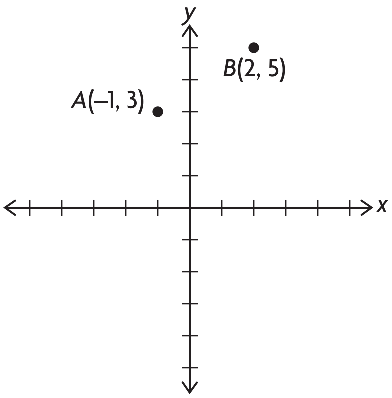
Obviously $|\overrightarrow{OB}|^2 = 100$, and so substituting gives

$$\begin{aligned} |\overrightarrow{OP}|^2 &= |\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2 \\ &= 125 + 100 \\ &= 225 \\ |\overrightarrow{OP}| &= \sqrt{225} \\ &= 15 \end{aligned}$$

19. To find a vector $\overrightarrow{AB}$ equivalent to $\overrightarrow{OP} = (-2, 3, 6)$, where $B(4, -2, 8)$, we need to move 2 units to the right of the *x*-coordinate for *B* (to $4 + 2 = 6$), 3 units to the left of the *y*-coordinate for *B* (to $-2 - 3 = -5$), and 6 units below the *z*-coordinate for *B* (to $8 - 6 = 2$). So we get the point $A(6, -5, 2)$. Indeed, notice that to get from *A* to *B* (which describes vector $\overrightarrow{AB}$), we move 2 units left in the *x*-coordinate, 3 units right in the *y*-coordinate, and 6 units up in the *z*-coordinate. This is equivalent to vector $\overrightarrow{OP} = (-2, 3, 6)$.

6.6 Operations with Algebraic Vectors in R^2, pp. 324–326

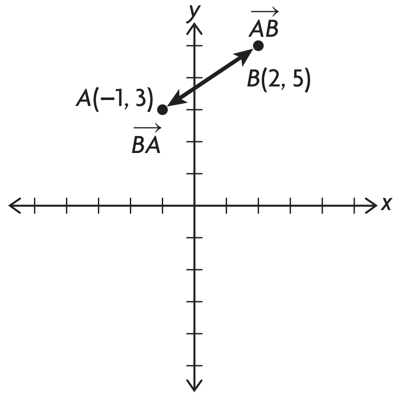
1.



a.

$$\begin{aligned} \overrightarrow{AB} &= (2, 5) - (-1, 3) \\ &= (3, 2) \\ \overrightarrow{BA} &= -\overrightarrow{AB} \\ &= -(3, 2) \\ &= (-3, -2) \end{aligned}$$

Here is a sketch of these two vectors in the *xy*-coordinate plane.



b.

$$\begin{aligned} |\overrightarrow{OA}| &= \sqrt{(-1)^2 + 3^2} \\ &= \sqrt{10} \\ &\doteq 3.16 \\ |\overrightarrow{OB}| &= \sqrt{2^2 + 5^2} \\ &= \sqrt{29} \\ &\doteq 5.39 \end{aligned}$$

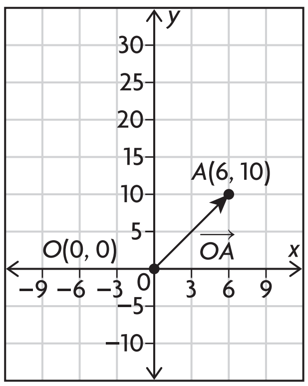
c.

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{3^2 + 2^2} \\ &= \sqrt{13} \\ &\doteq 3.61 \end{aligned}$$

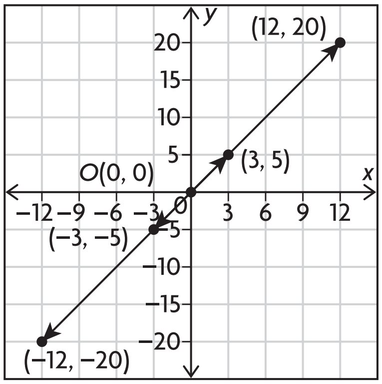
Also, since $\overrightarrow{BA} = -\overrightarrow{AB}$,

$$\begin{aligned} |\overrightarrow{BA}| &= |-\overrightarrow{AB}| \\ &= |-1| \cdot |\overrightarrow{AB}| \\ &= |\overrightarrow{AB}| \\ &= \sqrt{13} \\ &\doteq 3.61 \end{aligned}$$

2.



a.



b. The vectors with the same magnitude are

$\frac{1}{2}\overrightarrow{OA}$ and $-\frac{1}{2}\overrightarrow{OA}$,

$2\overrightarrow{OA}$ and $-2\overrightarrow{OA}$

3. $|\overrightarrow{OA}| = \sqrt{3^2 + (-4)^2}$
$= \sqrt{25}$
$= 5$

4. a. The $\vec{i}$-component will be equal to the first coordinate in component form, and so $a = -3$. Similarly, the $\vec{j}$-component will be equal to the second coordinate in component form, and so $b = 5$.

b. $|(-3, b)| = |(-3, 5)|$
$= \sqrt{(-3)^2 + 5^2}$
$= \sqrt{34}$
$\doteq 5.83$

5. a. $|\vec{a}| = \sqrt{(-60)^2 + 11^2}$
$= \sqrt{3721}$
$= 61$
$|\vec{b}| = \sqrt{(-40)^2 + (-9)^2}$
$= \sqrt{1681}$
$= 41$

b. $\vec{a} + \vec{b} = (-60, 11) + (-40, -9)$
$= (-100, 2)$
$|\vec{a} + \vec{b}| = \sqrt{(-100)^2 + 2^2}$
$= \sqrt{10\,004}$
$\doteq 100.02$
$\vec{a} - \vec{b} = (-60, 11) - (-40, -9)$
$= (-20, 20)$
$|\vec{a} - \vec{b}| = \sqrt{(-20)^2 + 20^2}$
$= \sqrt{800}$
$\doteq 28.28$

6. a. $2(-2, 3) + (2, 1) = (2(-2) + 2, 2(3) + 1)$
$= (-2, 7)$

b. $-3(4, -9) - 9(2, 3)$
$= (-3(4) - 9(2), -3(-9) - 9(3))$
$= (-30, 0)$

c. $-\frac{1}{2}(6, -2) + \frac{2}{3}(6, 15)$
$= \left(-\frac{1}{2}(6) + \frac{2}{3}(6), -\frac{1}{2}(-2) + \frac{2}{3}(15)\right)$
$= (1, 11)$

7. $\vec{x} = 2\vec{i} - \vec{j}, \vec{y} = -\vec{i} + 5\vec{j}$

a. $3\vec{x} - \vec{y} = 3(2\vec{i} - \vec{j}) - (-\vec{i} + 5\vec{j})$
$= (6 + 1)\vec{i} + (-3 - 5)\vec{j}$
$= 7\vec{i} - 8\vec{j}$

b. $-(\vec{x} + 2\vec{y}) + 3(-\vec{x} - 3\vec{y})$
$= -4\vec{x} - 11\vec{y}$
$= -4(2\vec{i} - \vec{j}) - 11(-\vec{i} + 5\vec{j})$
$= 3\vec{i} - 51\vec{j}$

c. $2(\vec{x} + 3\vec{y}) - 3(\vec{y} + 5\vec{x})$
$= -13\vec{x} + 3\vec{y}$
$= -13(2\vec{i} - \vec{j}) + 3(-\vec{i} + 5\vec{j})$
$= -29\vec{i} + 28\vec{j}$

8. a. $\vec{x} + \vec{y} = (2\vec{i} - \vec{j}) + (-\vec{i} + 5\vec{j})$
$= \vec{i} + 4\vec{j}$
$|\vec{x} + \vec{y}| = |\vec{i} + 4\vec{j}|$
$= \sqrt{1^2 + 4^2}$
$= \sqrt{17}$
$\doteq 4.12$

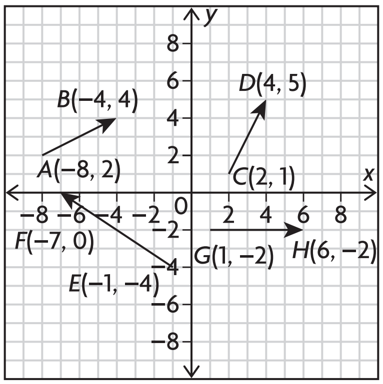
b. $\vec{x} - \vec{y} = (2\vec{i} - \vec{j}) - (-\vec{i} + 5\vec{j})$
$= 3\vec{i} - 6\vec{j}$
$|\vec{x} - \vec{y}| = |3\vec{i} - 6\vec{j}|$
$= \sqrt{3^2 + (-6)^2}$
$= \sqrt{45}$
$\doteq 6.71$

c. $2\vec{x} - 3\vec{y} = 2(2\vec{i} - \vec{j}) - 3(-\vec{i} + 5\vec{j})$
$= 7\vec{i} - 17\vec{j}$
$|2\vec{x} - 3\vec{y}| = |7\vec{i} - 17\vec{j}|$
$= \sqrt{7^2 + (-17)^2}$
$= \sqrt{338}$
$\doteq 18.38$

d. $|3\vec{y} - 2\vec{x}| = |-(2\vec{x} - 3\vec{y})|$
$= |-1||2\vec{x} - 3\vec{y}|$
$= |2\vec{x} - 3\vec{y}|$

so, from part c.,
$|3\vec{y} - 2\vec{x}| = |2\vec{x} - 3\vec{y}|$
$= \sqrt{338}$
$\doteq 18.38$

9.



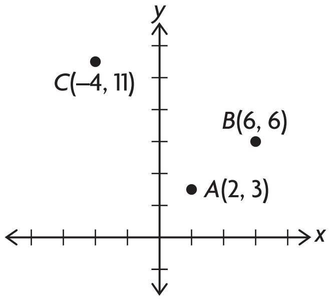
a. $\overrightarrow{AB} = (-4, 4) - (-8, 2)$
$= (4, 2)$
$\overrightarrow{CD} = (4, 5) - (2, 1)$
$= (2, 4)$
$\overrightarrow{EF} = (-7, 0) - (-1, -4)$
$= (-6, 4)$
$\overrightarrow{GH} = (6, -2) - (1, -2)$
$= (5, 0)$

b. $|\overrightarrow{AB}| = \sqrt{4^2 + 2^2}$
$= \sqrt{20}$
$\doteq 4.47$
$|\overrightarrow{CD}| = \sqrt{2^2 + 4^2}$
$= \sqrt{20}$
$\doteq 4.47$
$|\overrightarrow{EF}| = \sqrt{(-6)^2 + 4^2}$
$= \sqrt{52}$
$\doteq 7.21$
$|\overrightarrow{GH}| = \sqrt{5^2 + 0^2}$
$= \sqrt{25}$
$= 5$

10. a. By the parallelogram law of vector addition,
$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}$
$= (6, 3) + (11, -6)$
$= (17, -3)$
For the other vectors,
$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$
$= (6, 3) - (11, -6)$
$= (-5, 9)$
$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$
$= (17, -3) - (11, -6)$
$= (6, 3)$

b. $\overrightarrow{OA} = (6, 3)$
$= \overrightarrow{BC}$,
so obviously we will have $|\overrightarrow{OA}| = |\overrightarrow{BC}|$.
(It turns out that their common magnitude is $\sqrt{6^2 + 3^2} = \sqrt{45}$.)

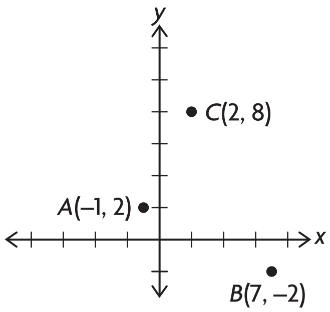
11. a.



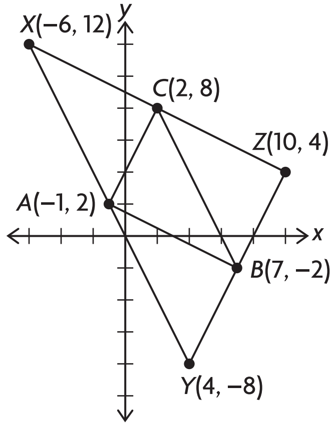
b. $\overrightarrow{AB} = (6, 6) - (2, 3)$
$= (4, 3)$
$|\overrightarrow{AB}| = \sqrt{4^2 + 3^2}$
$= \sqrt{25}$
$= 5$
$\overrightarrow{AC} = (-4, 11) - (2, 3)$
$= (-6, 8)$
$|\overrightarrow{AC}| = \sqrt{(-6)^2 + 8^2}$
$= \sqrt{100}$
$= 10$
$\overrightarrow{CB} = (6, 6) - (-4, 11)$
$= (10, -5)$
$|\overrightarrow{CB}| = \sqrt{10^2 + (-5)^2}$
$= \sqrt{125}$
$\doteq 11.18$

c. $|\overrightarrow{CB}|^2 = 125|\overrightarrow{AC}|^2 = 100, |\overrightarrow{AB}|^2 = 25$
Since $|\overrightarrow{CB}|^2 = |\overrightarrow{AC}|^2 + |\overrightarrow{AB}|^2$, the triangle is a right triangle.

12. a.



b.



c. As a first possibility for the fourth vertex, there is $X(x_1, x_2)$. From the sketch in part b., we see that we would then have

$$\overrightarrow{CX} = \overrightarrow{BA}$$
$$(x_1 - 2, x_2 - 8) = (-1 - 7, 2 - (-2))$$
$$= (-8, 4)$$
$$x_1 - 2 = -8$$
$$x_2 - 8 = 4$$

So $X(-6, 12)$. By similar reasoning for the other points labelled in the sketch in part b.,

$$\overrightarrow{AY} = \overrightarrow{CB}$$
$$(y_1 - (-1), y_2 - 2) = (7 - 2, -2 - 8)$$
$$= (5, -10)$$
$$y_1 + 1 = 5$$
$$y_2 - 2 = -10$$

So $Y(4, -8)$. Finally,

$$\overrightarrow{BZ} = \overrightarrow{AC}$$
$$(z_1 - 7, z_2 - (-2)) = (2 - (-1), 8 - 2)$$
$$= (3, 6)$$
$$z_1 - 7 = 3$$
$$z_2 + 2 = 6$$

So $Z(10, 4)$. In conclusion, the three possible locations for a fourth vertex in a parallelogram with vertices A, B, and C are $X(-6, 12)$, $Y(4, -8)$, and $Z(10, 4)$.

13. a. $3(x, 1) - 5(2, 3y) = (11, 33)$

$$(3x - 5(2), 3 - 5(3y)) = (11, 33)$$
$$(3x - 10, 3 - 15y) = (11, 33)$$
$$3x - 10 = 11$$
$$3 - 15y = 33$$

So $x = 7$ and $y = -2$.

b. $-2(x, x + y) - 3(6, y) = (6, 4)$

$$(-2x - 18, -2x - 5y) = (6, 4)$$
$$-2x - 18 = 6$$
$$-2x - 5y = 4$$

To solve for x, use

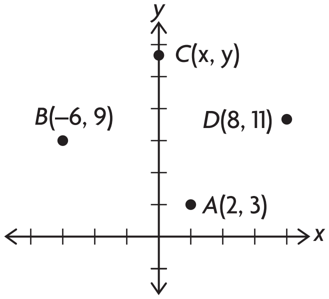
$$-2x - 18 = 6$$
$$x = -12$$

Substituting this into the last equation above, we can now solve for y.

$$-2(-12) - 5y = 4$$
$$y = 4$$

So $x = -12$ and $y = 4$.

14. a.



b. Because $ABCD$ is a rectangle, we will have

$$\overrightarrow{BC} = \overrightarrow{AD}$$
$$(x, y) - (-6, 9) = (8, 11) - (2, 3)$$
$$(x + 6, y - 9) = (6, 8)$$
$$x + 6 = 6$$
$$y - 9 = 8$$

So, $x = 0$ and $y = 17$, i.e., $C(0, 17)$.

15. a. Since $|\overrightarrow{PA}| = |\overrightarrow{PB}|$, and

$$\overrightarrow{PA} = (5, 0) - (a, 0)$$
$$= (5 - a, 0),$$
$$\overrightarrow{PB} = (0, 2) - (a, 0)$$
$$= (-a, 2),$$

this means that

$$(5 - a)^2 = (-a)^2 + 2^2$$
$$25 - 10a + a^2 = a^2 + 4$$
$$10a = 21$$
$$a = \frac{21}{10}$$

So $P\left(\frac{21}{10}, 0\right)$.

b. This point Q on the y-axis will be of the form $Q(0, b)$ for some real number b. Reasoning similarly to part a., we have

$$\overrightarrow{QA} = (5, 0) - (0, b)$$
$$= (5, -b)$$
$$\overrightarrow{QB} = (0, 2) - (0, b)$$
$$= (0, 2 - b)$$

So since $\left|\overrightarrow{QA}\right| = \left|\overrightarrow{QB}\right|$,

$$(-b)^2 + 5^2 = (2 - b)^2$$
$$b^2 + 25 = 4 - 4b + b^2$$
$$4b = -21$$
$$b = -\frac{21}{4}$$

So $Q\left(0, -\frac{21}{4}\right)$.

16. $\overrightarrow{QP}$ is in the direction opposite to $\overrightarrow{PQ}$, and

$$\begin{aligned}\overrightarrow{QP} &= \overrightarrow{OP} - \overrightarrow{OQ}\\ &= (11, 19) - (2, -21)\\ &= (9, 40)\\ |\overrightarrow{QP}| &= \sqrt{9^2 + 40^2}\\ &= \sqrt{1681}\\ &= 41\end{aligned}$$

A unit vector in the direction of $\overrightarrow{QP}$ is

$$\begin{aligned}\vec{u} &= \frac{1}{41}\overrightarrow{QP}\\ &= \left(\frac{9}{41}, \frac{40}{41}\right)\end{aligned}$$

Indeed, $\vec{u}$ is obviously in the same direction as $\overrightarrow{QP}$ (since $\vec{u}$ is a positive scalar multiple of $\overrightarrow{QP}$), and notice that

$$\begin{aligned}|\vec{u}| &= \sqrt{\left(\frac{9}{41}\right)^2 + \left(\frac{40}{41}\right)^2}\\ &= \sqrt{\frac{81 + 1600}{1681}}\\ &= 1\end{aligned}$$

17. a. O, P, and R can be thought of as the vertices of a triangle.

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{OR} - \overrightarrow{OP}\\ &= (-8, -1) - (-7, 24)\\ &= (-1, -25)\\ |\overrightarrow{PR}|^2 &= (-1)^2 + (-25)^2\\ &= 626\\ |\overrightarrow{OR}|^2 &= (-8)^2 + (-1)^2\\ &= 65\\ |\overrightarrow{OP}|^2 &= (-7)^2 + 24^2\\ &= 625\end{aligned}$$

By the cosine law, the angle, θ, between $\overrightarrow{OR}$ and $\overrightarrow{OP}$ satisfies

$$\begin{aligned}\cos\theta &= \frac{|\overrightarrow{OR}|^2 + |\overrightarrow{OP}|^2 - |\overrightarrow{PR}|^2}{2|\overrightarrow{OR}| \cdot |\overrightarrow{OP}|}\\ &= \frac{65 + 625 - 626}{2\sqrt{65} \cdot \sqrt{625}}\\ \theta &= \cos^{-1}\left(\frac{65 + 625 - 626}{2\sqrt{65} \cdot \sqrt{625}}\right)\\ &\doteq 80.9°\end{aligned}$$

So the angle between $\overrightarrow{OR}$ and $\overrightarrow{OP}$ is about 80.86°.

b. We found the vector $\overrightarrow{PR} = (-1, -25)$ in part a., so $\overrightarrow{RP} = -\overrightarrow{PR} = (1, 25)$ and

$$\begin{aligned}|\overrightarrow{RP}|^2 &= |\overrightarrow{PR}|^2\\ &= 626\end{aligned}$$

Also, by the parallelogram law of vector addition,

$$\begin{aligned}\overrightarrow{OQ} &= \overrightarrow{OR} + \overrightarrow{OP}\\ &= (-8, -1) + (-7, 24)\\ &= (-15, 23)\\ |\overrightarrow{OQ}|^2 &= (-15)^2 + 23^2\\ &= 754\end{aligned}$$

Placing $\overrightarrow{RP} = (1, 25)$ and $\overrightarrow{OQ} = (-15, 23)$ with their tails at the origin, a triangle is formed by joining the heads of these two vectors. The third side of this triangle is the vector

$$\begin{aligned}\vec{v} &= \overrightarrow{RP} - \overrightarrow{OQ}\\ &= (1, 25) - (-15, 23)\\ &= (16, 2)\\ |\vec{v}|^2 &= 16^2 + 2^2\\ &= 260\end{aligned}$$

Now by reasoning similar to part a., the cosine law implies that the angle, θ, between $\overrightarrow{RP}$ and $\overrightarrow{OQ}$ satisfies

$$\begin{aligned}\cos\theta &= \frac{|\overrightarrow{RP}|^2 + |\overrightarrow{OQ}|^2 - |\vec{v}|^2}{2|\overrightarrow{RP}| \cdot |\overrightarrow{OQ}|}\\ &= \frac{626 + 754 - 260}{2\sqrt{626} \cdot \sqrt{754}}\\ \theta &= \cos^{-1}\left(\frac{626 + 754 - 260}{2\sqrt{626} \cdot \sqrt{754}}\right)\\ &\doteq 35.4°\end{aligned}$$

So the angle between $\overrightarrow{RP}$ and $\overrightarrow{OQ}$ is about 35.40°. However, since we are discussing the diagonals of parallelogram $OPQR$ here, it would also have been appropriate to report the supplement of this angle, or about 180°−35.40° = 144.60°, as the angle between these vectors.

6.7 Operations with Vectors in R^3, pp. 332–333

1. a. $\overrightarrow{OA} = -1\vec{i} + 2\vec{j} + 4\vec{k}$

b. $|\overrightarrow{OA}| = \sqrt{(-1)^2 + 2^2 + 4^2} = \sqrt{21} \doteq 4.58$

2. $\overrightarrow{OB} = (3, 4, -4)$

$|\overrightarrow{OB}| = \sqrt{3^2 + 4^2 + (-4)^2} = \sqrt{41} \doteq 6.40$

3.
$$\begin{aligned}\vec{a} + \frac{1}{3}\vec{b} - \vec{c} &= (1, 3, -3) + (-1, 2, 4)\\ &\quad - (0, 8, 1)\\ &= (1 + (-1) - 0, 3 + 2 - 8,\\ &\quad (-3) + 4 - 1)\\ &= (0, -3, 0)\\ \left|\vec{a} + \frac{1}{3}\vec{b} - \vec{c}\right| &= \sqrt{0^2 + (-3)^2 + 0^2}\\ &= 3\end{aligned}$$

4. a. $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$
$= ((-3) + 2, 4 + 2, 12 + (-1))$
$= (-1, 6, 11)$

b. $|\overrightarrow{OA}| = \sqrt{(-3)^2 + 4^2 + 12^2} = 13$
$|\overrightarrow{OB}| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$
$|\overrightarrow{OP}| = \sqrt{(-1)^2 + 6^2 + 11^2} \doteq 12.57$

c. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= (2, 2, -1) - (-3, 4, 12)$
$= (2 - (-3), 2 - 4, (-1) - 12)$
$= (5, -2, -13)$
$|\overrightarrow{AB}| = \sqrt{5^2 + (-2)^2 + (-13)^2} = \sqrt{198} \doteq 14.07$
$\overrightarrow{AB}$ represents the vector from the tip of $\overrightarrow{OA}$ to the tip of $\overrightarrow{OB}$. It is the difference between the two vectors.

5. a. $\vec{x} - 2\vec{y} - \vec{z}$
$= (1, 4, -1) - 2(1, 3, -2) - (-2, 1, 0)$
$= (1, 4, -1) - (2, 6, -4) - (-2, 1, 0)$
$= (1 - 2 - (-2), 4 - 6 - 1, -1 - (-4) - 0)$
$= (1, -3, 3)$

b. $-2\vec{x} - 3\vec{y} + \vec{z}$
$= -2(1, 4, -1) - 3(1, 3, -2) + (-2, 1, 0)$
$= (-2, -8, 2) - (3, 9, -6) + (-2, 1, 0)$
$= (-2 - 3 - 2, -8 - 9 + 1, 2 + 6 + 0)$
$= (-7, -16, 8)$

c. $\frac{1}{2}\vec{x} - \vec{y} + 3\vec{z}$
$= \frac{1}{2}(1, 4, -1) - (1, 3, -2) + 3(-2, 1, 0)$
$= \left(\frac{1}{2}, 2, -\frac{1}{2}\right) - (1, 3, -2) + (-6, 3, 0)$
$= \left(\frac{1}{2} - 1 + (-6), 2 - 3 + 3, -\frac{1}{2} - (-2) + 0\right)$
$= \left(-\frac{13}{2}, 2, \frac{3}{2}\right)$

d. $3\vec{x} + 5\vec{y} + 3\vec{z}$
$= 3(1, 4, -1) + 5(1, 3, -2) + 3(-2, 1, 0)$
$= (3, 12, -3) + (5, 15, -10) + (-6, 3, 0)$
$= (3 + 5 - 6, 12 + 15 + 3, -3 - 10 + 0)$
$= (2, 30, -13)$

6. a. $\vec{p} + \vec{q} = (2\vec{i} - \vec{j} + \vec{k}) + (-\vec{i} - \vec{j} + \vec{k})$
$= (2 - 1)\vec{i} + (-1 - 1)\vec{j} + (1 + 1)\vec{k}$
$= \vec{i} - 2\vec{j} + 2\vec{k}$

b. $\vec{p} - \vec{q} = (2\vec{i} - \vec{j} + \vec{k}) - (-\vec{i} - \vec{j} + \vec{k})$
$= (2 + 1)\vec{i} + (-1 + 1)\vec{j} + (1 - 1)\vec{k}$
$= 3\vec{i} + 0\vec{j} + 0\vec{k}$

c. $2\vec{p} - 5\vec{q} = 2(2\vec{i} - \vec{j} + \vec{k}) - 5(-\vec{i} - \vec{j} + \vec{k})$
$= (4\vec{i} - 2\vec{j} + 2\vec{k}) - (-5\vec{i} - 5\vec{j} + 5\vec{k})$
$= (4 + 5)\vec{i} + (-2 + 5)\vec{j} + (2 - 5)\vec{k}$
$= 9\vec{i} + 3\vec{j} - 3\vec{k}$

d. $-2\vec{p} + 5\vec{q} = -2(2\vec{i} - \vec{j} + \vec{k}) + 5(-\vec{i} - \vec{j} + \vec{k})$
$= (-4\vec{i} + 2\vec{j} - 2\vec{k}) + (-5\vec{i} - 5\vec{j} + 5\vec{k})$
$= (-4 - 5)\vec{i} + (2 - 5)\vec{j} + (-2 + 5)\vec{k}$
$= -9\vec{i} - 3\vec{j} + 3\vec{k}$

7. a. $\vec{m} - \vec{n} = (2\vec{i} - \vec{k}) - (-2\vec{i} + \vec{j} + 2\vec{k})$
$= (2 - (-2))\vec{i} + (-1)\vec{j} + (-1 - 2)\vec{k}$
$= 4\vec{i} - \vec{j} - 3\vec{k}$
$|\vec{m} - \vec{n}| = \sqrt{4^2 + (-1)^2 + (-3)^2} = \sqrt{26} \doteq 5.10$

b. $\vec{m} + \vec{n} = (2\vec{i} - \vec{k}) + (-2\vec{i} + \vec{j} + 2\vec{k})$
$= (2 + (-2))\vec{i} + \vec{j} + (-1 + 2)\vec{k}$
$= 0\vec{i} + \vec{j} + \vec{k}$
$|\vec{m} + \vec{n}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2} \doteq 1.41$

c. $2\vec{m} + 3\vec{n} = 2(2\vec{i} - \vec{k}) + 3(-2\vec{i} + \vec{j} + 2\vec{k})$
$= (4\vec{i} - 2\vec{k}) + (-6\vec{i} + 3\vec{j} + 6\vec{k})$
$= (4 + (-6))\vec{i} + 3\vec{j} + (-2 + 6)\vec{k}$
$= -2\vec{i} + 3\vec{j} + 4\vec{k}$
$|2\vec{m} + 3\vec{n}| = \sqrt{(-2)^2 + 3^2 + 4^2} = \sqrt{29} \doteq 5.39$

d. $-5\vec{m} = -5(2\vec{i} - \vec{k}) = -10\vec{i} + 5\vec{k}$
$|-5\vec{m}| = \sqrt{(-10)^2 + (5)^2} = \sqrt{125} \doteq 11.18$

8. $\vec{x} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$
$+\ \vec{x} - \vec{y} = 3\vec{i} + 6\vec{j} - 7\vec{k}$
$2\vec{x} = 2\vec{i} + 8\vec{j} - 2\vec{k}$
Divide by 2 on both sides to get:
$\vec{x} = \vec{i} + 4\vec{j} - \vec{k}$
Plug this equation into the first given equation:
$\vec{i} + 4\vec{j} - \vec{k} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$
$\vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k} - (\vec{i} + 4\vec{j} - \vec{k})$
$\vec{y} = (-1 - 1)\vec{i} + (2 - 4)\vec{j} + (5 + 1)\vec{k}$
$\vec{y} = -2\vec{i} - 2\vec{j} + 6\vec{k}$

9. a. The vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ represent the *xy*-plane, *xz*-plane, and *yz*-plane, respectively. They are also the vector from the origin to points $(a, b, 0)$, $(a, 0, c)$, and $(0, b, c)$, respectively.

b. $\overrightarrow{OA} = a\vec{i} + b\vec{j} + 0\vec{k}$
$\overrightarrow{OB} = a\vec{i} + 0\vec{j} + c\vec{k}$
$\overrightarrow{OC} = 0\vec{i} + b\vec{j} + c\vec{k}$

c. $|\overrightarrow{OA}| = \sqrt{a^2 + b^2}$
$|\overrightarrow{OB}| = \sqrt{a^2 + c^2}$
$|\overrightarrow{OB}| = \sqrt{b^2 + c^2}$

d. $\overrightarrow{AB} = (a, 0, c) - (a, b, 0) = (0, -b, c)$

$\overrightarrow{AB}$ is a direction vector from A to B.

10. a. $|\overrightarrow{OA}| = \sqrt{(-2)^2 + (-6)^2 + 3^2} = \sqrt{49} = 7$

b. $|\overrightarrow{OB}| = \sqrt{(3)^2 + (-4)^2 + 12^2} = \sqrt{169} = 13$

c. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (3, -4, 12) - (-2, -6, 3)$

$= (3 - (-2), -4 - (-6), 12 - 3)$

$= (5, 2, 9)$

d. $|\overrightarrow{AB}| = \sqrt{5^2 + 2^2 + 9^2} = \sqrt{110} \doteq 10.49$

e. $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

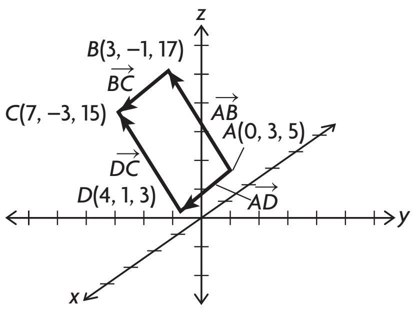
$= (-2, -6, 3) - (3, -4, 12)$

$= (-5, -2, -9)$

f. $|\overrightarrow{BA}| = \sqrt{(-5)^2 + (-2)^2 + (-9)^2}$

$= \sqrt{110} \doteq 10.49$

11.



In order to show that $ABCD$ is a parallelogram, we must show that $\overrightarrow{AB} = \overrightarrow{DC}$ or $\overrightarrow{BC} = \overrightarrow{AD}$. This will show they have the same direction, thus the opposite sides are parallel. By showing the vectors are equal they will have the same magnitude, implying the opposite sides having congruency.

$\overrightarrow{AB} = (3, -1, 17) - (0, 3, 5)$

$= (3 - 0, -1 - 3, 17 - 5)$

$= (3, -4, 12)$

$\overrightarrow{DC} = (7, -3, 15) - (4, 1, 3)$

$= (7 - 4, -3 - 1, 15 - 3)$

$= (3, -4, 12)$

Thus $\overrightarrow{AB} = \overrightarrow{DC}$. Do the calculations for the other pair as a check.

$\overrightarrow{BC} = (7, -3, 15) - (3, -1, 17)$

$= (7 - 3, -3 - (-1), 15 - 17)$

$= (4, -2, -2)$

$\overrightarrow{AD} = (4, 1, 3) - (0, 3, 5)$

$= (4 - 0, 1 - 3, 3 - 5)$

$= (4, -2, -2)$

So $\overrightarrow{BC} = \overrightarrow{AD}$.

We have shown $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{BC} = \overrightarrow{AD}$, so $ABCD$ is a parallelogram.

12. $2\vec{x} + \vec{y} - 2\vec{z}$

$= 2(-1, b, c) + (a, -2, c) - 2(-a, 6, c)$

$= (-2, 2b, 2c) + (a, -2, c) - (-2a, 12, 2c)$

$= (-2 + a + 2a, 2b - 2 - 12, 2c + c - 2c)$

$= (-2 + 3a, 2b - 14, c)$

$= (0, 0, 0)$

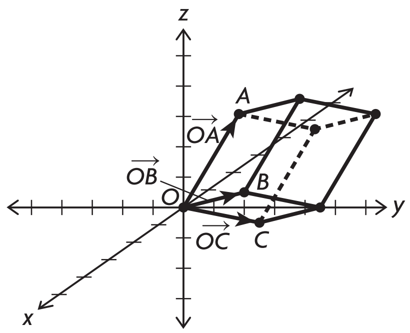
$-2 + 3a = 0; 2b - 14 = 0; c = 0$

$3a = 2; a = \frac{2}{3}$

$2b = 14; b = 7$

$c = 0$

13. a.



b. $V_1 = (0, 0, 0)$, the origin

$V_2 =$ end point of $\overrightarrow{OA} = (-2, 2, 5)$

$V_3 =$ end point of $\overrightarrow{OB} = (0, 4, 1)$

$V_4 =$ end point of $\overrightarrow{OC} = (0, 5, -1)$

$V_5 = \overrightarrow{OA} + \overrightarrow{OB} = (-2, 2, 5) + (0, 4, 1)$

$= (-2 + 0, 2 + 4, 5 + 1)$

$= (-2, 6, 6)$

$V_6 = \overrightarrow{OA} + \overrightarrow{OC} = (-2, 2, 5) + (0, 5, -1)$

$= (-2 + 0, 2 + 5, 5 - 1)$

$= (-2, 7, 4)$

$V_7 = \overrightarrow{OB} + \overrightarrow{OC} = (0, 4, 1) + (0, 5, -1)$

$= (0 + 0, 4 + 5, 1 - 1)$

$= (0, 9, 0)$

$V_8 = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$

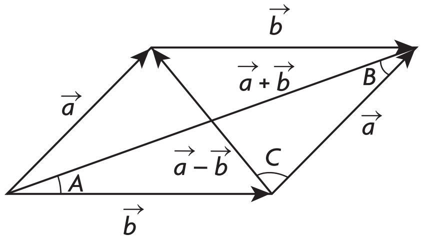
$= (-2, 2, 5) + (0, 9, 0)$ (by V_7)

$= (-2 + 0, 2 + 9, 5 + 0)$

$= (-2, 11, 5)$

14. Any point on the x-axis has y-coordinate 0 and z-coordinate 0. The z-coordinate of each of A and B is 3, so the z-component of the distance from the desired point is the same for each of A and B. The y-component of the distance from the desired point will be 1 for each of A and B, $12 = (-1)^2$. So, the x-coordinate of the desired point has to be halfway between the x-coordinates of A and B. The desired point is (1, 0, 0).

15.



To solve this problem, we must first consider the triangle formed by $\vec{a}$, $\vec{b}$, and $\vec{a}+\vec{b}$. We will use their magnitudes to solve for angle A, which will be used to solve for $\frac{1}{2}\vec{a}-\vec{b}$ in the triangle formed by $\vec{b}$, $\frac{1}{2}\vec{a}+\vec{b}$, and $\frac{1}{2}\vec{a}-\vec{b}$.

Using the cosine law, we see that:

$$\cos(A) = \frac{|\vec{b}|^2 + |\vec{a}+\vec{b}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{a}+\vec{b}|}$$
$$= \frac{25+49-9}{70}$$
$$= \frac{13}{14}$$

Now, consider the triangle formed by $\vec{b}$, $\frac{1}{2}\vec{a}+\vec{b}$, and $\frac{1}{2}\vec{a}-\vec{b}$. Using the cosine law again:

$$\cos(A) = \frac{|\vec{b}|^2 + \left(\frac{1}{2}|\vec{a}+\vec{b}|\right)^2 - \left(\frac{1}{2}|\vec{a}-\vec{b}|\right)^2}{|\vec{b}||\vec{a}+\vec{b}|}$$
$$\frac{13}{14} = \frac{\frac{149}{4} - \left(\frac{1}{2}|\vec{a}-\vec{b}|\right)^2}{35}$$
$$|\vec{a}-\vec{b}|^2 = -4\left(\frac{65}{2} - \frac{149}{4}\right)$$
$$|\vec{a}-\vec{b}|^2 = 19$$
$$|\vec{a}-\vec{b}| = \sqrt{19} \text{ or } 4.36$$

6.8 Linear Combinations and Spanning Sets, pp. 340–341

1. They are collinear, thus a linear combination is not applicable.
2. It is not possible to use $\vec{0}$ in a spanning set. Therefore, the remaining vectors only span R^2.
3. The set of vectors spanned by (0, 1) is $m(0, 1)$. If we let $m = -1$, then $m(0, 1) = (0, -1)$.
4. $\vec{i}$ spans the set $m(1, 0, 0)$. This is any vector along the x-axis. Examples: (2, 0, 0), $(-21, 0, 0)$
5. As in question 2, it is not possible to use $\vec{0}$ in a spanning set.
6. $\{(-1, 2), (-1, 1)\}$, $\{(2, -4), (-1, 1)\}$, $\{(-1, 1), (-3, 6)\}$ are all the possible spanning sets for R^2 with 2 vectors.
7. a. $2(2\vec{a} - 3\vec{b} + \vec{c}) = 4\vec{a} - 6\vec{b} + 2\vec{c}$
$= 4\vec{i} - 8\vec{j} - 6\vec{j} + 18\vec{k} + 2\vec{i} - 6\vec{j} + 4k$
$= 6\vec{i} - 20\vec{j} + 22\vec{k}$
$4(-\vec{a} + \vec{b} - \vec{c}) = -4\vec{a} + 4\vec{b} - 4\vec{c}$
$= -4\vec{i} + 8\vec{j} + 4\vec{j} - 12\vec{k} - 4\vec{i} + 12\vec{j} - 8k$
$= -8\vec{i} + 24\vec{j} - 20\vec{k}$
$(\vec{a} - \vec{c}) = \vec{i} - 2\vec{j} - \vec{i} + 3\vec{j} - 2\vec{k}$
$= \vec{j} - 2\vec{k}$
$2(2\vec{a} - 3\vec{b} + \vec{c}) - 4(-\vec{a} + \vec{b} - \vec{c}) + (\vec{a} - \vec{c})$
$= 6\vec{i} - 20\vec{j} + 22\vec{k} + 8\vec{i} - 24\vec{j} + 20\vec{k} + \vec{j} - 2\vec{k}$
$= 14\vec{i} - 43\vec{j} + 40\vec{k}$
b. $\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) = \vec{a} - 2\vec{b} - 4\vec{c}$
$= \vec{i} - 2\vec{j} - 2\vec{j} + 6\vec{k} - 4\vec{i} + 12\vec{j} - 8\vec{k}$
$= -3\vec{i} + 8\vec{j} - 2\vec{k}$
$\frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c}) = \vec{a} - 2\vec{b} + 3\vec{c}$
$= \vec{i} - 2\vec{j} - 2\vec{j} + 6\vec{k} + 3\vec{i} - 9\vec{j} + 6\vec{k}$
$= 4\vec{i} - 15\vec{j} + 12\vec{k}$
$\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) - \frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c})$
$= -3\vec{i} + 8\vec{j} - 2\vec{k} - 4\vec{i} + 15\vec{j} - 12\vec{k}$
$= -7\vec{i} + 23\vec{j} - 14\vec{k}$
8. $\{(1, 0, 0), (0, 1, 0)\}$:
$(-1, 2, 0) = -1(1, 0, 0) + 2(0, 1, 0)$
$(3, 4, 0) = 3(1, 0, 0) + 4(0, 1, 0)$
$\{(1, 1, 0), (0, 1, 0)\}$
$(-1, 2, 0) = -1(1, 1, 0) + 3(0, 1, 0)$
$(3, 4, 0) = 3(1, 1, 0) + (0, 1, 0)$
9. a. It is the set of vectors in the xy-plane.
b. $(-2, 4, 0) = -2(1, 0, 0) + 4(0, 1, 0)$
c. By part a. the vector is not in the xy-plane. There is no combination that would produce a number other than 0 for the z-component.
d. It would still only span the xy-plane. There would be no need for that vector.
10. Looking at the x-component:
$2a + 3c = 5$
The y-component:
$6 + 21 = b + c$
The z-component:
$2c + 3c = 15$
$5c = 15$
$c = 3$

Substituting this into the first and second equation:
$2a + 9 = 5$
$a = -2$
$27 = b + 3$
$b = 24$
11. $(-10, -34) = a(-1, 3) + b(1, 5)$
Looking at the x-component:
$-10 = -a + b \quad a = 10 + b$
Looking at the y-component:
$-34 = 3a + 5b$
Substituting in a:
$-34 = 30 + 3b + 5b$
$b = -8$
Substituting b into x-component equation:
$-10 = -a + (-8)$
$a = -2$
$(-10, -34) = -2(-1, 3) - 8(1, 5)$
12. a. $a(2, -1) + b(-1, 1) = (x, y)$
$x = 2a - b$
$b = 2a - x$
$y = -a + b$
Substitute in b:
$y = -a + 2a - x$
$a = x + y$
Substitute this back into the first equation:
$b = 2x + 2y - x$
$b = x + 2y$
b. Using the formulas in part a:
For $(2, -3)$:
$a = x + y = 2 - 3 = -1$
$b = x + 2y = 2 - 6 = -4$
$(2, -3) = -1(2, -1) - 4(-1, 1)$
For $(124, -5)$:
$a = 124 - 5 = 119$
$b = 124 - 10 = 114$
$(124, -5) = 119(2, -1) + 114(-1, 1)$
For $(4, -11)$
$a = 4 - 11 = -7$
$b = 4 - 22 = -18$
$(4, -11) = -7(2, -1) - 18(-1, 1)$
13. Try: $a(-1, 2, 3) + b(4, 1, -2)$
$= (-14, -1, 16)$
x components:
$-a + 4b = -14$
$a = 14 + 4b$
y components:
$2a + b = -1$
Substitute in a:
$28 + 8b + b = -1$
$b = -3$
Substitute this result into the x-components:
$a = 14 - 3 = 11$
Check by substituting into z-components:
$3a - 2b = 16$
$33 + 5 = 16$
Therefore:
$a(-1, 2, 3) + b(4, 1, -2) \neq (-14, -1, 16)$ for any a and b. They do not lie on the same plane.
b. $a(-1, 3, 4) + b(0, -1, 1) = (-3, 14, 7)$
x components:
$-a = -3$
$a = 3$
y components:
$3a - b = 14$
Substitute in a:
$9 - b = 14$
$b = -5$
Check with z components:
$4a + b = 7$
$12 - 5 = 7$
Since there exists an a and b to form a linear combination of 2 of the vectors to form the third, they lie on the same plane.
$3(-1, 3, 4) - 5(0, -1, 1) = (-3, 14, 7)$
14. Let vector $\vec{a} = (-1, 3, 4)$ and $\vec{b} = (-2, 3, -1)$ (vectors from the origin to points A and B, respectively). To determine x, we let $\vec{c}$ (vector from origin to C) be a linear combination of $\vec{a}$ and $\vec{b}$.
$a(-1, 3, 4) + b(-2, 3, -1) = (-5, 6, x)$
x components:
$-a - 2b = -5$
$a = 5 - 2b$
y components:
$3a + 3b = 6$
Substitute in a:
$15 - 6b + 3b = 6$
$b = 3$
Substitute in b in x component equation:
$a = 5 - 6 = -1$
z components:
$4a - b = x$
Substitute in a and b:
$x = -4 - 3 = -7$
15. $m = 2$, $n = 3$. Non-parallel vectors cannot be equal, unless their magnitudes equal 0.
16. Answers may vary. For example:
Try linear combinations of the 2 vectors such that

the z component equals 5. Then calculate what p and q would equal.
$-1(4, 1, 7) + 2(-1, 1, 6) = (-6, 1, 5)$
So $p = -6$ and $q = 1$
$5(4, 1, 5) - 5(-1, 1, 6) = (25, 0, 5)$
So $p = 25$ and $q = 0$
$(4, 1, 7) - \frac{1}{3}(-1, 1, 6) = \left(\frac{13}{3}, \frac{2}{3}, 5\right)$
So $p = \frac{13}{3}$ and $q = \frac{2}{3}$
17. As in question 15, non-parallel vectors. Their magnitudes must be 0 again to make the equality true.
$m^2 + 2m - 3 = (m - 1)(m + 3)$
$m = 1, -3$
$m^2 + m - 6 = (m - 2)(m + 3)$
$m = 2, -3$
So, when $m = -3$, their sum will be 0.

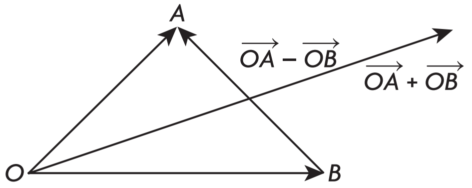
Review Exercise, pp. 344–347

1. a. false; Let $\vec{b} = -\vec{a} \neq 0$ then:
$|\vec{a} + \vec{b}| = |\vec{a} + (-\vec{a})|$
$= |0|$
$= 0 < |\vec{a}|$
b. true; $|\vec{a} + \vec{b}|$ and $|\vec{a} + \vec{c}|$ both represent the lengths of the diagonal of a parallelogram, the first with sides $\vec{a}$ and $\vec{b}$ and the second with sides $\vec{a}$ and $\vec{c}$; since both parallelograms have $\vec{a}$ as a side and diagonals of equal length $|\vec{b}| = |\vec{c}|$.
c. true; Subtracting $\vec{a}$ from both sides shows that $\vec{b} = \vec{c}$
d. true; Draw the parallelogram formed by $\overrightarrow{RF}$ and $\overrightarrow{SW}$. $\overrightarrow{FW}$ and $\overrightarrow{RS}$ are the opposite sides of a parallelogram and must be equal.
e. true; The distributive law for scalars
f. false; Let $\vec{b} = -\vec{a}$ and let $\vec{c} = \vec{d} \neq 0$. Then,
$|\vec{a}| = |-\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$
but $|\vec{a} + \vec{b}| = |\vec{a} + (-\vec{a})| = 0$
$|\vec{c} + \vec{d}| = |\vec{c} + \vec{c}| = |2\vec{c}|$
so $|\vec{a} + \vec{b}| \neq |\vec{c} + \vec{d}|$
2. a. Substitute the given values of $\vec{x}$, $\vec{y}$, and $\vec{z}$ into the expression $2\vec{x} - 3\vec{y} + 5\vec{z}$
$2\vec{x} - 3\vec{y} + 5\vec{z}$
$= 2(2\vec{a} - 3\vec{b} - 4\vec{c}) - 3\,(-2\vec{a} + 3\vec{b} + 3\vec{c})$
$\quad + 5(2\vec{a} - 3\vec{b} + 5\vec{c})$
$= 4\vec{a} - 6\vec{b} - 8\vec{c} + 6\vec{a} - 9\vec{b} - 9\vec{c} + 10\vec{a} - 15\vec{b}$
$\quad + 25\vec{c}$
$= 4\vec{a} + 6\vec{a} + 10\vec{a} - 6\vec{b} - 9\vec{b} - 15\vec{b} - 8\vec{c} - 9\vec{c}$
$\quad + 25\vec{c}$
$= 20\vec{a} - 30\vec{b} + 8\vec{c}$
b. Simplify the expression before substituting the given values of $\vec{x}$, $\vec{y}$, and $\vec{z}$
$3(-2\vec{x} - 4\vec{y} + \vec{z}) - (2\vec{x} - \vec{y} + \vec{z})$
$\quad - 2(-4\vec{x} - 5\vec{y} + \vec{z})$
$= -6\vec{x} - 12\vec{y} + 3\vec{z} - 2\vec{x} + \vec{y} - \vec{z} + 8\vec{x}$
$\quad + 10\vec{y} - 2\vec{z}$
$= -6\vec{x} - 2\vec{x} + 8\vec{x} - 12\vec{y} + \vec{y} + 10\vec{y} + 3\vec{z}$
$\quad - \vec{z} - 2\vec{z}$
$= 0\vec{x} - \vec{y} + 0\vec{z}$
$= -\vec{y}$
$= 2\vec{a} - 3\vec{b} - 3\vec{c}$
3. a. $\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$
$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$
$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
$= (-4 - (-2), 4 - 1, 8 - 2)$
$= (-2, 3, 6)$
$|\overrightarrow{XY}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(-2)^2 + (3)^2 + (6)^2}$
$= \sqrt{4 + 9 + 36}$
$= \sqrt{49}$
$= 7$
b. The components of a unit vector in the same direction as $\overrightarrow{XY}$ are $\frac{1}{7}(-2, 3, 6) = \left(-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$.
4. a. The position vector $\overrightarrow{OP}$ is equivalent to $\overrightarrow{YX}$.
$\overrightarrow{OP} = \overrightarrow{YX}$
$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$
$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
$= (-1 - 5, 2 - 5, 6 - 12)$
$= (-6, -3, -6)$
b. $|\overrightarrow{YX}| = \sqrt{(-6)^2 + (-3)^2 + (-6)^2}$
$= \sqrt{81}$
$= 9$
The components of a unit vector in the same direction as $\overrightarrow{YX}$ are $\frac{1}{9}(-6, -3, -6) = \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$
5. $-\overrightarrow{MN} = \overrightarrow{NM}$
$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$
$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
$= (2 - 8, 3 - 1, 5 - 2)$
$= (-6, 2, 3)$

$$|\overrightarrow{NM}| = \sqrt{(-6)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{49}$$
$$= 7$$

The components of the unit vector with the opposite direction to $\overrightarrow{MN}$ are $\frac{1}{7}(-6, 2, 3) = \left(-\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

6. a. The two diagonals can be found by calculating $\overrightarrow{OA} + \overrightarrow{OB}$ and $\overrightarrow{OA} - \overrightarrow{OB}$.



$$\overrightarrow{OA} + \overrightarrow{OB} = (3, 2, -6) + (-6, 6, -2)$$
$$= (3 + -6, 2 + 6, -6 + -2)$$
$$= (-3, 8, -8)$$
$$\overrightarrow{OA} - \overrightarrow{OB} = (3, 2, -6) + (-6, 6, -2)$$
$$= (3 - (-6), 2 - 6, -6 - (-2))$$
$$= (9, -4, -4)$$

b. To determine the angle between the sides of the parallelogram, calculate $|\overrightarrow{OA}|$, $|\overrightarrow{OB}|$, and $|\overrightarrow{OA} - \overrightarrow{OB}|$ and apply the cosine law.

$$|\overrightarrow{OA}| = \sqrt{(3)^2 + (2)^2 + (-6)^2}$$
$$= \sqrt{49}$$
$$= 7$$
$$|\overrightarrow{OB}| = \sqrt{(-6)^2 + (6)^2 + (-2)^2}$$
$$= \sqrt{76}$$
$$= 2\sqrt{19}$$
$$|\overrightarrow{OA} - \overrightarrow{OB}| = \sqrt{(9)^2 + (-4)^2 + (-4)^2}$$
$$= \sqrt{113}$$
$$\cos\theta = \frac{|\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2 - |\overrightarrow{OA} - \overrightarrow{OB}|^2}{2|\overrightarrow{OA}||\overrightarrow{OB}|}$$
$$\cos\theta = \frac{(7)^2 + (2\sqrt{19})^2 - (\sqrt{113})^2}{2(7)(2\sqrt{19})}$$
$$\cos\theta \doteq 0.098$$
$$\theta \doteq 84.4°$$

7. a. $|\overrightarrow{AB}| = \sqrt{(2 - (-1))^2 + (0 - 1)^2 + (3 - 1)^2}$
$$= \sqrt{(3)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{9 + 1 + 4}$$
$$= \sqrt{14}$$

$$|\overrightarrow{BC}| = \sqrt{(3 - 2)^2 + (3 - 0)^2 + (-4 - 3)^2}$$
$$= \sqrt{(1)^2 + (3)^2 + (-7)^2}$$
$$= \sqrt{1 + 9 + 49}$$
$$= \sqrt{59}$$
$$|\overrightarrow{CA}| = \sqrt{(-1 - 3)^2 + (1 - 3)^2 + (1 - (-4))^2}$$
$$= \sqrt{(-4)^2 + (-2)^2 + (5)^2}$$
$$= \sqrt{16 + 4 + 25}$$
$$= \sqrt{45}$$

Triangle ABC is a right triangle if and only if $|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = |\overrightarrow{BC}|^2$.

$$|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = (\sqrt{14})^2 + (\sqrt{45})^2$$
$$= 14 + 45$$
$$= 59$$
$$|\overrightarrow{BC}|^2 = (\sqrt{59})^2$$
$$= 59$$

So triangle ABC is a right triangle.

b. Area of a triangle $= \frac{1}{2}bh$. For triangle ABC the longest side $\overrightarrow{BC}$ is the hypotenuse, so $\overrightarrow{AB}$ and $\overrightarrow{CA}$ are the base and height of the triangle.

$$\text{Area} = \frac{1}{2}(|\overrightarrow{AB}|)(|\overrightarrow{CA}|)$$
$$= \frac{1}{2}\sqrt{14}\sqrt{45}$$
$$= \frac{1}{2}\sqrt{630}$$
$$= \frac{3}{2}\sqrt{70} \text{ or } 12.5$$

c. Perimeter of a triangle equals the sum of the sides.

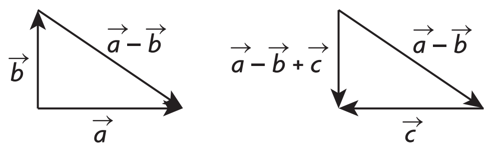
$$\text{Perimeter} = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CA}|$$
$$= \sqrt{14} + \sqrt{59} + \sqrt{45}$$
$$\doteq 18.13$$

d. The fourth vertex D is the head of the diagonal vector from A. To find $\overrightarrow{AD}$ take $\overrightarrow{AB} + \overrightarrow{AC}$.

$$\overrightarrow{AB} = (2 - (-1), 0 - 1, 3 - 1) = (3, -1, 2)$$
$$\overrightarrow{AC} = (3 - (-1), 3 - 1, -4 - 1) = (4, 2, -5)$$
$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$$
$$= (3 + 4, -1 + 2, 2 + (-5))$$
$$= (7, 1, -3)$$

So the fourth vertex is $D(-1 + 7, 1 + 1, 1 + (-3))$ or $D(6, 2, -2)$.

8. a.



b. Since the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$. So,

$$\begin{aligned}|\vec{a}+\vec{b}|^2 &= (4)^2 + (3)^2\\ &= 16 + 9\\ &= 25\\ |\vec{a}+\vec{b}| &= \sqrt{25} = 5\end{aligned}$$

9. Express $\vec{r}$ as a linear combination of $\vec{p}$ and $\vec{q}$:
Solve for a and b:

$$\begin{aligned}\vec{r} &= a\vec{p} + b\vec{q}\\ (-1, 2) &= a(-11, 7) + b(-3, 1)\\ (-1, 2) &= (-11a, 7a) + (-3b, b)\\ (-1, 2) &= (-11a - 3b, 7a + b)\end{aligned}$$

Solve the system of equations:

$$\begin{aligned}-1 &= -11a - 3b\\ 2 &= 7a + b\end{aligned}$$

Use the method of elimination:

$$\begin{aligned}3(2) &= 3(7a + b)\\ 6 &= 21a + 3b\\ + -1 &= -11a - 3b\\ \hline 5 &= 10a\\ \frac{1}{2} &= a\end{aligned}$$

By substitution, $b = -\frac{3}{2}$

Therefore $\frac{1}{2}(-11, 7) + -\frac{3}{2}(-3, 1) = (-1, 2)$

Express $\vec{q}$ as a linear combination of $\vec{p}$ and $\vec{r}$.
Solve for a and b:

$$\begin{aligned}\vec{q} &= a\vec{p} + b\vec{r}\\ (-3, 1) &= a(-11, 7) + b(-1, 2)\\ (-3, 1) &= (-11a, 7a) + (-b, 2b)\\ (-3, 1) &= (-11a - b, 7a + 2b)\end{aligned}$$

Solve the system of equations:

$$\begin{aligned}-3 &= -11a - b\\ 1 &= 7a + 2b\end{aligned}$$

Use the method of elimination:

$$\begin{aligned}2(-3) &= 2(-11a - b)\\ -6 &= -22a - 2b\\ +1 &= 7a + 2b\\ \hline -5 &= -15a\\ \frac{1}{3} &= a\end{aligned}$$

By substitution, $-\frac{2}{3} = b$

Therefore $\frac{1}{3}(-11, 7) + -\frac{2}{3}(-1, 2) = (-3, 1)$

Express $\vec{p}$ as a linear combination of $\vec{q}$ and $\vec{r}$.
Solve for a and b:

$$\begin{aligned}\vec{p} &= a\vec{q} + b\vec{r}\\ (-11, 7) &= a(-3, 1) + b(-1, 2)\\ (-11, 7) &= (-3a, a) + (-b, 2b)\\ (-11, 7) &= (-3a - b, a + 2b)\end{aligned}$$

Solve the system of equations:

$$\begin{aligned}-11 &= -3a - b\\ 7 &= a + 2b\end{aligned}$$

Use the method of elimination:

$$\begin{aligned}2(-11) &= 2(-3a - b)\\ -22 &= -6a - 2b\\ + \quad 7 &= a + 2b\\ \hline -15 &= -5a\\ 3 &= a\end{aligned}$$

By substitution, $2 = b$

Therefore $3(-3, 1) + 2(-1, 2) = (-11, 7)$

10. a. Let $P(x, y, z)$ be a point equidistant from A and B. Then $|\overrightarrow{PA}| = |\overrightarrow{PB}|$.

$$\begin{aligned}&(x-2)^2 + (y-(-1))^2 + (z-3)^2\\ &= (x-1)^2 + (y-2)^2 + (z-(-3))^2\end{aligned}$$

$$\begin{aligned}&x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 6z + 9\\ &= x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + 6z + 9\\ &-2x + 6y - 12z = 0\\ &x - 3y + 6z = 0\end{aligned}$$

b. (0, 0, 0) and $\left(1, \frac{1}{3}, 0\right)$ clearly satisfy the equation and are equidistant from A and B.

11. a.

$$\begin{aligned}(-24, 3, 25) &= 2(a, b, 4) + \frac{1}{2}(6, 8, c) - 3(7, c, -4)\\ (-24, 3, 25) &= (2a, 2b, 8) + \left(3, 4, \frac{c}{2}\right)\\ &\quad - (21, 3c, -12)\\ (-24, 3, 25) &= \left(2a - 18, 2b + 4 - 3c, \frac{c}{2} + 20\right)\end{aligned}$$

Solve the equations:

i. $$\begin{aligned}-24 &= 2a - 18\\ -6 &= 2a\\ -3 &= a\end{aligned}$$

ii. $$\begin{aligned}25 &= \frac{c}{2} + 20\\ 5 &= \frac{c}{2}\\ 10 &= c\end{aligned}$$

iii. $$\begin{aligned}3 &= 2b + 4 - 3c\\ 3 &= 2b + 4 - 3(18)\\ 3 &= 2b - 50\\ 53 &= 2b\\ 26.5 &= b\end{aligned}$$

b. $(3, -22, 54)$

$= 2\left(a, a, \frac{1}{2}a\right) + (3b, 0, -5c) + 2\left(c, \frac{3}{2}c, 0\right)$

$(3, -22, 54)$

$= (2a, 2a, a) + (3b, 0, -5c) + (2c, 3c, 0)$

$(3, -22, 54) = (2a + 3b + 2c, 2a + 3c, a - 5c)$

Solve the system of equations:

$-22 = 2a + 3c$

$54 = a - 5c$

Use the method of elimination:

$$\begin{aligned} -2(54) &= -2(a - 5c) \\ -108 &= -2a + 10c \\ + -22 &= 2a + 3c \\ \hline -130 &= 13c \\ -10 &= c \end{aligned}$$

By substitution, $8 = a$

Solve the equation:

$3 = 2a + 3b + 2c$

$3 = 2(8) + 3b + 2(-10)$

$3 = 16 + 3b - 20$

$3 = 3b - 4$

$7 = 3b$

$\frac{7}{3} = b$

12. a. Find $|\overrightarrow{AB}|, |\overrightarrow{BC}|, |\overrightarrow{CA}|$

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(2-1)^2 + (2-(-1))^2 + (2-1)^2} \\ &= \sqrt{(1)^2 + (3)^2 + (1)^2} \\ &= \sqrt{11} \end{aligned}$$

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{(4-2)^2 + (-2-2)^2 + (1-2)^2} \\ &= \sqrt{(2)^2 + (-4)^2 + (-1)^2} \\ &= \sqrt{21} \end{aligned}$$

$$\begin{aligned} |\overrightarrow{CA}| &= \sqrt{(4-1)^2 + (-2-(-1))^2 + (1-1)^2} \\ &= \sqrt{(3)^2 + (-1)^2} \\ &= \sqrt{10} \end{aligned}$$

Test $|\overrightarrow{AB}|, |\overrightarrow{BC}|, |\overrightarrow{CA}|$ in the Pythagorean theorem:

$$\begin{aligned} |\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 &= (\sqrt{11})^2 + (\sqrt{10})^2 \\ &= 11 + 10 \\ &= 21 \\ |\overrightarrow{BC}|^2 &= (\sqrt{21})^2 \\ &= 21 \end{aligned}$$

So triangle ABC is a right triangle.

b. Yes, $P(1, 2, 3)$, $Q(2, 4, 6)$, and $R(-1, -2, -3)$ are collinear because:

$2P = (2, 4, 6)$

$1Q = (2, 4, 6)$

$-2R = (2, 4, 6)$

13. a. Find $|\overrightarrow{AB}|, |\overrightarrow{BC}|, |\overrightarrow{CA}|$

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(1-3)^2 + (2-0)^2 + (5-4)^2} \\ &= \sqrt{(-2)^2 + (2)^2 + (1)^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{(2-1)^2 + (1-2)^2 + (3-5)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (-2)^2} \\ &= \sqrt{6} \end{aligned}$$

$$\begin{aligned} |\overrightarrow{CA}| &= \sqrt{(2-3)^2 + (1-0)^2 + (3-4)^2} \\ &= \sqrt{(-1)^2 + (1)^2 + (-1)^2} \\ &= \sqrt{3} \end{aligned}$$

Test $|\overrightarrow{AB}|, |\overrightarrow{BC}|, |\overrightarrow{CA}|$ in the Pythagorean theorem:

$$\begin{aligned} |\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2 &= (\sqrt{6})^2 + (\sqrt{3})^2 \\ &= 6 + 3 \\ &= 9 \\ |\overrightarrow{AB}|^2 &= (3)^2 \\ &= 9 \end{aligned}$$

So triangle ABC is a right triangle.

b. Since triangle ABC is a right triangle,

$\cos\angle ABC = \sqrt{\frac{6}{3}}$

14. a. $\overrightarrow{DA}, \overrightarrow{BC}$ and $\overrightarrow{EB}, \overrightarrow{ED}$

b. $\overrightarrow{DC}, \overrightarrow{AB}$ and $\overrightarrow{CE}, \overrightarrow{EA}$

c. $|\overrightarrow{AD}|^2 + |\overrightarrow{DC}|^2 = |\overrightarrow{AC}|^2$,

But $|\overrightarrow{AC}|^2 = |\overrightarrow{DB}|^2$

Therefore, $|\overrightarrow{AD}|^2 + |\overrightarrow{DC}|^2 = |\overrightarrow{DB}|^2$

15. a. $C(3, 0, 5)$; $P(3, 4, 5)$; $E(0, 4, 5)$; $F(0, 4, 0)$

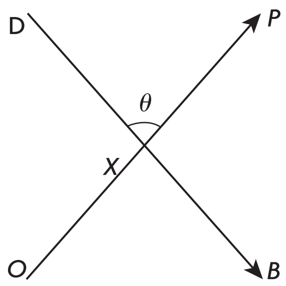
b. $\overrightarrow{DB} = (3 - 0, 4 - 0, 0 - 5)$

$= (3, 4, -5)$

$\overrightarrow{CF} = (0 - 3, 4 - 0, 0 - 5)$

$= (-3, 4, -5)$

c.

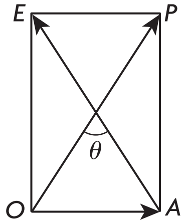


$|\overrightarrow{OD}| = 5$

$|\overrightarrow{DP}| = 5$ by the Pythagorean theorem

Thus $ODPB$ is a square and $\cos\theta = 0$, so the angle between the vectors is 90°.

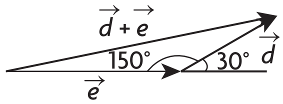
d.



$|\overrightarrow{OA}| = 3$, $|\overrightarrow{OP}| = \sqrt{5}$

$$\theta = 180° - 2(m\angle POA)$$
$$= 180° - 2\left(\cos^{-1}\left(\frac{3}{\sqrt{50}}\right)\right)$$
$$\doteq 50.2°$$

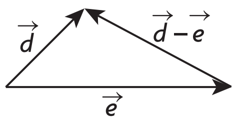
16. a.



Use the cosine law to evaluate $|\vec{d} + \vec{e}|$

$$|\vec{d} + \vec{e}|^2 = |\vec{d}|^2 + |\vec{e}|^2 - 2|\vec{d}||\vec{e}|\cos\theta$$
$$= (3)^2 + (5)^3 - 2(3)(5)\cos 150°$$
$$= 9 + 25 - 30\frac{-\sqrt{3}}{2}$$
$$\doteq 59.98$$
$$|\vec{d} + \vec{e}| \doteq \sqrt{59.98}$$
$$\doteq 7.74$$

b.

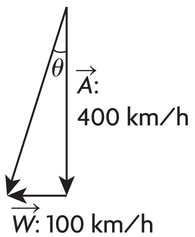


Use the cosine law to evaluate $|\vec{d} - \vec{e}|$

$$|\vec{d} - \vec{e}|^2 = |\vec{d}|^2 + |\vec{e}|^2 - 2|\vec{d}||\vec{e}|\cos\theta$$
$$= (3)^2 + (5)^3 - 2(3)(5)\cos 30°$$
$$= 9 + 25 - 30\frac{\sqrt{3}}{2}$$
$$\doteq 8.02$$
$$|\vec{d} - \vec{e}| \doteq \sqrt{8.02}$$
$$\doteq 2.83$$

c. $|\vec{e} - \vec{d}| = |-(\vec{d} - \vec{e})| = |\vec{d} - \vec{e}| \doteq 2.83$

17. a.



Let $\vec{A}$ represent the air speed of the airplane and let $\vec{W}$ represent the velocity of the wind. In one hour, the plane will travel $|\vec{A} + \vec{W}|$ kilometers. Because $\vec{A}$ and $\vec{W}$ make a right angle, use the Pythagorean theorem:

$$|\vec{A} + \vec{W}|^2 = |\vec{A}|^2 + |\vec{W}|^2$$
$$= (400)^2 + (100)^2$$
$$= 170000$$
$$|\vec{A} + \vec{W}| = \sqrt{170000}$$
$$\doteq 412.3 \text{ km}$$

So in 3 hours, the plane will travel

$3(412.3)\text{km} \doteq 1236.9$ km

b. $\tan\theta = \dfrac{|\vec{W}|}{|\vec{A}|}$

$$= \frac{100}{400}$$
$$\theta = \tan^{-1}\left(\frac{1}{4}\right)$$
$$\doteq 14.0°$$

The direction of the airplane is S14.0°W.

18. a. Any pair of nonzero, noncollinear vectors will span R^2. To show that (2, 3) and (3, 5) are noncollinear, show that there does not exist any number k such that $k(2, 3) = (3, 5)$. Solve the system of equations:

$2k = 3$

$3k = 5$

Solving both equations gives two different values for k, $\frac{3}{2}$ and $\frac{5}{3}$, so (2, 3) and (3, 5) are noncollinear and thus span R^2

b. $(323, 795) = m(2, 3) + n(3, 5)$

$(323, 795) = (2m, 3m) + (3n, 5n)$

$(323, 795) = (2m + 3n, 3m + 5n)$

Solve the system of equations:

$323 = 2m + 3n$

$795 = 3m + 5n$

Use the method of elimination:

$$-3(323) = -3(2m + 3n)$$
$$2(795) = 2(3m + 5n)$$
$$-969 = -6m - 9n$$
$$+\quad \underline{1590 = 6m + 10n}$$
$$621 = n$$

By substitution, $m = -770$.

19. a. Find a and b such that

$(5, 9, 14) = a(-2, 3, 1) + b(3, 1, 4)$

$(5, 9, 14) = (-2a, 3a, a) + (3b, b, 4b)$

$(5, 9, 14) = (-2a + 3b, 3a + b, a + 4b)$

i. $5 = -2a + 3b$

ii. $9 = 3a + b$

iii. $14 = a + 4b$

Use the method of elimination with **i.** and **iii.**

$$\begin{aligned} 2(14) &= 2(a + 4b) \\ 28 &= 2a + 8b \\ +\quad 5 &= -2a + 3b \\ \hline 33 &= 11b \\ 3 &= b \end{aligned}$$

By substitution, $a = 2$.

$\vec{a}$ lies in the plane determined by $\vec{b}$ and $\vec{c}$ because it can be written as a linear combination of $\vec{b}$ and $\vec{c}$.

b. If vector $\vec{a}$ is in the span of $\vec{b}$ and $\vec{c}$, then $\vec{a}$ can be written as a linear combination of $\vec{b}$ and $\vec{c}$. Find m and n such that

$$\begin{aligned} (-13, 36, 23) &= m(-2, 3, 1) + n(3, 1, 4) \\ &= (-2m, 3m, m) + (3n, n, 4n) \\ &= (-2m + 3n, 3m + n, m + 4n) \end{aligned}$$

Solve the system of equations:

$$\begin{aligned} -13 &= -2m + 3n \\ 36 &= 3m + n \\ 23 &= m + 4n \end{aligned}$$

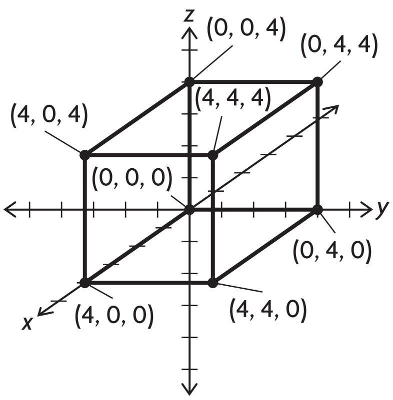
Use the method of elimination:

$$\begin{aligned} 2(23) &= 2(m + 4n) \\ 46 &= 2m + 8n \\ + -13 &= -2m + 3n \\ \hline 33 &= 11n \\ 3 &= n \end{aligned}$$

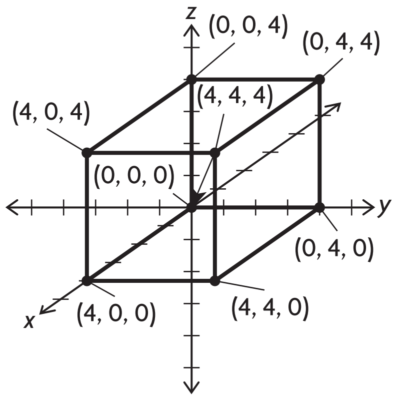
By substitution, $m = 11$.

So, vector $\vec{a}$ is in the span of $\vec{b}$ and $\vec{c}$.

20. a.



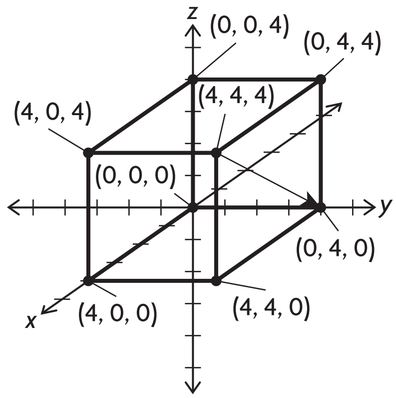
b.



$\overrightarrow{PO} = (4, 4, 4)$ so,

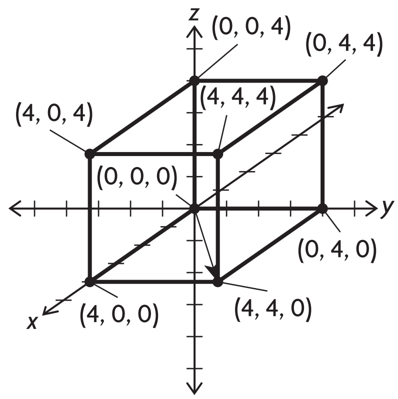
$\overrightarrow{OP} = -\overrightarrow{PO} = -(4, 4, 4) = (-4, -4, -4)$

c.



The vector $\overrightarrow{PQ}$ from $P(4, 4, 4)$ to $Q(0, 4, 0)$ can be written as $\overrightarrow{PQ} = (-4, 0, -4)$.

d.



The vector with the coordinates (4, 4, 0).

21. $|2(\vec{a} + \vec{b} - \vec{c}) - (\vec{a} + 2\vec{b}) + 3(\vec{a} - \vec{b} + \vec{c})|$

$= |2\vec{a} + 2\vec{b} - 2\vec{c} - \vec{a} - 2\vec{b} + 3\vec{a} - 3\vec{b} + 3\vec{c}|$

$= |4\vec{a} - 3\vec{b} + \vec{c}|$

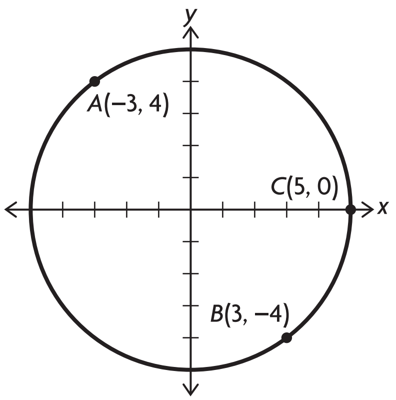
$= |4(1, 1, -1) - 3(2, -1, 3) + (2, 0, 13)|$

$= |(4, 4, -4) + (-6, 3, -9) + (2, 0, 13)|$

$= |(0, 7, 0)|$

$= 7$

22.



a. $|\overrightarrow{AB}| = 10$ because it is the diameter of the circle.

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{(5-3)^2 + (0-(-4))^2} \\ &= \sqrt{(2)^2 + (4)^2} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \text{ or } 4.47 \end{aligned}$$

$$\begin{aligned} |\overrightarrow{CA}| &= \sqrt{(5-(-3))^2 + (0-4)^2} \\ &= \sqrt{(8)^2 + (-4)^2} \\ &= \sqrt{80} \text{ or } 8.94 \end{aligned}$$

b. If *A, B,* and *C* are vertices of a right triangle, then

$$|\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2 = |\overrightarrow{AB}|^2$$

$$\begin{aligned} |\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2 &= (2\sqrt{5})^2 + (\sqrt{80})^2 \\ &= 20 + 80 \\ &= 100 \\ |\overrightarrow{AB}|^2 &= 10^2 \\ &= 100 \end{aligned}$$

So, triangle *ABC* is a right triangle.

23. a. $\overrightarrow{FL} = \overrightarrow{FG} + \overrightarrow{GH} + \overrightarrow{HL} = \vec{a} + \vec{b} + \vec{c}$

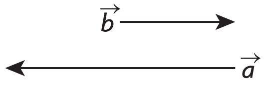
b. $\overrightarrow{MK} = \overrightarrow{JK} - \overrightarrow{JM} = \vec{a} - \vec{b}$

c. $\overrightarrow{HJ} = \overrightarrow{HG} + \overrightarrow{GF} + \overrightarrow{FJ} = -\vec{b} - \vec{a} + \vec{c}$

d. $\overrightarrow{IH} + \overrightarrow{KJ} = \overrightarrow{FG} + \overrightarrow{GF} = 0$

e. $\overrightarrow{IK} - \overrightarrow{IH} = \overrightarrow{HK} = \overrightarrow{IJ} = \vec{b} - \vec{c}$

24.



25. a. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$ by the Pythagorean theorem

b. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$ by the Pythagorean theorem

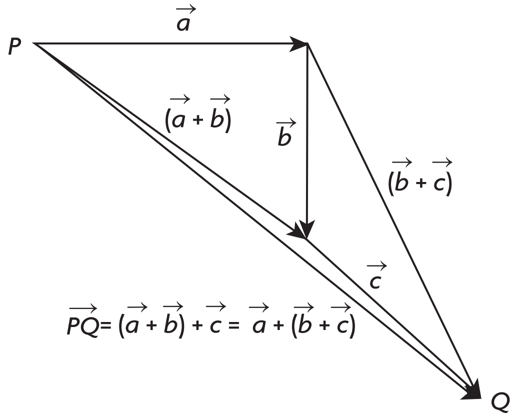
c. $\sqrt{4|\vec{a}|^2 + 9|\vec{b}|^2}$ by the Pythagorean theorem

26. Case 1 If $\vec{b}$ and $\vec{c}$ are collinear, then $2\vec{b} + 4\vec{c}$ is also collinear with both $\vec{b}$ and $\vec{c}$. But $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, so $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Case 2 If $\vec{b}$ and $\vec{c}$ are not collinear, then by spanning sets, $\vec{b}$ and $\vec{c}$ span a plane in R^3, and $\overline{2\vec{b} + 4\vec{c}}$ is in that plane. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then it is perpendicular to the plane and all vectors in the plane. So, $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Chapter 6 Test, p. 348

1. Let *P* be the tail of $\vec{a}$ and let *Q* be the head of $\vec{c}$. The vector sums $[\vec{a} + (\vec{b} + \vec{c})]$ and $[(\vec{a} + \vec{b}) + \vec{c}]$ can be depicted as in the diagram below, using the triangle law of addition. We see that $\overrightarrow{PQ} = \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$. This is the associative property for vector addition.



2. a. $\overrightarrow{AB} = (6 - (-2), 7 - 3, 3 - (-5)) = (8, 4, 8)$

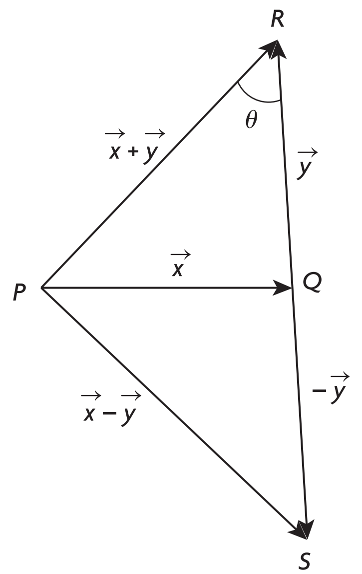
b. $|\overrightarrow{AB}| = \sqrt{8^2 + 4^2 + 8^2} = 12$

c. $\overrightarrow{BA} = (-1)\overrightarrow{AB} = (-8, -4, -8)$;

$|\overrightarrow{BA}| = |\overrightarrow{AB}| = 12$; unit vector in direction of

$$\begin{aligned} |\overrightarrow{BA}| &= \frac{1}{|\overrightarrow{BA}|}\overrightarrow{BA} \\ &= \frac{1}{12}(-8, -4, -8) \\ &= \left(-\frac{2}{3} - \frac{1}{3}, -\frac{2}{3}\right) \end{aligned}$$

3. Let $\vec{x} = \overrightarrow{PQ}$, $\vec{y} = \overrightarrow{QR}$, and $-\vec{y} = \overrightarrow{QS}$, as in the diagram below. Note that $|\overrightarrow{RS}| = |2\vec{y}| = 6$ and that triangle *PQR* and triangle *PRS* share angle θ.



By the cosine law:

$$\cos\theta = \frac{|\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}|^2}{2|\vec{y}||\vec{x}+\vec{y}|} \text{ and}$$

$$\cos\theta = \frac{|2\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}-\vec{y}|^2}{2|2\vec{y}||\vec{x}+\vec{y}|}.$$

Hence,

$$\frac{|\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}|^2}{2|\vec{y}||\vec{x}+\vec{y}|}$$

$$= \frac{|2\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}-\vec{y}|^2}{2|2\vec{y}||\vec{x}+\vec{y}|}$$

$$2(|\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}|^2)$$

$$= |2\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}-\vec{y}|^2$$

$$|\vec{x}-\vec{y}|^2 = 2|\vec{y}|^2 - |\vec{x}+\vec{y}|^2 + 2|\vec{x}|^2$$

$$|\vec{x}-\vec{y}| = \sqrt{2|\vec{y}|^2 - |\vec{x}+\vec{y}|^2} + \sqrt{2|\vec{x}|^2}$$

$$|\vec{x}-\vec{y}| = \sqrt{2(3)^2 - (\sqrt{17})^2} + \sqrt{2(3)^2}$$

$$|\vec{x}-\vec{y}| = \sqrt{19}$$

4. a. We have $3\vec{x} - 2\vec{y} = \vec{a}$ and $5\vec{x} - 3\vec{y} = \vec{b}$. Multiplying the first equation by -3 and the second equation by 2 yields: $-9\vec{x} + 6\vec{y} = -3\vec{a}$ and $10\vec{x} - 6\vec{y} = 2\vec{b}$. Adding these equations, we have: $\vec{x} = 2\vec{b} - 3\vec{a}$. Substituting this into the first equation yields: $3(2\vec{b} - 3\vec{a}) - 2\vec{y} = \vec{a}$. Simplifying, we have: $\vec{y} = 3\vec{b} - 5\vec{a}$.

b. First, conduct scalar multiplication on the third vector, yielding:
$(2, -1, c) + (a, b, 1) - (6, 3a, 12) = (-3, 1, 2c)$.
Now, each of the three components corresponds to an equation. First, $2 + a - 6 = -3$, which implies $a = 1$. Second, $-1 + b - 3a = 1$. Substituting $a = 1$ and simplifying yields $b = 5$. Third, $c + 1 - 12 = 2c$, so $c = -11$.

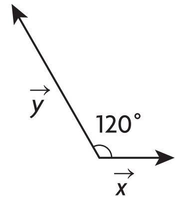
5. a. $\vec{a}$ and $\vec{b}$ span R^2, because any vector (x, y) in R^2 can be written as a linear combination of $\vec{a}$ and $\vec{b}$. These two vectors are not multiples of each other.

b. First, conduct scalar multiplication on the vectors, yielding: $(-2p, 3p) + (3q, -q) = (13, -9)$. Now, each component corresponds to an equation. First, $-2p + 3q = 13$. Second, $3p - q = -9$. Multiplying the second equation by 3 and adding the result to the first equation yields: $7p = -14$, which implies $p = -2$. Substituting this into the first equation and simplifying yields $q = 3$.

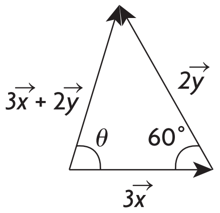
6. a. $\vec{a} = m\vec{b} + n\vec{c}$
$(1, 12, -29) = m(3, 1, 4) + n(1, 2, -3)$
$(1, 12, -29) = (3m, m, 4m) + (n, 2n, -3n)$
Each of the three components corresponds to an equation. First, $1 = 3m + n$. Second, $12 = m + 2n$. Third, $-29 = 4m - 3n$. Multiplying the first equation by -2 and adding the result to the second equation yields $m = -2$. Substituting $m = -2$ into the first equation yields $n = 7$. Since $m = -2$ and $n = 7$ also solves the third component's equation, $\vec{a} = m\vec{b} + n\vec{c}$ for $m = -2$ and $n = 7$. Hence, $\vec{a}$ can be written as a linear combination of $\vec{b}$ and $\vec{c}$.

b. $\vec{r} = m\vec{p} + n\vec{q}$
$(16, 11, -24) = m(-2, 3, 4) + n(4, 1, -6)$
$(16, 11, -24) = (-2m, 3m, 4m) + (4n, n, -6n)$
Each of the three components corresponds to an equation. First, $16 = -2m + 4n$. Second, $11 = 3m + n$. Third, $-24 = 4m - 6n$. Multiplying the first equation by 2 and adding the result to the third equation yields $n = 4$. Substituting $n = 4$ into the first equation yields $m = 0$. We have that $n = 4$ and $m = 0$ is the *unique* solution to the first and third equations, but $n = 4$ and $m = 0$ does not solve the second equation. Hence, this system of equations has no solution, and $\vec{r}$ cannot be written as a linear combination of $\vec{p}$ and $\vec{q}$. In other words, $\vec{r}$ does not lie in the plane determined by $\vec{p}$ and $\vec{q}$.

7. $\vec{x}$ and $\vec{y}$ have magnitudes of 1 and 2, respectively, and have an angle of 120° between them, as depicted in the picture below.



Since 60° is the complement of 120° $3\vec{x} + 2\vec{y}$ can be depicted as below.



By the cosine law:

$|3\vec{x} + 2\vec{y}|^2 = |3\vec{x}|^2 + |2\vec{y}|^2 - 2|3\vec{x}||2\vec{y}| \cos 60$

$|3\vec{x} + 2\vec{y}|^2 = 9|\vec{x}|^2 + 4|\vec{y}|^2 - 6|\vec{x}||\vec{y}|$

$|3\vec{x} + 2\vec{y}|^2 = 9 + 16 - 12$

$|3\vec{x} + 2\vec{y}| = \sqrt{13}$ or 3.61

The direction of $3\vec{x} + 2\vec{y}$ is θ, the angle from $\vec{x}$. This can be computed from the sine law:

$$\frac{|3\vec{x} + 2\vec{y}|}{\sin 60} = \frac{|2\vec{y}|}{\sin \theta}$$

$$\sin \theta = \frac{|2\vec{y}| \sin 60}{|3\vec{x} + 2\vec{y}|}$$

$$\theta = \sin^{-1}\left(\frac{|2\vec{y}| \sin 60}{|3\vec{x} + 2\vec{y}|}\right)$$

$$\theta = \sin^{-1}\left(\frac{(4) \sin 60}{\sqrt{13}}\right)$$

$\theta \doteq 73.9°$ relative to x

8. $\overrightarrow{DE} = \overrightarrow{CE} - \overrightarrow{CD}$

$\overrightarrow{DE} = \vec{b} - \vec{a}$

Also,

$\overrightarrow{BA} = \overrightarrow{CA} - \overrightarrow{CB}$

$\overrightarrow{BA} = 2\vec{b} - 2\vec{a}$

Thus,

$\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BA}$

CHAPTER 6
Introduction to Vectors

Review of Prerequisite Skills, p. 273

1. a. $\frac{\sqrt{3}}{2}$ **c.** $\frac{1}{2}$ **e.** $\frac{\sqrt{2}}{2}$

b. $-\sqrt{3}$ **d.** $\frac{\sqrt{3}}{2}$ **f.** 1

2. Find BC using the Pythagorean theorem, $AC^2 = AB^2 + BC^2$.

$$\begin{aligned} BC^2 &= AC^2 - AB^2 \\ &= 10^2 - 6^2 \\ &= 64 \\ BC &= 8 \end{aligned}$$

Next, use the ratio $\tan A = \frac{\text{opposite}}{\text{adjacent}}$.

$$\begin{aligned} \tan A &= \frac{BC}{AB} \\ &= \frac{8}{6} \\ &= \frac{4}{3} \end{aligned}$$

3. a. To solve ΔABC, find measures of the sides and angles whose values are not given: AB, $\angle B$, and $\angle C$. Find AB using the Pythagorean theorem, $BC^2 = AB^2 + AC^2$.

$$\begin{aligned} AB^2 &= BC^2 - AC^2 \\ &= (37.0)^2 - (22.0)^2 \\ &= 885 \\ AB &= \sqrt{885} \\ &\doteq 29.7 \end{aligned}$$

Find $\angle B$ using the ratio $\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$.

$$\begin{aligned} \sin B &= \frac{AC}{BC} \\ &= \frac{22.0}{37.0} \\ \angle B &\doteq 36.5° \\ \angle C &= 90° - \angle B \\ \angle C &= 90° - 36.5° \\ \angle C &\doteq 53.5° \end{aligned}$$

b. Find measures of the angles whose values are not given. Find $\angle A$ using the cosine law, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

$$\begin{aligned} &= \frac{5^2 + 8^2 - 10^2}{2(5)(8)} \\ &= \frac{-11}{80} \\ \angle A &\doteq 97.9° \end{aligned}$$

Find $\angle B$ using the sine law.

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin A}{a} \\ \frac{\sin B}{5} &= \frac{\sin (97.9°)}{10} \\ \sin B &\doteq 0.5 \\ \angle B &\doteq 29.7° \end{aligned}$$

Find $\angle C$ using the sine law.

$$\begin{aligned} \frac{\sin C}{c} &= \frac{\sin A}{a} \\ \frac{\sin C}{8} &= \frac{\sin (97.9°)}{10} \\ \sin C &\doteq 0.8 \\ \angle C &\doteq 52.4° \end{aligned}$$

4. Since the sum of the internal angles of a triangle equals 180°, determine the measure of $\angle Z$ using $\angle X = 60°$ and $\angle Y = 70°$.

$$\begin{aligned} \angle Z &= 180° - (\angle X + \angle Y) \\ &= 180° - (60° + 70°) \\ &= 50° \end{aligned}$$

Find XY and YZ using the sine law.

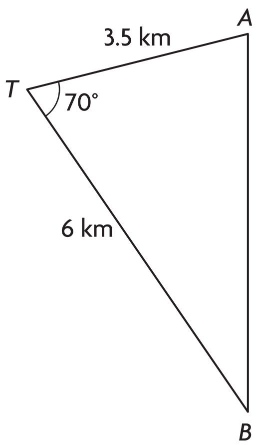
$$\begin{aligned} \frac{XY}{\sin Y} &= \frac{XY}{\sin Z} \\ \frac{XY}{\sin 70°} &= \frac{6}{\sin 50°} \\ XZ &\doteq 7.36 \\ \frac{YZ}{\sin X} &= \frac{XY}{\sin Z} \\ \frac{YZ}{\sin 60°} &= \frac{6}{\sin 50°} \\ YZ &\doteq 6.78 \end{aligned}$$

5. Find each angle using the cosine law.

$$\begin{aligned} \cos R &= \frac{RS^2 + RT^2 - ST^2}{2(RS)(RT)} \\ &= \frac{4^2 + 7^2 - 5^2}{2(4)(7)} \end{aligned}$$

$$= \frac{5}{7}$$
$$\angle R \doteq 44°$$
$$\cos S = \frac{RS^2 + ST^2 - RT^2}{2(RS)(ST)}$$
$$= \frac{4^2 + 5^2 - 7^2}{2(4)(5)}$$
$$= -\frac{1}{5}$$
$$\angle S \doteq 102°$$
$$\cos T = \frac{RT^2 + ST^2 - RS^2}{2(RT)(ST)}$$
$$= \frac{7^2 + 5^2 - 4^2}{2(7)(5)}$$
$$= \frac{29}{35}$$
$$\angle T \doteq 34°$$

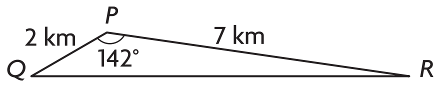
6.



Find AB (the distance between the airplanes) using the cosine law.

$$AB^2 = AT^2 + BT^2 - 2(AT)(BT)\cos T$$
$$= (3.5\text{ km})^2 + (6\text{ km})^2 - 2(3.5\text{ km})(6\text{ km})\cos 70°$$
$$\doteq 33.89\text{ km}^2$$
$$AB \doteq 5.82\text{ km}$$

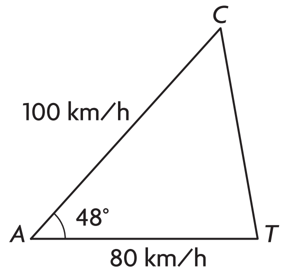
7.



Find QR using the cosine law.

$$QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos P$$
$$= (2\text{ km})^2 + (7\text{ km})^2 - 2(2\text{ km})(7\text{ km})\cos 142°$$
$$\doteq 75.06\text{ km}^2$$
$$QR \doteq 8.66\text{ km}$$

8.



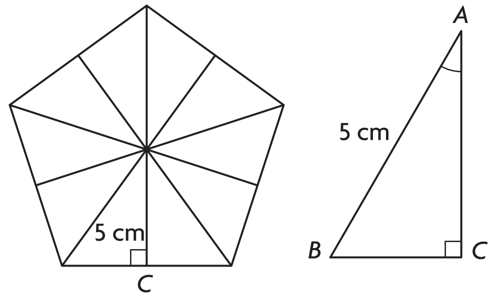
Find AC and AT using the speed of each vehicle and the elapsed time (in hours) until it was located, distance = speed × time.

$$AC = 100\text{ km/h} \times \frac{1}{4}\text{ h}$$
$$= 25\text{ km}$$
$$AT = 80\text{ km/h} \times \frac{1}{3}\text{ h}$$
$$= 26\frac{2}{3}\text{ km}$$

Find CT using the cosine law.

$$CT^2 = AC^2 + AT^2 - 2(AC)(AT)\cos A$$
$$= (25\text{ km})^2 + \left(26\frac{2}{3}\text{ km}\right)^2 - 2(25\text{ km})\left(26\frac{2}{3}\text{ km}\right)\cos 48°$$
$$\doteq 443.94\text{ km}^2$$
$$CT \doteq 21.1\text{ km}$$

9.



The pentagon can be divided into 10 congruent right triangles with height AC and base BC.

$$10 \times \angle A = 360°$$
$$\angle A = 36°$$

Find AC and BC using trigonometric ratios.

$$AC = AB \times \cos A$$
$$= 5\cos 36°$$
$$\doteq 4.0\text{ cm}$$
$$BC = AB \times \sin A$$
$$= 5\sin 36°$$
$$\doteq 2.9\text{ cm}$$

The area of the pentagon is the sum of the areas of the 10 right triangles. Use the area of $\triangle ABC$ to determine the area of the pentagon.

$$\begin{aligned}\text{Area}_{\text{pentagon}} &= 10 \times \frac{1}{2}(BC)(AC)\\ &= 10 \times \frac{1}{2}(2.9\text{ cm})(4.0\text{ cm})\\ &= 59.4\text{ cm}^2\end{aligned}$$

6.1 An Introduction to Vectors, pp. 279–281

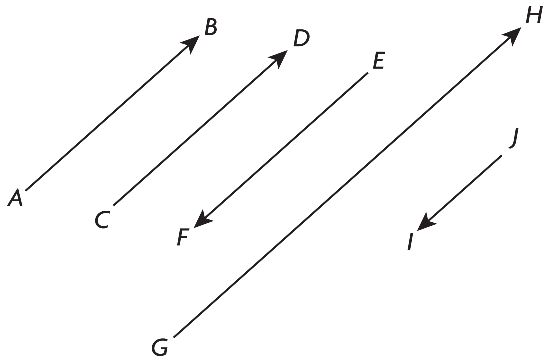
1. a. False. Two vectors with the same magnitude can have different directions, so they are not equal.
b. True. Equal vectors have the same direction and the same magnitude.
c. False. Equal or opposite vectors must be parallel and have the same magnitude. If two parallel vectors have different magnitude, they cannot be equal or opposite.
d. False. Equal or opposite vectors must be parallel and have the same magnitude. Two vectors with the same magnitude can have directions that are not parallel, so they are not equal or opposite.
2. Vectors must have a magnitude and direction. For some scalars, it is clear what is meant by just the number. Other scalars are related to the magnitude of a vector.

- Height is a scalar. Height is the distance (see below) from one end to the other end. No direction is given.
- Temperature is a scalar. Negative temperatures are below freezing, but this is not a direction.
- Weight is a vector. It is the force (see below) of gravity acting on your mass.
- Mass is a scalar. There is no direction given.
- Area is a scalar. It is the amount space inside a two-dimensional object. It does not have direction.
- Volume is a scalar. It is the amount of space inside a three-dimensional object. No direction is given.
- Distance is a scalar. The distance between two points does not have direction.
- Displacement is a vector. Its magnitude is related to the scalar distance, but it gives a direction.
- Speed is a scalar. It is the rate of change of distance (a scalar) with respect to time, but does not give a direction.
- Force is a vector. It is a push or pull in a certain direction.
- Velocity is a vector. It is the rate of change of displacement (a vector) with respect to time. Its magnitude is related to the scalar speed.

3. Answers may vary. For example: Friction resists the motion between two surfaces in contact by acting in the opposite direction of motion.

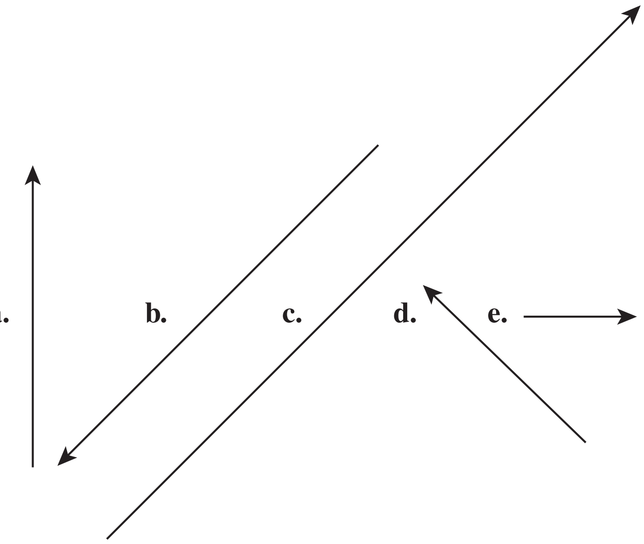
- A rolling ball stops due to friction which resists the direction of motion.
- A swinging pendulum stops due to friction resisting the swinging pendulum.

4. Answers may vary. For example:
a. $\overrightarrow{AD} = \overrightarrow{BC}$; $\overrightarrow{AB} = \overrightarrow{DC}$; $\overrightarrow{AE} = \overrightarrow{EC}$; $\overrightarrow{DE} = \overrightarrow{EB}$
b. $\overrightarrow{AD} = -\overrightarrow{CB}$; $\overrightarrow{AB} = -\overrightarrow{CD}$; $\overrightarrow{AE} = -\overrightarrow{CE}$; $\overrightarrow{ED} = -\overrightarrow{EB}$; $\overrightarrow{DA} = -\overrightarrow{BC}$
c. $\overrightarrow{AC}$ & $\overrightarrow{DB}$; $\overrightarrow{AE}$ & $\overrightarrow{EB}$; $\overrightarrow{EC}$ & $\overrightarrow{DE}$; $\overrightarrow{AB}$ & $\overrightarrow{CB}$
5.



a. $\overrightarrow{AB} = \overrightarrow{CD}$
b. $\overrightarrow{AB} = -\overrightarrow{EF}$
c. $|\overrightarrow{AB}| = |\overrightarrow{EF}|$ but $\overrightarrow{AB} \neq \overrightarrow{EF}$
d. $\overrightarrow{GH} = 2\overrightarrow{AB}$
e. $\overrightarrow{AB} = -2\overrightarrow{JI}$

6.



7. a. 100 km/h, south
b. 50 km/h, west
c. 100 km/h, northeast
d. 25 km/h, northwest
e. 60 km/h, east

$$\begin{aligned}\text{Area}_{\text{pentagon}} &= 10 \times \frac{1}{2}(BC)(AC)\\ &= 10 \times \frac{1}{2}(2.9\text{ cm})(4.0\text{ cm})\\ &= 59.4\text{ cm}^2\end{aligned}$$

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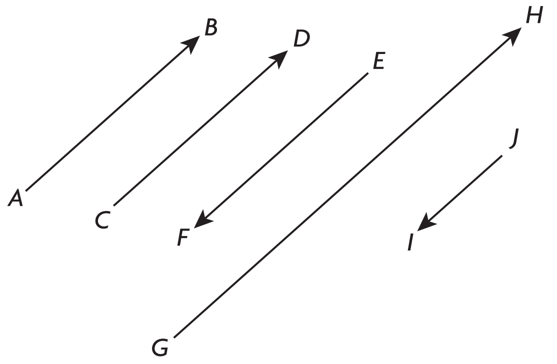
c. $\overrightarrow{AC}$ & $\overrightarrow{DB}$; $\overrightarrow{AE}$ & $\overrightarrow{EB}$; $\overrightarrow{EC}$ & $\overrightarrow{DE}$; $\overrightarrow{AB}$ & $\overrightarrow{CB}$
5.



a. $\overrightarrow{AB} = \overrightarrow{CD}$
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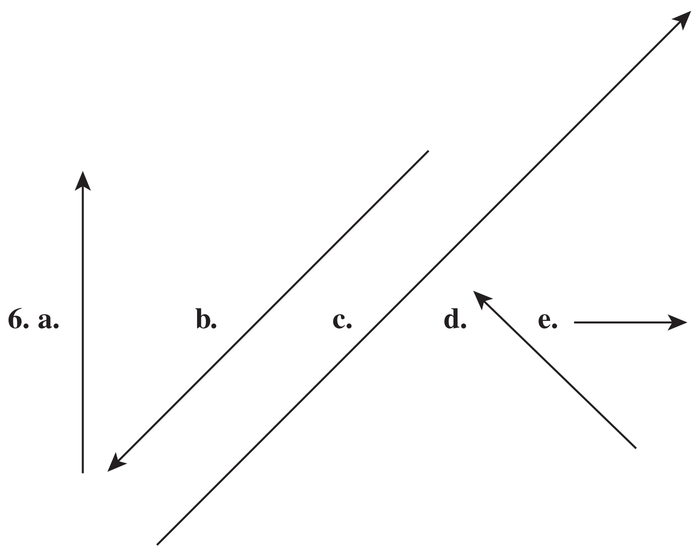
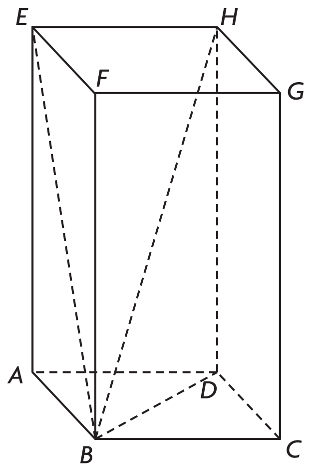
6.



7. a. 100 km/h, south
b. 50 km/h, west
c. 100 km/h, northeast
d. 25 km/h, northwest
e. 60 km/h, east

8. a. 400 km/h, due south
b. 70 km/h, southwesterly
c. 30 km/h southeasterly
d. 25 km/h, due east
9. a. i. False. They have equal magnitude, but opposite direction.
ii. True. They have equal magnitude.
iii. True. The base has sides of equal length, so the vectors have equal magnitude.
iv. True. They have equal magnitude and direction.
b.



To calculate $|\overrightarrow{BD}|$, $|\overrightarrow{BE}|$ and $|\overrightarrow{BH}|$, find the lengths of their corresponding line segments BD, BE and BH using the Pythagorean theorem.

$$\begin{aligned} BD^2 &= AB^2 + AD^2 \\ &= 3^2 + 3^2 \\ BD &= \sqrt{18} \\ BE^2 &= AB^2 + AE^2 \\ &= 3^2 + 8^2 \\ BE &= \sqrt{73} \\ BH^2 &= BD^2 + DH^2 \\ &= (\sqrt{18})^2 + 8^2 \\ BH &= \sqrt{82} \end{aligned}$$

10. a. The tangent vector describes James's velocity at that moment. At point A his speed is 15 km/h and he is heading north. The tangent vector shows his velocity is 15 km/h, north.
b. The length of the vector represents the magnitude of James's velocity at that point. James's speed is the same as the magnitude of James's velocity.
c. The magnitude of James's velocity (his speed) is constant, but the direction of his velocity changes at every point.
d. Point C
e. This point is halfway between D and A, which is $\frac{7}{8}$ of the way around the circle. Since he is running 15 km/h and the track is 1 km in circumference, he can run around the track 15 times in one hour. That means each lap takes him 4 minutes. $\frac{7}{8}$ of 4 minutes is 3.5 minutes.
f. When he has travelled $\frac{3}{8}$ of a lap, James will be halfway between B and C and will be heading southwest.
11. a. Find the magnitude of $\overrightarrow{AB}$ using the distance formula.

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(x_A - x_B)^2 + (y_B - y_A)^2} \\ &= \sqrt{(-4+1)^2 + (3-2)^2} \\ &= \sqrt{10} \text{ or } 3.16 \end{aligned}$$

b. $\overrightarrow{CD} = \overrightarrow{AB}$. $\overrightarrow{AB}$ moves from $A(-4, 2)$ to $B(-1, 3)$ or $(x_B, y_B) = (x_A + 3, y_A + 1)$. Use this to find point D.

$$\begin{aligned} (x_D, y_D) &= (x_C + 3, y_C + 1) \\ &= (-6 + 3, 0 + 1) \\ &= (-3, 1) \end{aligned}$$

c. $\overrightarrow{EF} = \overrightarrow{AB}$. Find point E using

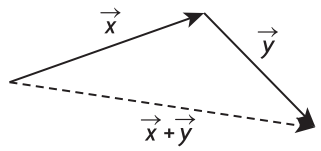
$$\begin{aligned} (x_A, y_A) &= (x_B - 3, y_B - 1). \\ (x_E, y_E) &= (x_F - 3, y_F - 1) \\ &= (3 - 3, -2 - 1) \\ &= (0, -3) \end{aligned}$$

d. $\overrightarrow{GH} = -\overrightarrow{AB}$, and moves in the opposite direction as $\overrightarrow{AB}$.

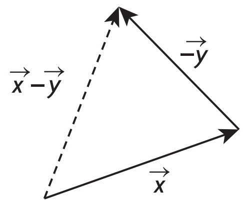
$$\begin{aligned} (x_H, y_H) &= (x_G - 3, y_G - 1). \\ (x_H, y_H) &= (x_G - 3, y_G - 1) \\ &= (3 - 3, 1 - 1) \\ &= (0, 0) \end{aligned}$$

6.2 Vector Addition, pp. 290–292

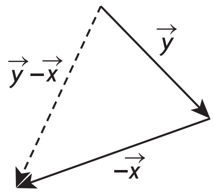
1. a.



b.



c.



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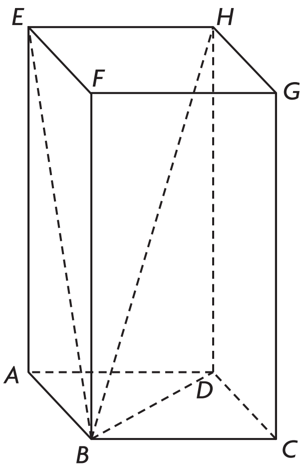

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6.2 Vector Addition, pp. 290–292

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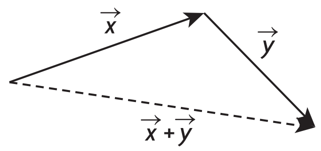
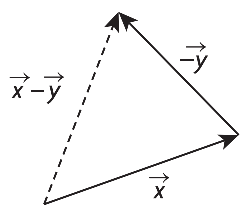
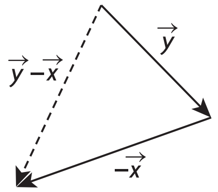


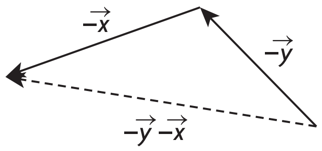
b.



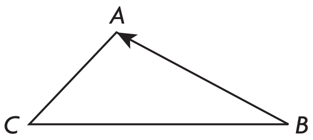
c.



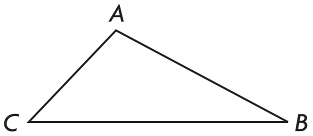
d.



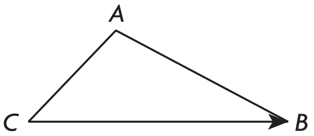
2. a. $\overrightarrow{BA}$



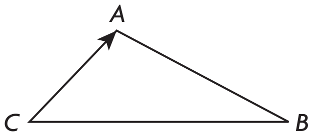
b. $\vec{0}$



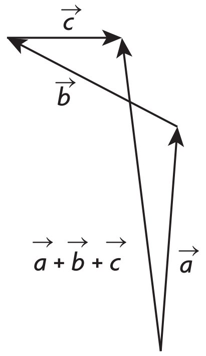
c. $\overrightarrow{CB}$



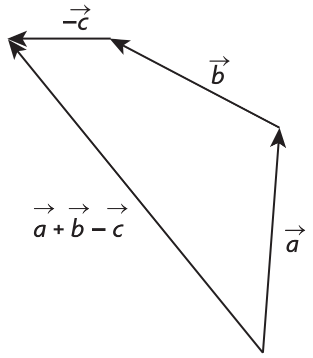
d. $\overrightarrow{CA}$



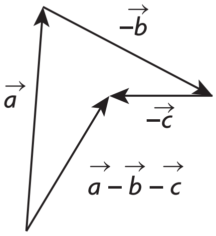
3. a.



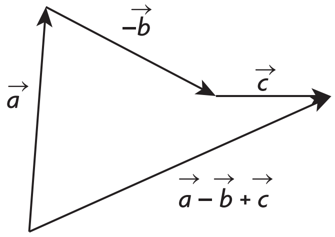
b.



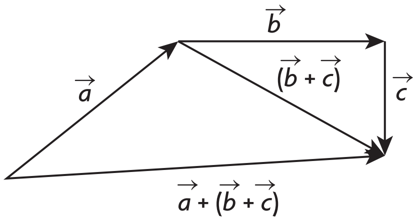
c.



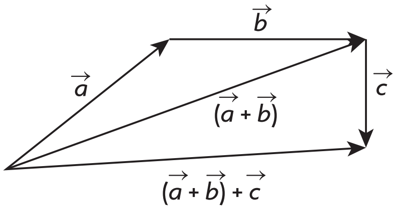
d.



4. a.

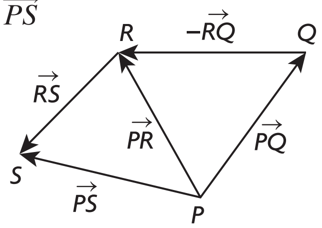


b.

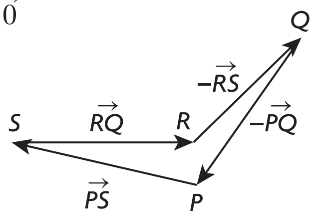


c. The resultant vectors are the same. The order in which you add vectors does not matter.
$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

5. a. $\overrightarrow{PS}$

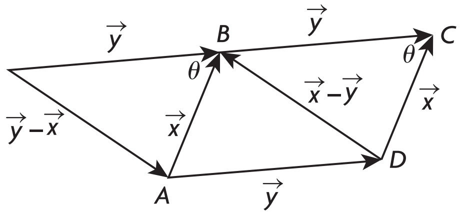


b. $\vec{0}$

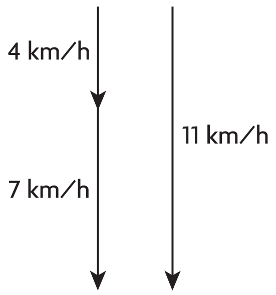


6. $\vec{x} + \vec{y} = \overrightarrow{MR} + \overrightarrow{RS}$
$= \overrightarrow{MS}$
$\vec{z} + \vec{t} = \overrightarrow{ST} + \overrightarrow{TQ}$
$= \overrightarrow{SQ}$
so
$(\vec{x} + \vec{y}) + (\vec{z} + \vec{t}) = \overrightarrow{MS} + \overrightarrow{SQ}$
$= \overrightarrow{MQ}$

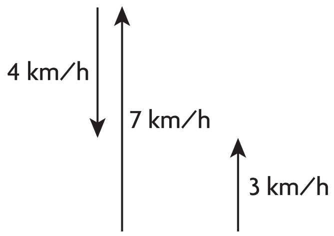
7. a. $-\vec{x}$
b. $\vec{y}$
c. $\vec{x} + \vec{y}$
d. $-\vec{x} + \vec{y}$
e. $\vec{x} + \vec{y} + \vec{z}$
f. $-\vec{x} - \vec{y}$
g. $-\vec{x} + \vec{y} + \vec{z}$
h. $-\vec{x} - \vec{z}$
8. a.



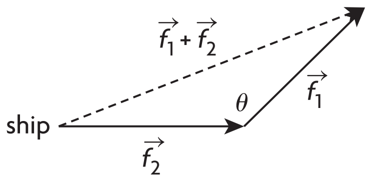
b. See the figure in part a. for the drawn vectors.
$|\vec{y} - \vec{x}|^2 = |\vec{y}|^2 + |\vec{x}|^2 - 2|\vec{y}||-\vec{x}| \cos(\theta)$ and
$|-\vec{x}| = |\vec{x}|$, so $|\vec{y} - \vec{x}|^2 = |\vec{x} - \vec{y}|^2$
9. a. Maria's velocity is 11 km/h downstream.
b.



c.



Maria's speed is 3 km/h.
10. a.

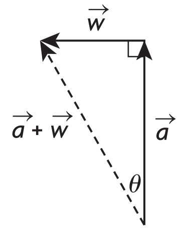


b. The vectors form a triangle with side lengths $|\vec{f_1}|$, $|\vec{f_2}|$ and $|\vec{f_1} + \vec{f_2}|$. Find $|\vec{f_1} + \vec{f_2}|$ using the cosine law.

$$|\vec{f_1} + \vec{f_2}|^2 = |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}| \cos(\theta)$$

$$|\vec{f_1} + \vec{f_2}| = \sqrt{|\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos(\theta)}$$

11.



Find $|\vec{a} + \vec{w}|$ using the Pythagorean theorem.

$$\begin{aligned}|\vec{a} + \vec{w}|^2 &= |\vec{a}|^2 + |\vec{w}|^2 \\ &= (150 \text{ km/h})^2 + (80 \text{ km/h})^2 \\ &= 28\,900 \\ |\vec{a} + \vec{w}|^2 &= 170\end{aligned}$$

Find the direction of $\vec{a} + \vec{w}$ using the ratio

$$\tan(\theta) = \frac{|\vec{w}|}{|\vec{a}|}$$

$$\begin{aligned}\theta &= \tan^{-1}\frac{80 \text{ km/h}}{150 \text{ km/h}} \\ &\doteq \text{N } 28.1° \text{ W}\end{aligned}$$

$\vec{a} + \vec{w} = 170$ km/h, N 28.1° W
12. $\vec{x}$, $\vec{y}$, and $\vec{x} + \vec{y}$ form a right triangle. Find $|\vec{x} + \vec{y}|$ using the Pythagorean theorem.

$$\begin{aligned}|\vec{x} + \vec{y}|^2 &= |\vec{x}|^2 + |\vec{y}|^2 \\ &= 7^2 + 24^2 \\ &= 625 \\ |\vec{x} + \vec{y}| &= 25\end{aligned}$$

Find the angle between $\vec{x}$ and $\vec{x} + \vec{y}$ using the ratio

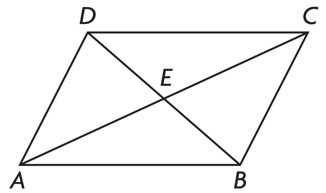
$$\tan(\theta) = \frac{|\vec{y}|}{|\vec{x}|}$$

$$\begin{aligned}\theta &= \tan^{-1}\frac{24}{7} \\ &\doteq 73.7°\end{aligned}$$

13. Find $|\overrightarrow{AB} + \overrightarrow{AC}|$ using the cosine law and the supplement to the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

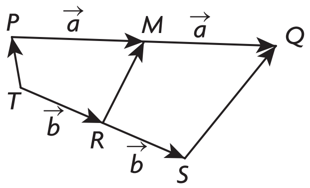
$$\begin{aligned}&|\overrightarrow{AB} + \overrightarrow{AC}|^2 \\ &= |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{AC}| \cos(30°) \\ &= 1^2 + 1^2 - 2(1)(1)\frac{\sqrt{3}}{2} \\ &= 2 - \sqrt{3} \\ &|\overrightarrow{AB} + \overrightarrow{AC}| \doteq 0.52\end{aligned}$$

14.



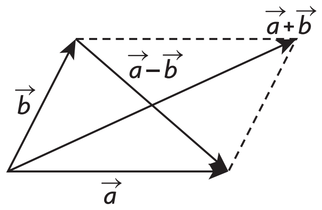
The diagonals of a parallelogram bisect each other.
So $\overrightarrow{EA} = -\overrightarrow{EC}$ and $\overrightarrow{ED} = -\overrightarrow{EB}$.
Therefore, $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \vec{0}$.

15.



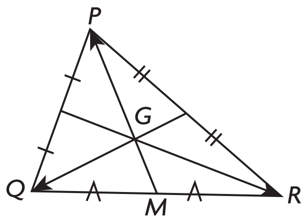
Multiple applications of the Triangle Law for adding vectors show that
$\overrightarrow{RM} + \vec{b} = \vec{a} + \overrightarrow{TP}$ (since both are equal to the undrawn vector $\overrightarrow{TM}$), and that
$\overrightarrow{RM} + \vec{a} = \vec{b} + \overrightarrow{SQ}$ (since both are equal to the undrawn vector $\overrightarrow{RQ}$)
Adding these two equations gives
$$2\,\overrightarrow{RM} + \vec{a} + \vec{b} = \vec{a} + \vec{b} + \overrightarrow{TP} + \overrightarrow{SQ}$$
$$2\,\overrightarrow{RM} = \overrightarrow{TP} + \overrightarrow{SQ}$$

16. $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ represent the diagonals of a parallelogram with sides $\vec{a}$ and $\vec{b}$.



Since $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ and the only parallelogram with equal diagonals is a rectangle, the parallelogram must also be a rectangle.

17.

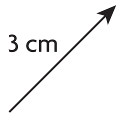


Let point M be defined as shown. Two applications of the Triangle Law for adding vectors show that
$\overrightarrow{GQ} + \overrightarrow{QM} + \overrightarrow{MG} = \vec{0}$
$\overrightarrow{GR} + \overrightarrow{RM} + \overrightarrow{MG} = \vec{0}$
Adding these two equations gives
$\overrightarrow{GQ} + \overrightarrow{QM} + 2\,\overrightarrow{MG} + \overrightarrow{GR} + \overrightarrow{RM} = \vec{0}$
From the given information,
$2\,\overrightarrow{MG} = \overrightarrow{GP}$ and
$\overrightarrow{QM} + \overrightarrow{RM} = \vec{0}$ (since they are opposing vectors of equal length), so
$\overrightarrow{GQ} + \overrightarrow{GP} + \overrightarrow{GR} = \vec{0}$, as desired.

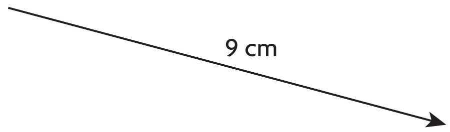
6.3 Multiplication of a Vector by a Scalar, pp. 298–301

1. A vector cannot equal a scalar.

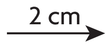
2. a.



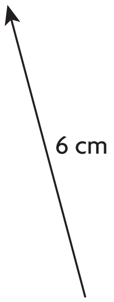
b.



c.

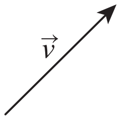


d.

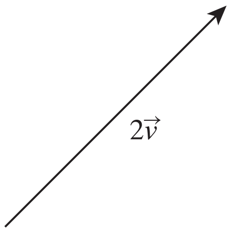


3. E25°N describes a direction that is 25° toward the north of due east (90° east of north), in other words $90° - 25° = 65°$ toward the east of due north. N65°E and "a bearing of 65°" both describe a direction that is 65° toward the east of due north. So, each is describing the same direction in a different way.

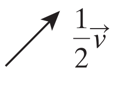
4. Answers may vary. For example:



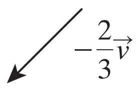
a.



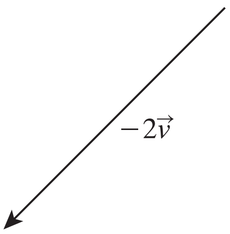
b.



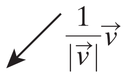
c.

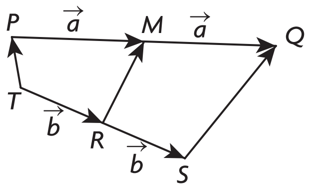


d.



e.



15.


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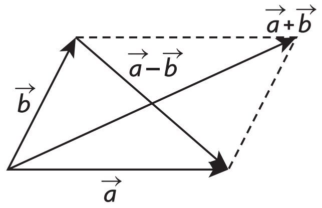
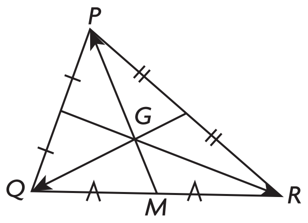
$2\,\overrightarrow{RM} = \overrightarrow{TP} + \overrightarrow{SQ}$

16. $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ represent the diagonals of a parallelogram with sides $\vec{a}$ and $\vec{b}$.



Since $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ and the only parallelogram with equal diagonals is a rectangle, the parallelogram must also be a rectangle.

17.


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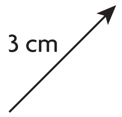
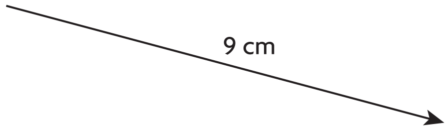
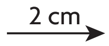
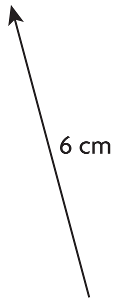
6.3 Multiplication of a Vector by a Scalar, pp. 298–301

1. A vector cannot equal a scalar.

2. a.


b.


c.


d.


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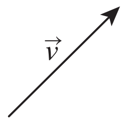
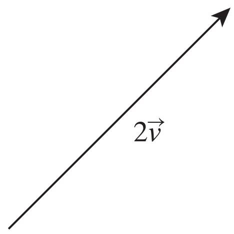
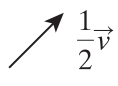
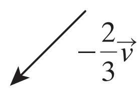
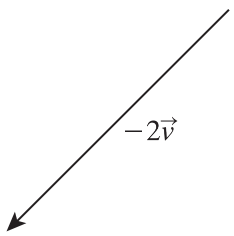
4. Answers may vary. For example:

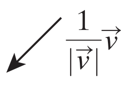


a.


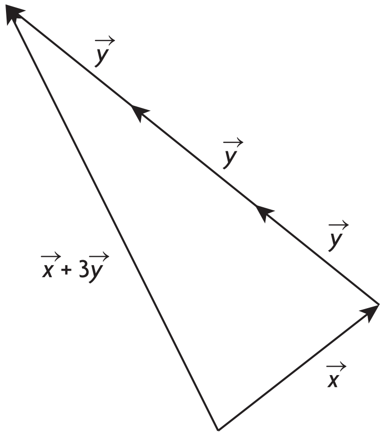
b.


c.


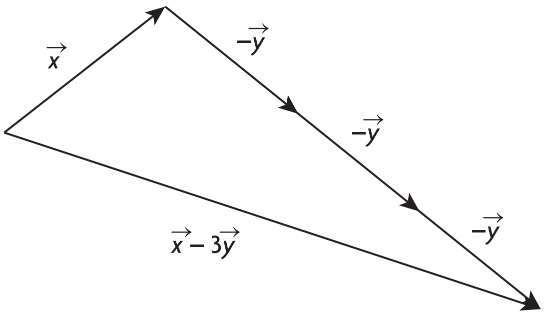
d.


e.


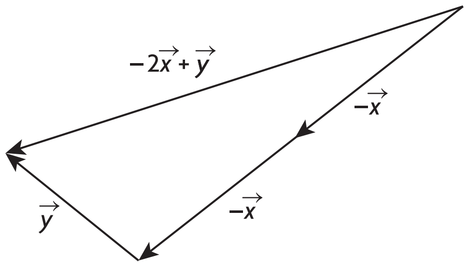
5. a.



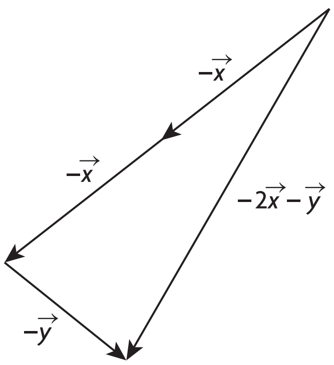
b.



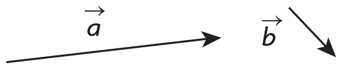
c.



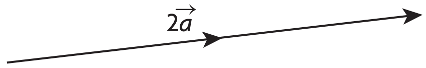
d.



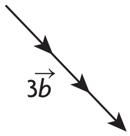
6. Answers may vary. For example:



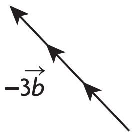
a.



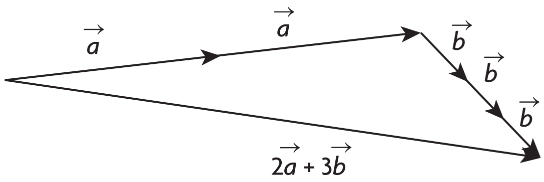
b.



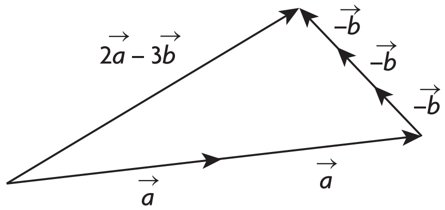
c.



d.



e.



7. a.

$$\vec{c} = 2\vec{a}, \vec{b} = \frac{3}{2}\vec{a}$$

$$m\vec{c} + n\vec{b} = \vec{0}$$

$$m(2\vec{a}) + n\left(\frac{3}{2}\vec{a}\right) = \vec{0}$$

$$m(4\vec{a}) + n\,(3\vec{a}) = \vec{0}$$

$m = 3$ and $n = -4$ satisfy the equation, as does any multiple of the pair $(3, -4)$. There are infinitely many values possible.

b.

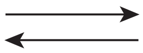
$$\vec{c} = 2\vec{a}, \vec{b} = \frac{3}{2}\vec{a}$$

$$d\vec{a} + e\vec{b} + f\vec{c} = \vec{0}$$

$$d\vec{a} + e\left(\frac{3}{2}\vec{a}\right) + f(2\vec{a}) = \vec{0}$$

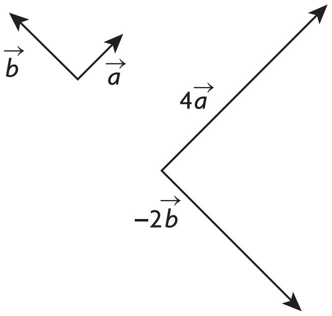
$$2d\vec{a} + 3e\vec{a} + 4f\vec{a} = \vec{0}$$

$d = 2$, $e = 0$, and $f = -1$ satisfy the equation, as does any multiple of the triple $(2, 0, -1)$. There are infinitely many values possible.

8.  or 

$\vec{a}$ and $\vec{b}$ are collinear, so $\vec{a} = k\vec{b}$, where k is a nonzero scalar. Since $|\vec{a}| = |\vec{b}|$, k can only be -1 or 1.

9.



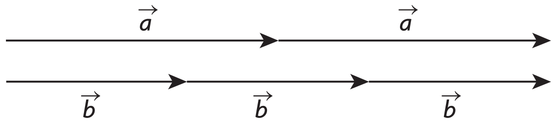
Yes

10. Two vectors are collinear if and only if they can be related by a scalar multiple. In this case $\vec{a} \neq k\vec{b}$
a. collinear
b. not collinear
c. not collinear
d. collinear

11. a. $\frac{1}{|\vec{x}|}\vec{x}$ is a vector with length 1 unit in the same direction as $\vec{x}$.

b. $-\frac{1}{|\vec{x}|}\vec{x}$ is a vector with length 1 unit in the opposite direction of $\vec{x}$.

12.



$$m = \frac{2}{3}$$

13. a. $-\frac{2}{3}\vec{a}$

b. $\frac{1}{3}\vec{a}$

c. $\frac{1}{3}|\vec{a}|$

d. $\frac{2}{3}|\vec{a}|$

e. $\frac{4}{3}\vec{a}$

14. $\vec{x}$ and $\vec{y}$ make an angle of 90°, so you may find $|2\vec{x} + \vec{y}|$ using the Pythagorean theorem.

$$|2\vec{x} + \vec{y}|^2 = |2\vec{x}|^2 + |\vec{y}|^2$$
$$= 2^2 + 1^2$$
$$|2\vec{x} + \vec{y}| = \sqrt{5} \text{ or } 2.24$$

Find the direction of $2\vec{x} + \vec{y}$ using the ratio

$$\tan(\theta) = \frac{|\vec{y}|}{|2\vec{x}|}$$
$$\theta = \tan^{-1}\frac{1}{2}$$
$$\doteq 26.6° \text{ from } \vec{x} \text{ towards } 2\vec{x} + \vec{y}$$

15. Find $|2\vec{x} + \vec{y}|$ using the cosine law, and the supplement to the angle between $\vec{x}$ and $\vec{y}$.

$$|2\vec{x} + \vec{y}|^2 = |2\vec{x}|^2 + |\vec{y}|^2 - 2|2\vec{x}||\vec{y}|\cos(150°)$$
$$= 2^2 + 1^2 - 2(2)(1)\frac{-\sqrt{3}}{2}$$
$$|2\vec{x} + \vec{y}| \doteq 2.91$$

Find the direction of $2\vec{x} + \vec{y}$ using the sine law.

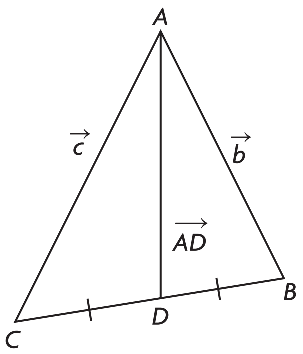
$$\frac{\sin\theta}{|\vec{y}|} = \frac{\sin(150°)}{|2\vec{x} + \vec{y}|}$$
$$\sin\theta \doteq (1)\frac{\frac{1}{2}}{2.91}$$
$$\theta \doteq 9.9° \text{ from } \vec{x} \text{ towards } \vec{y}$$

16. $\vec{b} = \frac{1}{|\vec{a}|}\vec{a}$

$$|\vec{b}| = \left|\frac{1}{|\vec{a}|}\vec{a}\right|$$
$$|\vec{b}| = \frac{1}{|\vec{a}|}|\vec{a}|$$
$$|\vec{b}| = 1$$

$\vec{b}$ is a positive multiple of $\vec{a}$, so it points in the same direction as $\vec{a}$ and has magnitude 1. It is a unit vector in the same direction as $\vec{a}$.

17.

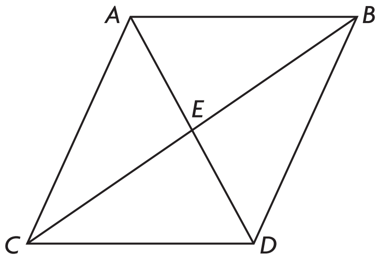


$$\overrightarrow{AD} = \vec{c} + \overrightarrow{CD}$$
$$\overrightarrow{AD} = \vec{b} + \overrightarrow{BD}$$
$$2\overrightarrow{AD} = \vec{c} + \vec{b} + \overrightarrow{CD} + \overrightarrow{BD}$$

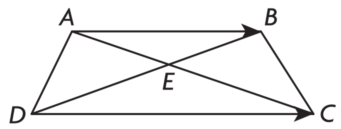
But $\overrightarrow{CD} + \overrightarrow{BD} = 0$.
So $2\overrightarrow{AD} = \vec{c} + \vec{b}$, or $\overrightarrow{AD} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{b}$.

18. $\overrightarrow{PM} = \vec{a}$ and $\overrightarrow{PN} = \vec{b}$
so $\overrightarrow{MN} = \overrightarrow{PN} - \overrightarrow{PM}$
$= \vec{b} - \vec{a}$
$\overrightarrow{PQ} = 2\vec{a}$ and $\overrightarrow{PR} = 2\vec{b}$
so $\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$
$= 2\vec{b} - 2\vec{a}$
Notice that
$2\overrightarrow{MN} = 2\vec{b} - 2\vec{a}$
$= \overrightarrow{QR}$
We can conclude that $\overrightarrow{QR}$ is parallel to $\overrightarrow{MN}$ and $|\overrightarrow{QR}| = 2|\overrightarrow{MN}|$.

19.



Answers may vary. For example:
a. $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{CD}$
b. $\vec{u} = \overrightarrow{AD}$ and $\vec{v} = \overrightarrow{AE}$
c. $\vec{u} = \overrightarrow{AC}$ and $\vec{v} = \overrightarrow{DB}$
d. $\vec{u} = \overrightarrow{ED}$ and $\vec{v} = \overrightarrow{AD}$
20. a. Since the magnitude of $\vec{x}$ is three times the magnitude of $\vec{y}$ and because the given sum is 0, $m\vec{x}$ must be in the opposite direction of $n\vec{y}$ and $|n| = 3|m|$.
b. Whether $\vec{x}$ and $\vec{y}$ are collinear or not, $m = 0$ and $n = 0$ will make the given equation true.
21. a. $\overrightarrow{CD} = \vec{b} - \vec{a}$
b. $\overrightarrow{BE} = 2\vec{b} - 2\vec{a}$
$= 2(\vec{b} - \vec{a})$
$= 2\overrightarrow{CD}$
The two are therefore parallel (collinear) and $|\overrightarrow{BE}| = 2|\overrightarrow{CD}|$
22.

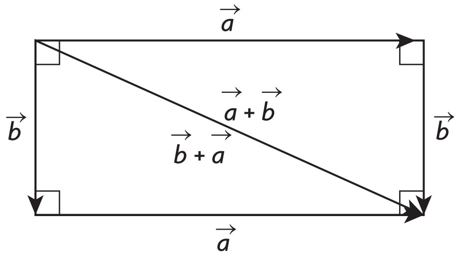


Applying the triangle law for adding vectors shows that
$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$
The given information states that
$$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{DC}$$
$$\frac{3}{2}\overrightarrow{AB} = \overrightarrow{DC}$$
By the properties of trapezoids, this gives
$\frac{3}{2}\overrightarrow{AE} = \overrightarrow{EC}$, and since
$\overrightarrow{AC} = \overrightarrow{AE} + \overrightarrow{EC}$, the original equation gives
$$\overrightarrow{AE} + \frac{3}{2}\overrightarrow{AE} = \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}$$
$$\frac{5}{2}\overrightarrow{AE} = \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}$$
$$\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AD} + \frac{3}{5}\overrightarrow{AB}$$

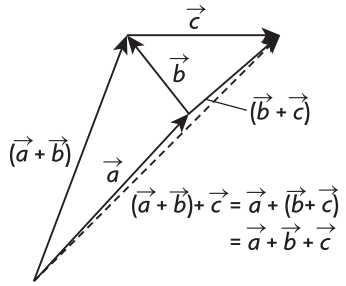
6.4 Properties of Vectors, pp. 306–307

1. a. 0
b. 1
c. $\vec{0}$
d. 1

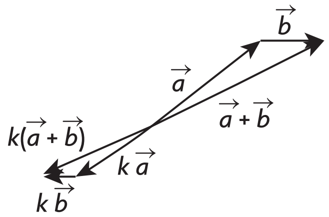
2. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



3.



4. Answers may vary. For example:



5. $\overrightarrow{PQ} = \overrightarrow{RQ} + \overrightarrow{SR} + \overrightarrow{TS} + \overrightarrow{PT}$
$= (\overrightarrow{RQ} + \overrightarrow{SR}) + (\overrightarrow{TS} + \overrightarrow{PT})$
$= (\overrightarrow{SR} + \overrightarrow{RQ}) + (\overrightarrow{PT} + \overrightarrow{TS})$
$= \overrightarrow{SQ} + \overrightarrow{PS}$
$= \overrightarrow{PS} + \overrightarrow{SQ}$
$= \overrightarrow{PQ}$
6. a. $\overrightarrow{EC}$
b. $\vec{0}$
c. Yes, the diagonals of a rectangular prism are of equal length
7. $= 3\vec{a} - 6\vec{b} - 15\vec{c} - 6\vec{a} + 12\vec{b} - 6\vec{c} - \vec{a} + 3\vec{b} - 3\vec{c}$
$= -4\vec{a} + 9\vec{b} - 24\vec{c}$
8. a. $= 6\vec{i} - 8\vec{j} + 2\vec{k} + 6\vec{i} - 9\vec{j} + 3\vec{k}$
$= 12\vec{i} - 17\vec{j} + 5\vec{k}$
b. $= 3\vec{i} - 4\vec{j} + \vec{k} - 10\vec{i} + 15\vec{j} - 5\vec{k}$
$= -7\vec{i} + 11\vec{j} - 4\vec{k}$
c. $= 2(3\vec{i} - 4\vec{j} + \vec{k} + 6\vec{i} - 9\vec{j} + 3\vec{k})$
$-3(-6\vec{i} + 8\vec{j} - 2\vec{k} + 14\vec{i} - 21\vec{j} + 7\vec{k})$
$= -6\vec{i} + 13\vec{j} - 7\vec{k}$
9. Solve the first equation for $\vec{x}$.
$$\vec{x} = \frac{1}{2}\vec{a} - \frac{3}{2}\vec{y}$$

Answers may vary. For example:

a. $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{CD}$
b. $\vec{u} = \overrightarrow{AD}$ and $\vec{v} = \overrightarrow{AE}$
c. $\vec{u} = \overrightarrow{AC}$ and $\vec{v} = \overrightarrow{DB}$
d. $\vec{u} = \overrightarrow{ED}$ and $\vec{v} = \overrightarrow{AD}$

20. a. Since the magnitude of $\vec{x}$ is three times the magnitude of $\vec{y}$ and because the given sum is 0, $m\vec{x}$ must be in the opposite direction of $n\vec{y}$ and $|n| = 3|m|$.

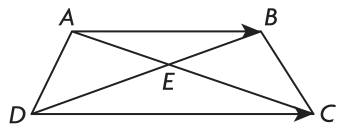
b. Whether $\vec{x}$ and $\vec{y}$ are collinear or not, $m = 0$ and $n = 0$ will make the given equation true.

21. a. $\overrightarrow{CD} = \vec{b} - \vec{a}$

b. $\overrightarrow{BE} = 2\vec{b} - 2\vec{a}$
$= 2(\vec{b} - \vec{a})$
$= 2\overrightarrow{CD}$

The two are therefore parallel (collinear) and $|\overrightarrow{BE}| = 2|\overrightarrow{CD}|$

22.



Applying the triangle law for adding vectors shows that

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

The given information states that

$$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{DC}$$

$$\frac{3}{2}\overrightarrow{AB} = \overrightarrow{DC}$$

By the properties of trapezoids, this gives

$\frac{3}{2}\overrightarrow{AE} = \overrightarrow{EC}$, and since

$\overrightarrow{AC} = \overrightarrow{AE} + \overrightarrow{EC}$, the original equation gives

$$\overrightarrow{AE} + \frac{3}{2}\overrightarrow{AE} = \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}$$

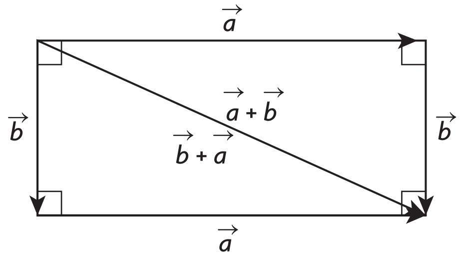
$$\frac{5}{2}\overrightarrow{AE} = \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}$$

$$\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AD} + \frac{3}{5}\overrightarrow{AB}$$

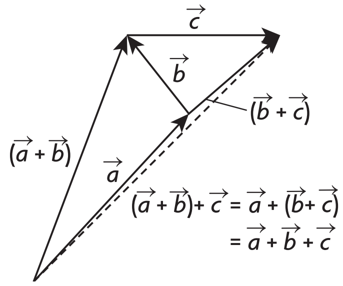
6.4 Properties of Vectors, pp. 306–307

1. a. 0
b. 1
c. $\vec{0}$
d. 1

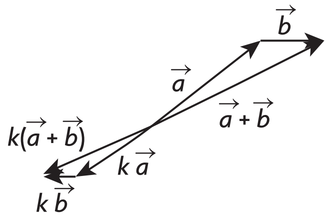
2. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



3.



4. Answers may vary. For example:



5. $\overrightarrow{PQ} = \overrightarrow{RQ} + \overrightarrow{SR} + \overrightarrow{TS} + \overrightarrow{PT}$
$= (\overrightarrow{RQ} + \overrightarrow{SR}) + (\overrightarrow{TS} + \overrightarrow{PT})$
$= (\overrightarrow{SR} + \overrightarrow{RQ}) + (\overrightarrow{PT} + \overrightarrow{TS})$
$= \overrightarrow{SQ} + \overrightarrow{PS}$
$= \overrightarrow{PS} + \overrightarrow{SQ}$
$= \overrightarrow{PQ}$

6. a. $\overrightarrow{EC}$
b. $\vec{0}$
c. Yes, the diagonals of a rectangular prism are of equal length

7. $= 3\vec{a} - 6\vec{b} - 15\vec{c} - 6\vec{a} + 12\vec{b} - 6\vec{c} - \vec{a} + 3\vec{b} - 3\vec{c}$
$= -4\vec{a} + 9\vec{b} - 24\vec{c}$

8. a. $= 6\vec{i} - 8\vec{j} + 2\vec{k} + 6\vec{i} - 9\vec{j} + 3\vec{k}$
$= 12\vec{i} - 17\vec{j} + 5\vec{k}$

b. $= 3\vec{i} - 4\vec{j} + \vec{k} - 10\vec{i} + 15\vec{j} - 5\vec{k}$
$= -7\vec{i} + 11\vec{j} - 4\vec{k}$

c. $= 2(3\vec{i} - 4\vec{j} + \vec{k} + 6\vec{i} - 9\vec{j} + 3\vec{k})$
$-3(-6\vec{i} + 8\vec{j} - 2\vec{k} + 14\vec{i} - 21\vec{j} + 7\vec{k})$
$= -6\vec{i} + 13\vec{j} - 7\vec{k}$

9. Solve the first equation for $\vec{x}$.

$$\vec{x} = \frac{1}{2}\vec{a} - \frac{3}{2}\vec{y}$$

Substitute into the second equation.

$$6\vec{b} = -\left(\frac{1}{2}\vec{a} - \frac{3}{2}\vec{y}\right) + 5\vec{y}$$

$$\vec{y} = \frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}$$

Lastly, find $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.

$$\vec{x} = \frac{1}{2}\vec{a} - \frac{3}{2}\left(\frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}\right)$$

$$= \frac{5}{13}\vec{a} - \frac{18}{13}\vec{b}$$

10. $\vec{a} = \vec{x} - \vec{y}$

$$= \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z} - (\vec{b} + \vec{z})$$

$$= \frac{2}{3}\vec{y} - \frac{2}{3}\vec{z} - \vec{b}$$

$$= \frac{2}{3}(\vec{y} - \vec{z}) - \vec{b}$$

$$= \frac{2}{3}\vec{b} - \vec{b}$$

$$= -\frac{1}{3}\vec{b}$$

11. a. $\overrightarrow{AG} = \vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{BH} = -\vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{CE} = -\vec{a} - \vec{b} + \vec{c}$

$\overrightarrow{DF} = \vec{a} - \vec{b} + \vec{c}$

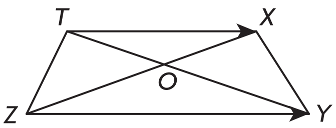
b. $|\overrightarrow{AG}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= |-\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= |\overrightarrow{BH}|^2$

Therefore, $|\overrightarrow{AG}| = |\overrightarrow{BH}|$

12.



Applying the triangle law for adding vectors shows that

$\overrightarrow{TY} = \overrightarrow{TZ} + \overrightarrow{ZY}$

The given information states that

$\overrightarrow{TX} = 2\,\overrightarrow{ZY}$

$\frac{1}{2}\overrightarrow{TX} = \overrightarrow{ZY}$

By the properties of trapezoids, this gives $\frac{1}{2}\overrightarrow{TO} = \overrightarrow{OY}$, and since $\overrightarrow{TY} = \overrightarrow{TO} + \overrightarrow{OY}$, the original equation gives

$$\overrightarrow{TO} + \frac{1}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$$

$$\frac{3}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$$

$$\overrightarrow{TO} = \frac{2}{3}\overrightarrow{TZ} + \frac{1}{3}\overrightarrow{TX}$$

Mid-Chapter Review, pp. 308–309

1. a. $\overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{BA} = \overrightarrow{CD}$

$\overrightarrow{AD} = \overrightarrow{BC}$

$\overrightarrow{CB} = \overrightarrow{DA}$

There is not enough information to determine if there is a vector equal to $\overrightarrow{AP}$.

b. $|\overrightarrow{PD}| = |\overrightarrow{DA}|$

$= |\overrightarrow{BC}|$ (parallelogram)

2. a. $\overrightarrow{RV}$

b. $\overrightarrow{RV}$

c. $\overrightarrow{PS}$

d. $\overrightarrow{RU}$

e. $\overrightarrow{PS}$

f. $\overrightarrow{PQ}$

3. a. Find $|\vec{a} + \vec{b}|$ using the cosine law, and the supplement to the angle between the vectors.

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos 60°$

$= 3^2 + 4^2 - 2(3)(4)\frac{1}{2}$

$= 3$

$|\vec{a} + \vec{b}| = \sqrt{3}$

b. Find θ using the ratio

$$\tan\theta = \frac{|\vec{b}|}{|\vec{a}|}$$

$$= \frac{4}{3}$$

$$\theta = \tan^{-1}\frac{4}{3}$$

$$\doteq 53°$$

4. $t = 4$ or $t = -4$

5. In quadrilateral $PQRS$, look at $\triangle PQR$. Joining the midpoints B and C creates a vector $\overrightarrow{BC}$ that is parallel to $\overrightarrow{PR}$ and half the length of $\overrightarrow{PR}$. Look at $\triangle SPR$. Joining the midpoints A and D creates a vector $\overrightarrow{AD}$ that is parallel to $\overrightarrow{PR}$ and half the length of $\overrightarrow{PR}$. $\overrightarrow{BC}$ is parallel to $\overrightarrow{AD}$ and equal in length to $\overrightarrow{AD}$. Therefore, $ABCD$ is a parallelogram.

6. a. Find $|\vec{u} - \vec{v}|$ using the cosine law. Note $|-\vec{v}| = |\vec{v}|$ and the angle between $\vec{u}$ and $-\vec{v}$ is 120°.

$|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |-\vec{v}|^2 - 2|\vec{u}||-\vec{v}| \cos 60°$

Substitute into the second equation.

$$6\vec{b} = -\left(\frac{1}{2}\vec{a} - \frac{3}{2}\vec{y}\right) + 5\vec{y}$$

$$\vec{y} = \frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}$$

Lastly, find $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.

$$\vec{x} = \frac{1}{2}\vec{a} - \frac{3}{2}\left(\frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}\right)$$

$$= \frac{5}{13}\vec{a} - \frac{18}{13}\vec{b}$$

10. $\vec{a} = \vec{x} - \vec{y}$

$$= \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z} - (\vec{b} + \vec{z})$$

$$= \frac{2}{3}\vec{y} - \frac{2}{3}\vec{z} - \vec{b}$$

$$= \frac{2}{3}(\vec{y} - \vec{z}) - \vec{b}$$

$$= \frac{2}{3}\vec{b} - \vec{b}$$

$$= -\frac{1}{3}\vec{b}$$

11. a. $\overrightarrow{AG} = \vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{BH} = -\vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{CE} = -\vec{a} - \vec{b} + \vec{c}$

$\overrightarrow{DF} = \vec{a} - \vec{b} + \vec{c}$

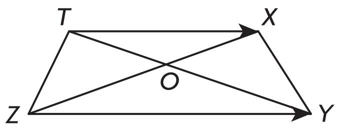
b. $|\overrightarrow{AG}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= |-\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= |\overrightarrow{BH}|^2$

Therefore, $|\overrightarrow{AG}| = |\overrightarrow{BH}|$

12.



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$$\overrightarrow{TO} + \frac{1}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$$

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$$\overrightarrow{TO} = \frac{2}{3}\overrightarrow{TZ} + \frac{1}{3}\overrightarrow{TX}$$

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c. $\overrightarrow{PS}$

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e. $\overrightarrow{PS}$

f. $\overrightarrow{PQ}$

3. a. Find $|\vec{a} + \vec{b}|$ using the cosine law, and the supplement to the angle between the vectors.

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 60°$

$= 3^2 + 4^2 - 2(3)(4)\frac{1}{2}$

$= 3$

$|\vec{a} + \vec{b}| = \sqrt{3}$

b. Find θ using the ratio

$$\tan\theta = \frac{|\vec{b}|}{|\vec{a}|}$$

$$= \frac{4}{3}$$

$$\theta = \tan^{-1}\frac{4}{3}$$

$$\doteq 53°$$

4. $t = 4$ or $t = -4$

5. In quadrilateral $PQRS$, look at $\triangle PQR$. Joining the midpoints B and C creates a vector $\overrightarrow{BC}$ that is parallel to $\overrightarrow{PR}$ and half the length of $\overrightarrow{PR}$. Look at $\triangle SPR$. Joining the midpoints A and D creates a vector $\overrightarrow{AD}$ that is parallel to $\overrightarrow{PR}$ and half the length of $\overrightarrow{PR}$. $\overrightarrow{BC}$ is parallel to $\overrightarrow{AD}$ and equal in length to $\overrightarrow{AD}$. Therefore, $ABCD$ is a parallelogram.

6. a. Find $|\vec{u} - \vec{v}|$ using the cosine law. Note $|-\vec{v}| = |\vec{v}|$ and the angle between $\vec{u}$ and $-\vec{v}$ is 120°.

$|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |-\vec{v}|^2 - 2|\vec{u}||-\vec{v}|\cos 60°$

$$= 8^2 + 10^2 - 2(8)(10)\left(\frac{1}{2}\right)$$

$|\vec{u} - \vec{v}| = 2\sqrt{21}$

b. Find the direction of $\vec{u} - \vec{v}$ using the sine law.

$$\frac{\sin\theta}{|-\vec{v}|} = \frac{\sin 60°}{|\vec{u} - \vec{v}|}$$

$$\sin\theta = \frac{5}{\sqrt{21}}\sin 60°$$

$$\theta = \sin^{-1}\frac{5}{\sqrt{28}}$$

$$\doteq 71°$$

c. $\frac{1}{|\vec{u} + \vec{v}|}(\vec{u} + \vec{v}) = \frac{1}{2\sqrt{21}}(\vec{u} + \vec{v})$

d. Find $|5\vec{u} + 2\vec{v}|$ using the cosine law.

$$|5\vec{u} + 2\vec{v}|^2 = |5\vec{u}|^2 + |2\vec{v}|^2 - 2|5\vec{u}||2\vec{v}|\cos 120°$$

$$= 40^2 + 20^2 - 2(40)(20)\left(-\frac{1}{2}\right)$$

$|5\vec{u} + 2\vec{v}| = 20\sqrt{7}$

7. Find $|2\vec{p} - \vec{q}|$ using the cosine law.

$$|2\vec{p} - \vec{q}|^2 = |2\vec{p}|^2 + |-\vec{q}|^2 - 2|2\vec{p}||-\vec{q}|\cos 60°$$

$$= 2^2 + 1^2 - 2(2)(1)\left(\frac{1}{2}\right) = 3$$

8. $|\vec{m} + \vec{n}| = |\vec{m}| - |\vec{n}|$

9. $\overrightarrow{BC} = -\vec{y}$

$\overrightarrow{DC} = \vec{x}$

$\overrightarrow{BD} = -\vec{x} - \vec{y}$

$\overrightarrow{AC} = \vec{x} - \vec{y}$

10. Construct a parallelogram with sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$. Since the diagonals bisect each other, $2\overrightarrow{OB}$ is the diagonal equal to $\overrightarrow{OA} + \overrightarrow{OC}$. Or $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ and $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$. So, $\overrightarrow{OB} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$. And $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$. Now $\overrightarrow{OB} = \overrightarrow{OA} + \frac{1}{2}\left(\overrightarrow{OC} - \overrightarrow{OA}\right)$ Multiplying by 2 gives $2\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$.

11. $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$

$3\vec{x} - \vec{y} + 2\vec{y} = \overrightarrow{AD}$

$3\vec{x} + \vec{y} = \overrightarrow{AD}$

$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\vec{x} + \overrightarrow{BD} = 3\vec{x} + \vec{y}$

$\overrightarrow{BD} = 2\vec{x} + \vec{y}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\vec{x} + \overrightarrow{BC} = 3\vec{x} - \vec{y}$

$\overrightarrow{BC} = 2\vec{x} - \vec{y}$

12. The air velocity of the airplane $(\vec{V}_{\text{air}})$ and the wind velocity $(\vec{W})$ have opposite directions.

$\vec{V}_{\text{ground}} = \vec{V}_{\text{air}} - \vec{W}$

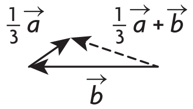
$= 460$ km/h due south

13. a. $\overrightarrow{PT}$

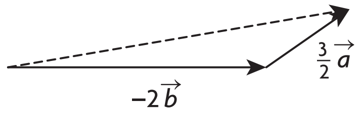
b. $\overrightarrow{PT}$

c. $\overrightarrow{SR}$

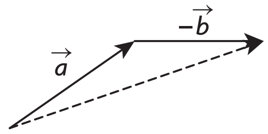
14. a.



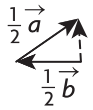
b.



c.



d.



15. $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$

$= 3\vec{b} - \vec{a}$

$\overrightarrow{RS} = \overrightarrow{QS} - \overrightarrow{QR}$

$= -3\vec{a}$

6.5 Vectors in R^2 and R^3, pp. 316–318

1. No, as the y-coordinate is not a real number.

2. a. We first arrange the x-, y-, and z-axes (each a copy of the real line) in a way so that each pair of axes are perpendicular to each other (i.e., the x- and y-axes are arranged in their usual way to form the xy-plane, and the z-axis passes through the origin of the xy-plane and is perpendicular to this plane). This is easiest viewed as a "right-handed system," where, from the viewer's perspective, the positive z-axis points upward, the positive x-axis points out of the page, and the positive y-axis points rightward in the plane of the page. Then, given point $P(a, b, c)$, we locate this point's unique position by moving a units along the x-axis, then from there b units parallel to the y-axis, and finally c units parallel to the z-axis. It's associated unique position vector is determined by drawing a vector with tail at the origin $O(0, 0, 0)$ and head at P.

b. Since this position vector is unique, its coordinates are unique. Therefore $a = -4$, $b = -3$, and $c = -8$.

3. a. Since A and B are really the same point, we can equate their coordinates. Therefore $a = 5$, $b = -3$, and $c = 8$.

$$= 8^2 + 10^2 - 2(8)(10)\left(\frac{1}{2}\right)$$

$$|\vec{u} - \vec{v}| = 2\sqrt{21}$$

b. Find the direction of $\vec{u} - \vec{v}$ using the sine law.

$$\frac{\sin\theta}{|-\vec{v}|} = \frac{\sin 60°}{|\vec{u} - \vec{v}|}$$

$$\sin\theta = \frac{5}{\sqrt{21}}\sin 60°$$

$$\theta = \sin^{-1}\frac{5}{\sqrt{28}}$$

$$\doteq 71°$$

c. $\frac{1}{|\vec{u} + \vec{v}|}(\vec{u} + \vec{v}) = \frac{1}{2\sqrt{21}}(\vec{u} + \vec{v})$

d. Find $|5\vec{u} + 2\vec{v}|$ using the cosine law.

$$|5\vec{u} + 2\vec{v}|^2 = |5\vec{u}|^2 + |2\vec{v}|^2 - 2|5\vec{u}||2\vec{v}|\cos 120°$$

$$= 40^2 + 20^2 - 2(40)(20)\left(-\frac{1}{2}\right)$$

$$|5\vec{u} + 2\vec{v}| = 20\sqrt{7}$$

7. Find $|2\vec{p} - \vec{q}|$ using the cosine law.

$$|2\vec{p} - \vec{q}|^2 = |2\vec{p}|^2 + |-\vec{q}|^2 - 2|2\vec{p}||-\vec{q}|\cos 60°$$

$$= 2^2 + 1^2 - 2(2)(1)\left(\frac{1}{2}\right) = 3$$

8. $|\vec{m} + \vec{n}| = |\vec{m}| - |\vec{n}|$

9. $\overrightarrow{BC} = -\vec{y}$

$\overrightarrow{DC} = \vec{x}$

$\overrightarrow{BD} = -\vec{x} - \vec{y}$

$\overrightarrow{AC} = \vec{x} - \vec{y}$

10. Construct a parallelogram with sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$. Since the diagonals bisect each other, $2\overrightarrow{OB}$ is the diagonal equal to $\overrightarrow{OA} + \overrightarrow{OC}$. Or $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ and $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$. So, $\overrightarrow{OB} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$. And $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$. Now $\overrightarrow{OB} = \overrightarrow{OA} + \frac{1}{2}\left(\overrightarrow{OC} - \overrightarrow{OA}\right)$ Multiplying by 2 gives $2\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$.

11. $\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$

$3\vec{x} - \vec{y} + 2\vec{y} = \overrightarrow{AD}$

$3\vec{x} + \vec{y} = \overrightarrow{AD}$

$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\vec{x} + \overrightarrow{BD} = 3\vec{x} + \vec{y}$

$\overrightarrow{BD} = 2\vec{x} + \vec{y}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\vec{x} + \overrightarrow{BC} = 3\vec{x} - \vec{y}$

$\overrightarrow{BC} = 2\vec{x} - \vec{y}$

12. The air velocity of the airplane ($\vec{V}_{\text{air}}$) and the wind velocity ($\vec{W}$) have opposite directions.

$\vec{V}_{\text{ground}} = \vec{V}_{\text{air}} - \vec{W}$

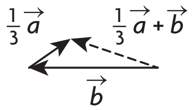
$= 460$ km/h due south

13. a. $\overrightarrow{PT}$

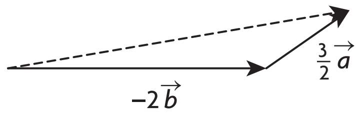
b. $\overrightarrow{PT}$

c. $\overrightarrow{SR}$

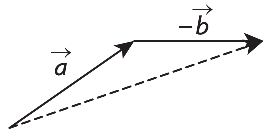
14. a.



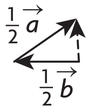
b.



c.



d.



15. $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$

$= 3\vec{b} - \vec{a}$

$\overrightarrow{RS} = \overrightarrow{QS} - \overrightarrow{QR}$

$= -3\vec{a}$

6.5 Vectors in R^2 and R^3, pp. 316–318

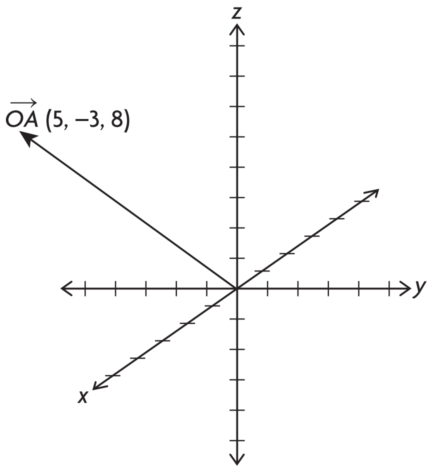
1. No, as the y-coordinate is not a real number.

2. a. We first arrange the x-, y-, and z-axes (each a copy of the real line) in a way so that each pair of axes are perpendicular to each other (i.e., the x- and y-axes are arranged in their usual way to form the xy-plane, and the z-axis passes through the origin of the xy-plane and is perpendicular to this plane). This is easiest viewed as a "right-handed system," where, from the viewer's perspective, the positive z-axis points upward, the positive x-axis points out of the page, and the positive y-axis points rightward in the plane of the page. Then, given point $P(a, b, c)$, we locate this point's unique position by moving a units along the x-axis, then from there b units parallel to the y-axis, and finally c units parallel to the z-axis. It's associated unique position vector is determined by drawing a vector with tail at the origin $O(0, 0, 0)$ and head at P.

b. Since this position vector is unique, its coordinates are unique. Therefore $a = -4$, $b = -3$, and $c = -8$.

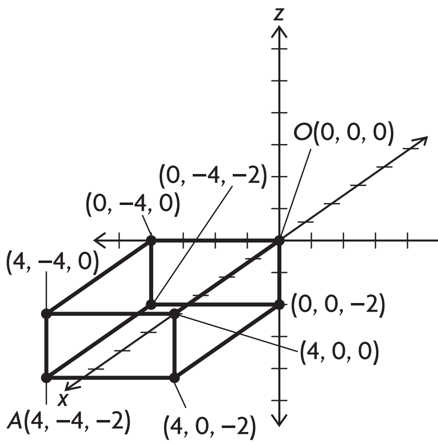
3. a. Since A and B are really the same point, we can equate their coordinates. Therefore $a = 5$, $b = -3$, and $c = 8$.

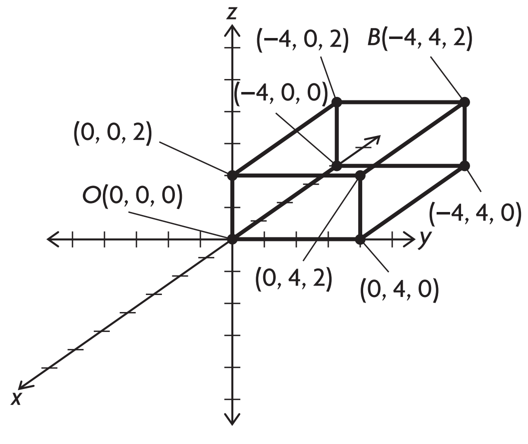
b. From part **a.**, $A(5, -3, 8)$, so $\overrightarrow{OA} = (5, -3, 8)$. Here is a depiction of this vector.

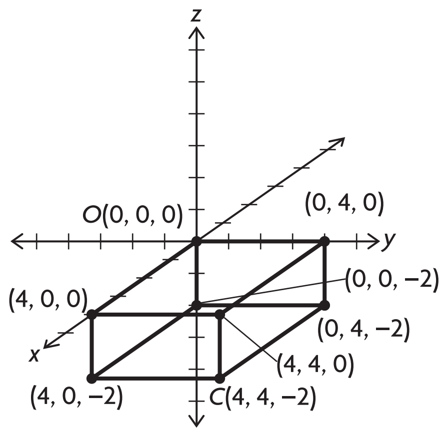


4. This is not an acceptable vector in I^3 as the z-coordinate is not an integer. However, since all of the coordinates are real numbers, this is acceptable as a vector in R^3.

5.







6. a. $A(0, -1, 0)$ is located on the y-axis. $B(0, -2, 0)$, $C(0, 2, 0)$, and $D(0, 10, 0)$ are three other points on this axis.

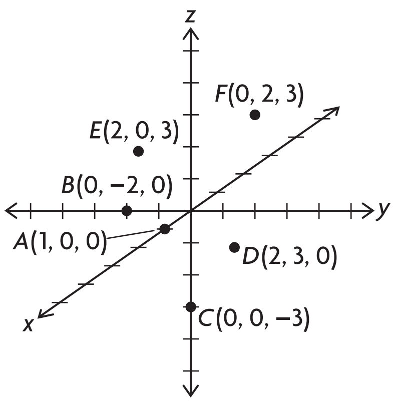
b. $\overrightarrow{OA} = (0, -1, 0)$, the vector with tail at the origin $O(0, 0, 0)$ and head at A.

7. a. Answers may vary. For example:
$\overrightarrow{OA} = (0, 0, 1)$, $\overrightarrow{OB} = (0, 0, -1)$,
$\overrightarrow{OC} = (0, 0, -5)$

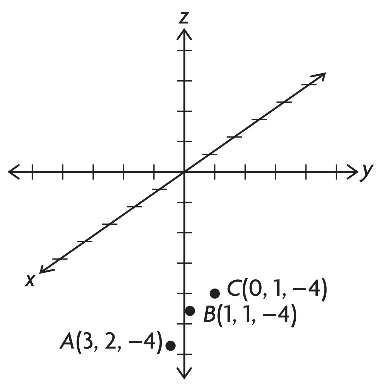
b. Yes, these vectors are collinear (parallel), as they all lie on the same line, in this case the z-axis.

c. A general vector lying on the z-axis would be of the form $\overrightarrow{OA} = (0, 0, a)$ for any real number a. Therefore, this vector would be represented by placing the tail at O, and the head at the point $(0, 0, a)$ on the z-axis.

8.

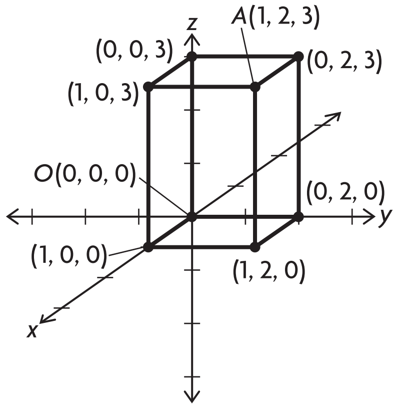


9. a.

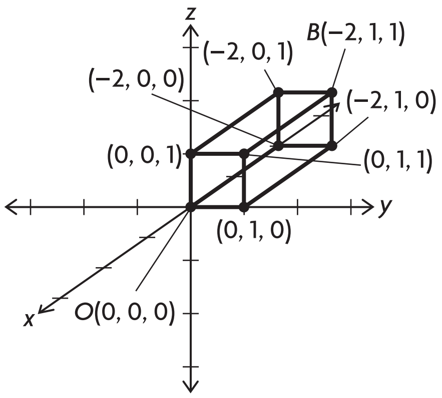


b. Every point on the plane containing points A, B, and C has z-coordinate equal to -4. Therefore, the equation of the plane containing these points is $z = -4$ (a plane parallel to the xy-plane through the point $z = -4$).

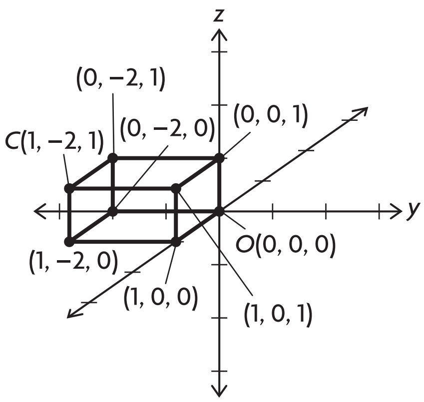
10. a.



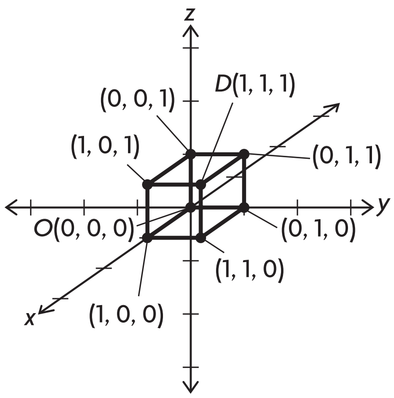
b.



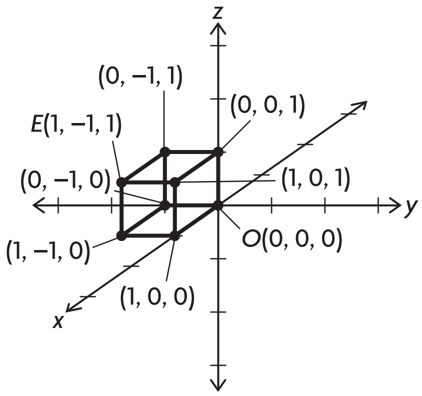
c.



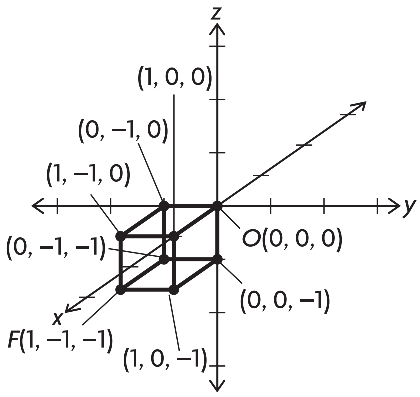
d.



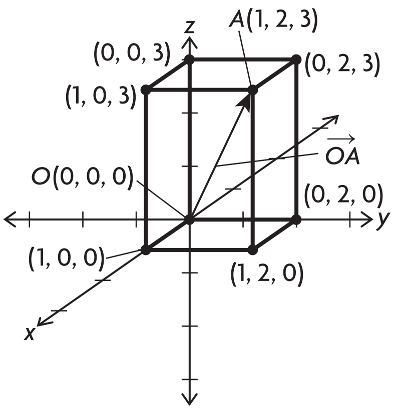
e.



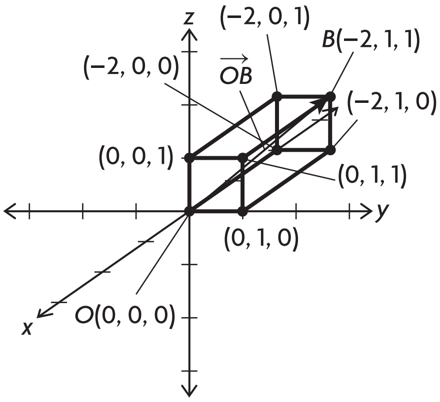
f.



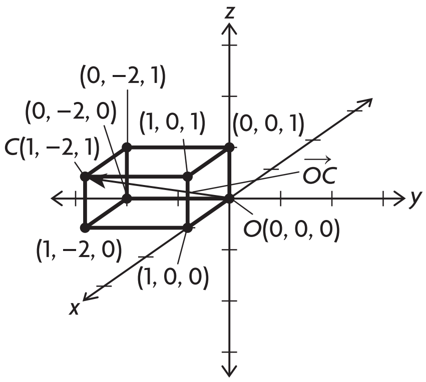
11. a. $\overrightarrow{OA} = (1, 2, 3)$



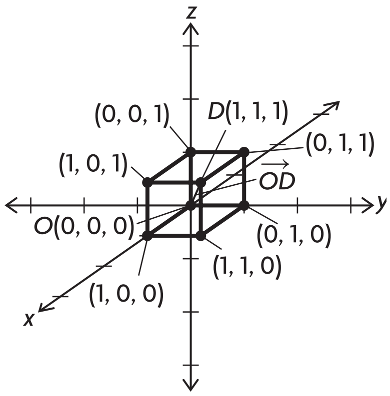
b. $\overrightarrow{OB} = (-2, 1, 1)$



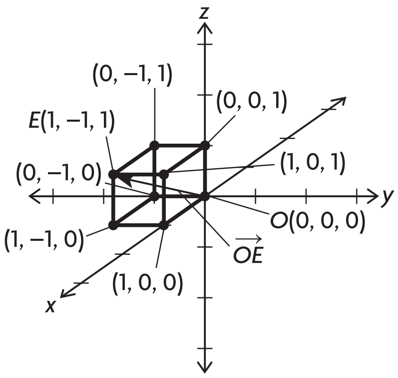
c. $\overrightarrow{OC} = (1, -2, 1)$



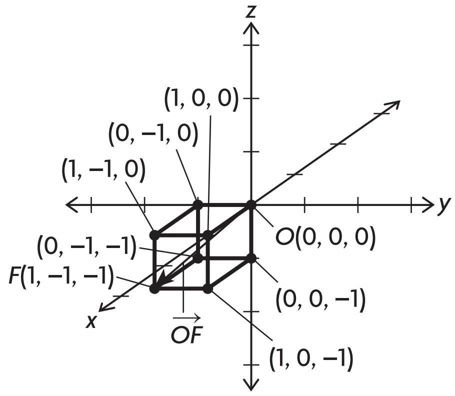
d. $\overrightarrow{OD} = (1, 1, 1)$



e. $\overrightarrow{OE} = (1, -1, 1)$



f. $\overrightarrow{OF} = (1, -1, -1)$



12. a. Since P and Q represent the same point, we can equate their y- and z-coordinates to get the system of equations

$$a - c = 6$$
$$a = 11$$

Substituting this second equation into the first gives

$$11 - c = 6$$
$$c = 5$$

So $a = 11$ and $c = 5$.

b. Since P and Q represent the same point in R^3, they will have the same associated position vector, i.e. $\overrightarrow{OP} = \overrightarrow{OQ}$. So, since these vectors are equal, they will certainly have equal magnitudes, i.e. $|\overrightarrow{OP}| = |\overrightarrow{OQ}|$.

13. $P(x, y, 0)$ represents a general point on the xy-plane, since the z-coordinate is 0. Similarly, $Q(x, 0, z)$ represents a general point in the xz-plane, and $R(0, y, z)$ represents a general point in the yz-plane.

14. a. Every point on the plane containing points M, N, and P has y-coordinate equal to 0. Therefore, the equation of the plane containing these points is $y = 0$ (this is just the xz-plane).

b. The plane $y = 0$ contains the origin $O(0, 0, 0)$, and so since it also contains the points M, N, and P as well, it will contain the position vectors associated with these points joining O (tail) to the given point (head). That is, the plane $y = 0$ contains the vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$, and $\overrightarrow{OP}$.

15. a. $A(-2, 0, 0)$, $B(-2, 4, 0)$, $C(0, 4, 0)$, $D(0, 0, -7)$, $E(0, 4, -7)$, $F(-2, 0, -7)$

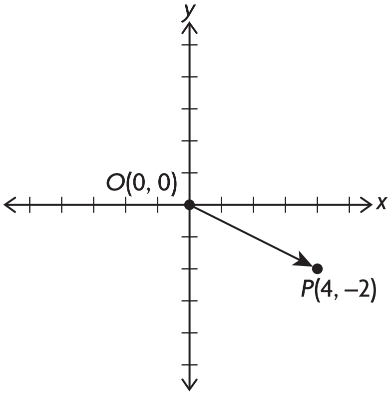
b. $\overrightarrow{OA} = (-2, 0, 0), \overrightarrow{OB} = (-2, 4, 0),$
$\overrightarrow{OC} = (0, 4, 0), \overrightarrow{OD} = (0, 0, -7),$
$\overrightarrow{OE} = (0, 4, -7), \overrightarrow{OF} = (-2, 0, -7)$

c. Rectangle $DEPF$ is 7 units below the xy-plane.

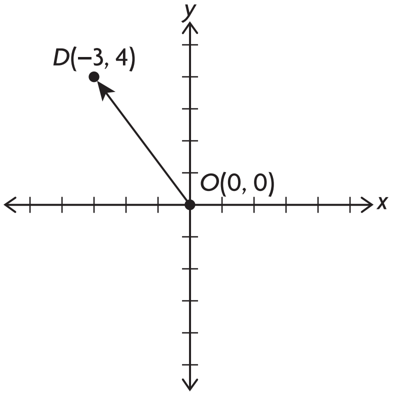
d. Every point on the plane containing points B, C, E, and P has y-coordinate equal to 4. Therefore, the equation of the plane containing these points is $y = 4$ (a plane parallel to the xz-plane through the point $y = 4$).

e. Every point contained in rectangle $BCEP$ has y-coordinate equal to 4, and so is of the form $(x, 4, z)$ where x and z are real numbers such that $-2 \le x \le 0$ and $-7 \le z \le 0$.

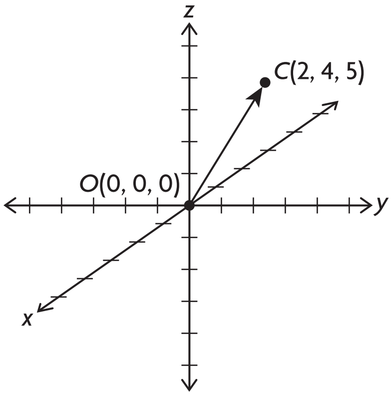
16. a.



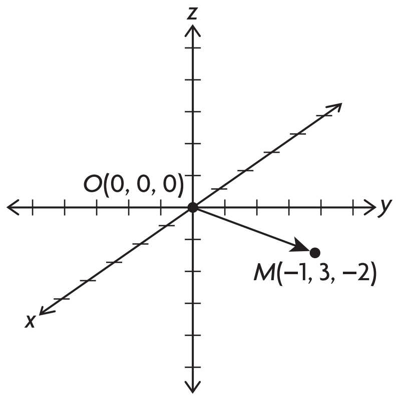
b.



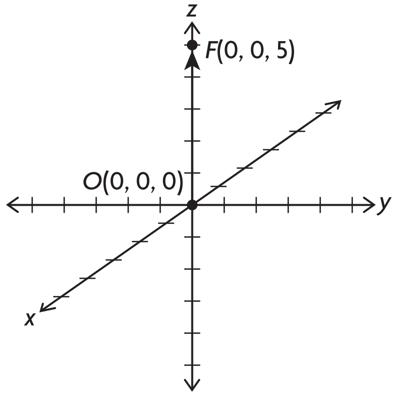
c.



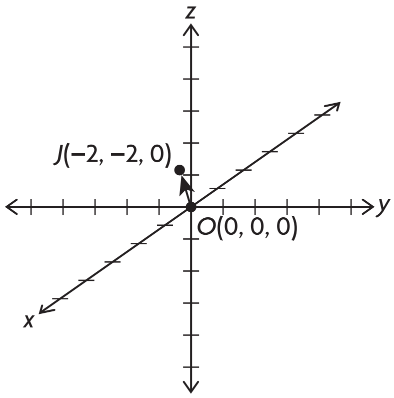
d.



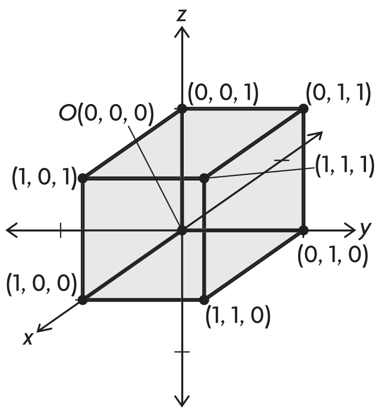
e.



f.



17. The following box illustrates the three dimensional solid consisting of the set of all points (x, y, z) such that $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$.



18. First, $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$ by the triangle law of vector addition, where $\overrightarrow{OA} = (5, -10, 0)$, $\overrightarrow{OB} = (0, 0, -10)$, $\overrightarrow{OP}$ and $\overrightarrow{OA}$ are drawn in standard position (starting from the origin $O(0, 0, 0)$), and $\overrightarrow{OB}$ is drawn starting from the head of $\overrightarrow{OA}$. Notice that $\overrightarrow{OA}$ lies in the *xy*-plane, and $\overrightarrow{OB}$ is perpendicular to the *xy*-plane (so is perpendicular to $\overrightarrow{OA}$). So, $\overrightarrow{OP}$, $\overrightarrow{OA}$, and $\overrightarrow{OB}$ form a right triangle and, by the Pythagorean theorem, $|\overrightarrow{OP}|^2 = |\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2$
Similarly, $\overrightarrow{OA} = \vec{a} + \vec{b}$ by the triangle law of vector addition, where $\vec{a} = (5, 0, 0)$ and $\vec{b} = (0, -10, 0)$, and these three vectors form a right triangle as well. So,

$$\begin{aligned} |\overrightarrow{OA}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 \\ &= 25 + 100 \\ &= 125 \end{aligned}$$

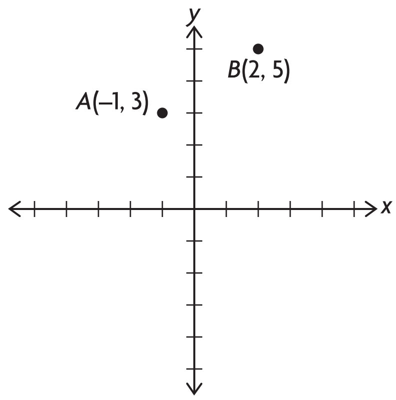
Obviously $|\overrightarrow{OB}|^2 = 100$, and so substituting gives

$$\begin{aligned} |\overrightarrow{OP}|^2 &= |\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2 \\ &= 125 + 100 \\ &= 225 \\ |\overrightarrow{OP}| &= \sqrt{225} \\ &= 15 \end{aligned}$$

19. To find a vector $\overrightarrow{AB}$ equivalent to $\overrightarrow{OP} = (-2, 3, 6)$, where $B(4, -2, 8)$, we need to move 2 units to the right of the *x*-coordinate for B (to $4 + 2 = 6$), 3 units to the left of the *y*-coordinate for B (to $-2 - 3 = -5$), and 6 units below the *z*-coordinate for B (to $8 - 6 = 2$). So we get the point $A(6, -5, 2)$. Indeed, notice that to get from A to B (which describes vector $\overrightarrow{AB}$), we move 2 units left in the *x*-coordinate, 3 units right in the *y*-coordinate, and 6 units up in the *z*-coordinate. This is equivalent to vector $\overrightarrow{OP} = (-2, 3, 6)$.

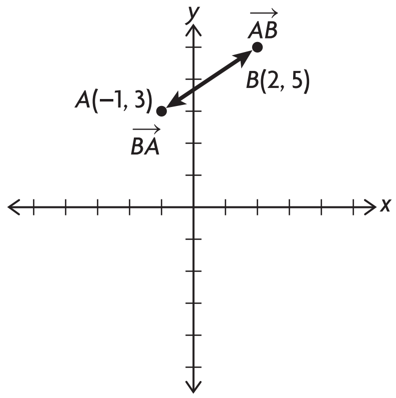
6.6 Operations with Algebraic Vectors in R^2, pp. 324–326

1.



a.
$$\begin{aligned} \overrightarrow{AB} &= (2, 5) - (-1, 3) \\ &= (3, 2) \\ \overrightarrow{BA} &= -\overrightarrow{AB} \\ &= -(3, 2) \\ &= (-3, -2) \end{aligned}$$

Here is a sketch of these two vectors in the *xy*-coordinate plane.



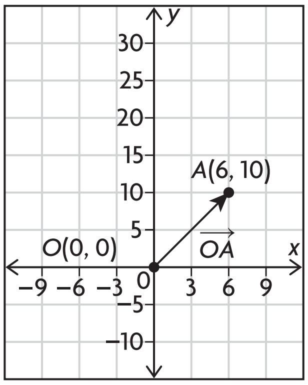
b.
$$\begin{aligned} |\overrightarrow{OA}| &= \sqrt{(-1)^2 + 3^2} \\ &= \sqrt{10} \\ &\doteq 3.16 \\ |\overrightarrow{OB}| &= \sqrt{2^2 + 5^2} \\ &= \sqrt{29} \\ &\doteq 5.39 \end{aligned}$$

c.
$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{3^2 + 2^2} \\ &= \sqrt{13} \\ &\doteq 3.61 \end{aligned}$$

Also, since $\overrightarrow{BA} = -\overrightarrow{AB}$,

$$\begin{aligned} |\overrightarrow{BA}| &= |-\overrightarrow{AB}| \\ &= |-1| \cdot |\overrightarrow{AB}| \\ &= |\overrightarrow{AB}| \\ &= \sqrt{13} \\ &\doteq 3.61 \end{aligned}$$

2.



18. First, $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$ by the triangle law of vector addition, where $\overrightarrow{OA} = (5, -10, 0)$, $\overrightarrow{OB} = (0, 0, -10)$, $\overrightarrow{OP}$ and $\overrightarrow{OA}$ are drawn in standard position (starting from the origin $O(0, 0, 0)$), and $\overrightarrow{OB}$ is drawn starting from the head of $\overrightarrow{OA}$. Notice that $\overrightarrow{OA}$ lies in the xy-plane, and $\overrightarrow{OB}$ is perpendicular to the xy-plane (so is perpendicular to $\overrightarrow{OA}$). So, $\overrightarrow{OP}$, $\overrightarrow{OA}$, and $\overrightarrow{OB}$ form a right triangle and, by the Pythagorean theorem, $|\overrightarrow{OP}|^2 = |\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2$
Similarly, $\overrightarrow{OA} = \vec{a} + \vec{b}$ by the triangle law of vector addition, where $\vec{a} = (5, 0, 0)$ and $\vec{b} = (0, -10, 0)$, and these three vectors form a right triangle as well. So,

$$\begin{aligned} |\overrightarrow{OA}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 \\ &= 25 + 100 \\ &= 125 \end{aligned}$$

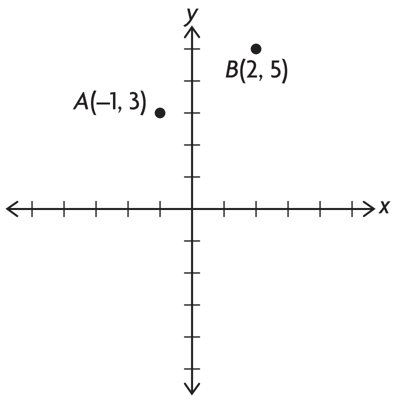
Obviously $|\overrightarrow{OB}|^2 = 100$, and so substituting gives

$$\begin{aligned} |\overrightarrow{OP}|^2 &= |\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2 \\ &= 125 + 100 \\ &= 225 \\ |\overrightarrow{OP}| &= \sqrt{225} \\ &= 15 \end{aligned}$$

19. To find a vector $\overrightarrow{AB}$ equivalent to $\overrightarrow{OP} = (-2, 3, 6)$, where $B(4, -2, 8)$, we need to move 2 units to the right of the x-coordinate for B (to $4 + 2 = 6$), 3 units to the left of the y-coordinate for B (to $-2 - 3 = -5$), and 6 units below the z-coordinate for B (to $8 - 6 = 2$). So we get the point $A(6, -5, 2)$. Indeed, notice that to get from A to B (which describes vector $\overrightarrow{AB}$), we move 2 units left in the x-coordinate, 3 units right in the y-coordinate, and 6 units up in the z-coordinate. This is equivalent to vector $\overrightarrow{OP} = (-2, 3, 6)$.

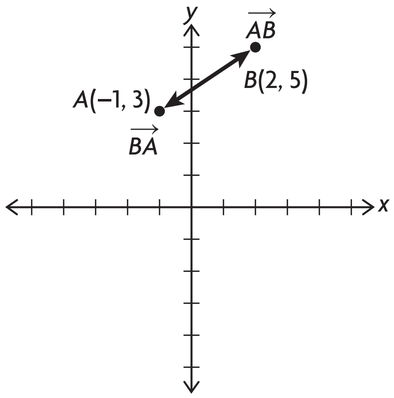
6.6 Operations with Algebraic Vectors in R^2, pp. 324–326

1.



a.
$$\begin{aligned} \overrightarrow{AB} &= (2, 5) - (-1, 3) \\ &= (3, 2) \\ \overrightarrow{BA} &= -\overrightarrow{AB} \\ &= -(3, 2) \\ &= (-3, -2) \end{aligned}$$

Here is a sketch of these two vectors in the xy-coordinate plane.



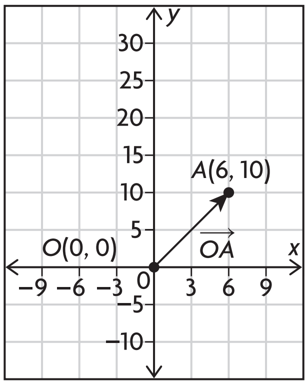
b.
$$\begin{aligned} |\overrightarrow{OA}| &= \sqrt{(-1)^2 + 3^2} \\ &= \sqrt{10} \\ &\doteq 3.16 \\ |\overrightarrow{OB}| &= \sqrt{2^2 + 5^2} \\ &= \sqrt{29} \\ &\doteq 5.39 \end{aligned}$$

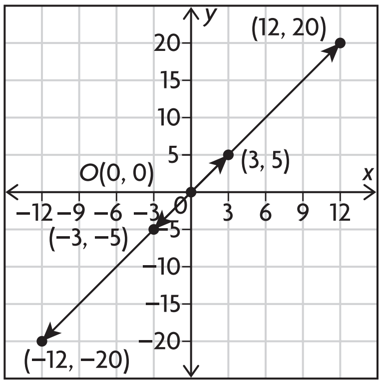
c.
$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{3^2 + 2^2} \\ &= \sqrt{13} \\ &\doteq 3.61 \end{aligned}$$

Also, since $\overrightarrow{BA} = -\overrightarrow{AB}$,

$$\begin{aligned} |\overrightarrow{BA}| &= |-\overrightarrow{AB}| \\ &= |-1| \cdot |\overrightarrow{AB}| \\ &= |\overrightarrow{AB}| \\ &= \sqrt{13} \\ &\doteq 3.61 \end{aligned}$$

2.



a. 

b. The vectors with the same magnitude are

$\frac{1}{2}\overrightarrow{OA}$ and $-\frac{1}{2}\overrightarrow{OA}$,

$2\overrightarrow{OA}$ and $-2\overrightarrow{OA}$

3. $|\overrightarrow{OA}| = \sqrt{3^2 + (-4)^2}$
$= \sqrt{25}$
$= 5$

4. a. The $\vec{i}$-component will be equal to the first coordinate in component form, and so $a = -3$. Similarly, the $\vec{j}$-component will be equal to the second coordinate in component form, and so $b = 5$.

b. $|(-3, b)| = |(-3, 5)|$
$= \sqrt{(-3)^2 + 5^2}$
$= \sqrt{34}$
$\doteq 5.83$

5. a. $|\vec{a}| = \sqrt{(-60)^2 + 11^2}$
$= \sqrt{3721}$
$= 61$
$|\vec{b}| = \sqrt{(-40)^2 + (-9)^2}$
$= \sqrt{1681}$
$= 41$

b. $\vec{a} + \vec{b} = (-60, 11) + (-40, -9)$
$= (-100, 2)$
$|\vec{a} + \vec{b}| = \sqrt{(-100)^2 + 2^2}$
$= \sqrt{10\,004}$
$\doteq 100.02$
$\vec{a} - \vec{b} = (-60, 11) - (-40, -9)$
$= (-20, 20)$
$|\vec{a} - \vec{b}| = \sqrt{(-20)^2 + 20^2}$
$= \sqrt{800}$
$\doteq 28.28$

6. a. $2(-2, 3) + (2, 1) = (2(-2) + 2, 2(3) + 1)$
$= (-2, 7)$

b. $-3(4, -9) - 9(2, 3)$
$= (-3(4) - 9(2), -3(-9) - 9(3))$
$= (-30, 0)$

c. $-\frac{1}{2}(6, -2) + \frac{2}{3}(6, 15)$
$= \left(-\frac{1}{2}(6) + \frac{2}{3}(6), -\frac{1}{2}(-2) + \frac{2}{3}(15)\right)$
$= (1, 11)$

7. $\vec{x} = 2\vec{i} - \vec{j}, \vec{y} = -\vec{i} + 5\vec{j}$

a. $3\vec{x} - \vec{y} = 3(2\vec{i} - \vec{j}) - (-\vec{i} + 5\vec{j})$
$= (6 + 1)\vec{i} + (-3 - 5)\vec{j}$
$= 7\vec{i} - 8\vec{j}$

b. $-(\vec{x} + 2\vec{y}) + 3(-\vec{x} - 3\vec{y})$
$= -4\vec{x} - 11\vec{y}$
$= -4(2\vec{i} - \vec{j}) - 11(-\vec{i} + 5\vec{j})$
$= 3\vec{i} - 51\vec{j}$

c. $2(\vec{x} + 3\vec{y}) - 3(\vec{y} + 5\vec{x})$
$= -13\vec{x} + 3\vec{y}$
$= -13(2\vec{i} - \vec{j}) + 3(-\vec{i} + 5\vec{j})$
$= -29\vec{i} + 28\vec{j}$

8. a. $\vec{x} + \vec{y} = (2\vec{i} - \vec{j}) + (-\vec{i} + 5\vec{j})$
$= \vec{i} + 4\vec{j}$
$|\vec{x} + \vec{y}| = |\vec{i} + 4\vec{j}|$
$= \sqrt{1^2 + 4^2}$
$= \sqrt{17}$
$\doteq 4.12$

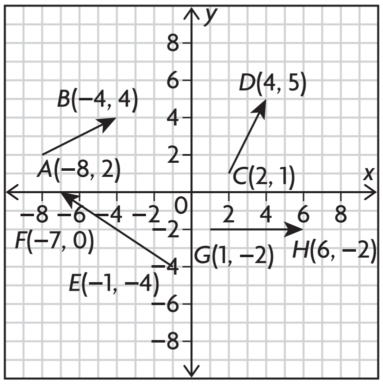
b. $\vec{x} - \vec{y} = (2\vec{i} - \vec{j}) - (-\vec{i} + 5\vec{j})$
$= 3\vec{i} - 6\vec{j}$
$|\vec{x} - \vec{y}| = |3\vec{i} - 6\vec{j}|$
$= \sqrt{3^2 + (-6)^2}$
$= \sqrt{45}$
$\doteq 6.71$

c. $2\vec{x} - 3\vec{y} = 2(2\vec{i} - \vec{j}) - 3(-\vec{i} + 5\vec{j})$
$= 7\vec{i} - 17\vec{j}$
$|2\vec{x} - 3\vec{y}| = |7\vec{i} - 17\vec{j}|$
$= \sqrt{7^2 + (-17)^2}$
$= \sqrt{338}$
$\doteq 18.38$

d. $|3\vec{y} - 2\vec{x}| = |-(2\vec{x} - 3\vec{y})|$
$= |-1||2\vec{x} - 3\vec{y}|$
$= |2\vec{x} - 3\vec{y}|$

so, from part c.,
$|3\vec{y} - 2\vec{x}| = |2\vec{x} - 3\vec{y}|$
$= \sqrt{338}$
$\doteq 18.38$

9.



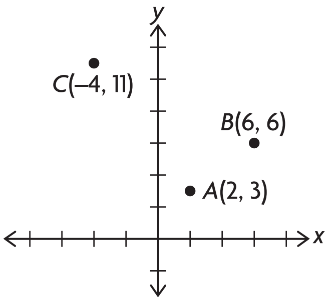
a. $\overrightarrow{AB} = (-4, 4) - (-8, 2)$
$= (4, 2)$
$\overrightarrow{CD} = (4, 5) - (2, 1)$
$= (2, 4)$
$\overrightarrow{EF} = (-7, 0) - (-1, -4)$
$= (-6, 4)$
$\overrightarrow{GH} = (6, -2) - (1, -2)$
$= (5, 0)$

b. $|\overrightarrow{AB}| = \sqrt{4^2 + 2^2}$
$= \sqrt{20}$
$\doteq 4.47$
$|\overrightarrow{CD}| = \sqrt{2^2 + 4^2}$
$= \sqrt{20}$
$\doteq 4.47$
$|\overrightarrow{EF}| = \sqrt{(-6)^2 + 4^2}$
$= \sqrt{52}$
$\doteq 7.21$
$|\overrightarrow{GH}| = \sqrt{5^2 + 0^2}$
$= \sqrt{25}$
$= 5$

10. a. By the parallelogram law of vector addition,
$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}$
$= (6, 3) + (11, -6)$
$= (17, -3)$
For the other vectors,
$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$
$= (6, 3) - (11, -6)$
$= (-5, 9)$
$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$
$= (17, -3) - (11, -6)$
$= (6, 3)$

b. $\overrightarrow{OA} = (6, 3)$
$= \overrightarrow{BC}$,
so obviously we will have $|\overrightarrow{OA}| = |\overrightarrow{BC}|$.
(It turns out that their common magnitude is $\sqrt{6^2 + 3^2} = \sqrt{45}$.)

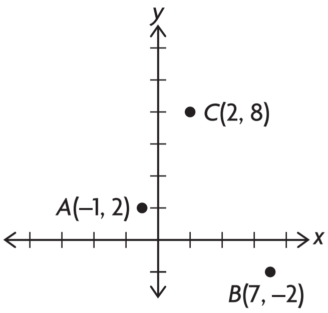
11. a.



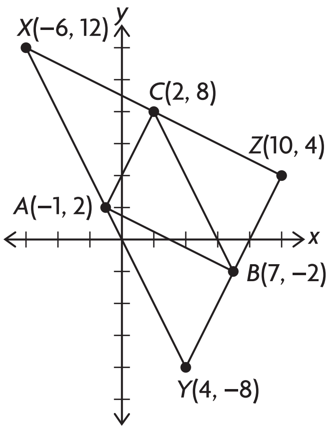
b. $\overrightarrow{AB} = (6, 6) - (2, 3)$
$= (4, 3)$
$|\overrightarrow{AB}| = \sqrt{4^2 + 3^2}$
$= \sqrt{25}$
$= 5$
$\overrightarrow{AC} = (-4, 11) - (2, 3)$
$= (-6, 8)$
$|\overrightarrow{AC}| = \sqrt{(-6)^2 + 8^2}$
$= \sqrt{100}$
$= 10$
$\overrightarrow{CB} = (6, 6) - (-4, 11)$
$= (10, -5)$
$|\overrightarrow{CB}| = \sqrt{10^2 + (-5)^2}$
$= \sqrt{125}$
$\doteq 11.18$

c. $|\overrightarrow{CB}|^2 = 125 |\overrightarrow{AC}|^2 = 100, |\overrightarrow{AB}|^2 = 25$
Since $|\overrightarrow{CB}|^2 = |\overrightarrow{AC}|^2 + |\overrightarrow{AB}|^2$, the triangle is a right triangle.

12. a.



b.



c. As a first possibility for the fourth vertex, there is $X(x_1, x_2)$. From the sketch in part b., we see that we would then have

$$\overrightarrow{CX} = \overrightarrow{BA}$$
$$(x_1 - 2, x_2 - 8) = (-1 - 7, 2 - (-2))$$
$$= (-8, 4)$$
$$x_1 - 2 = -8$$
$$x_2 - 8 = 4$$

So $X(-6, 12)$. By similar reasoning for the other points labelled in the sketch in part b.,

$$\overrightarrow{AY} = \overrightarrow{CB}$$
$$(y_1 - (-1), y_2 - 2) = (7 - 2, -2 - 8)$$
$$= (5, -10)$$
$$y_1 + 1 = 5$$
$$y_2 - 2 = -10$$

So $Y(4, -8)$. Finally,

$$\overrightarrow{BZ} = \overrightarrow{AC}$$
$$(z_1 - 7, z_2 - (-2)) = (2 - (-1), 8 - 2)$$
$$= (3, 6)$$
$$z_1 - 7 = 3$$
$$z_2 + 2 = 6$$

So $Z(10, 4)$. In conclusion, the three possible locations for a fourth vertex in a parallelogram with vertices A, B, and C are $X(-6, 12)$, $Y(4, -8)$, and $Z(10, 4)$.

13. a. $3(x, 1) - 5(2, 3y) = (11, 33)$

$$(3x - 5(2), 3 - 5(3y)) = (11, 33)$$
$$(3x - 10, 3 - 15y) = (11, 33)$$
$$3x - 10 = 11$$
$$3 - 15y = 33$$

So $x = 7$ and $y = -2$.

b. $-2(x, x + y) - 3(6, y) = (6, 4)$

$$(-2x - 18, -2x - 5y) = (6, 4)$$
$$-2x - 18 = 6$$
$$-2x - 5y = 4$$

To solve for x, use

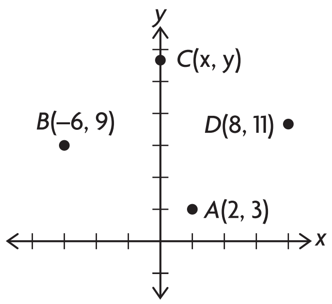
$$-2x - 18 = 6$$
$$x = -12$$

Substituting this into the last equation above, we can now solve for y.

$$-2(-12) - 5y = 4$$
$$y = 4$$

So $x = -12$ and $y = 4$.

14. a.



b. Because $ABCD$ is a rectangle, we will have

$$\overrightarrow{BC} = \overrightarrow{AD}$$
$$(x, y) - (-6, 9) = (8, 11) - (2, 3)$$
$$(x + 6, y - 9) = (6, 8)$$
$$x + 6 = 6$$
$$y - 9 = 8$$

So, $x = 0$ and $y = 17$, i.e., $C(0, 17)$.

15. a. Since $|\overrightarrow{PA}| = |\overrightarrow{PB}|$, and

$$\overrightarrow{PA} = (5, 0) - (a, 0)$$
$$= (5 - a, 0),$$
$$\overrightarrow{PB} = (0, 2) - (a, 0)$$
$$= (-a, 2),$$

this means that

$$(5 - a)^2 = (-a)^2 + 2^2$$
$$25 - 10a + a^2 = a^2 + 4$$
$$10a = 21$$
$$a = \frac{21}{10}$$

So $P\left(\frac{21}{10}, 0\right)$.

b. This point Q on the y-axis will be of the form $Q(0, b)$ for some real number b. Reasoning similarly to part a., we have

$$\overrightarrow{QA} = (5, 0) - (0, b)$$
$$= (5, -b)$$
$$\overrightarrow{QB} = (0, 2) - (0, b)$$
$$= (0, 2 - b)$$

So since $|\overrightarrow{QA}| = |\overrightarrow{QB}|$,

$$(-b)^2 + 5^2 = (2 - b)^2$$
$$b^2 + 25 = 4 - 4b + b^2$$
$$4b = -21$$
$$b = -\frac{21}{4}$$

So $Q\left(0, -\frac{21}{4}\right)$.

16. $\overrightarrow{QP}$ is in the direction opposite to $\overrightarrow{PQ}$, and

$$\begin{aligned}\overrightarrow{QP} &= \overrightarrow{OP} - \overrightarrow{OQ}\\ &= (11, 19) - (2, -21)\\ &= (9, 40)\\ |\overrightarrow{QP}| &= \sqrt{9^2 + 40^2}\\ &= \sqrt{1681}\\ &= 41\end{aligned}$$

A unit vector in the direction of $\overrightarrow{QP}$ is

$$\begin{aligned}\vec{u} &= \frac{1}{41}\overrightarrow{QP}\\ &= \left(\frac{9}{41}, \frac{40}{41}\right)\end{aligned}$$

Indeed, $\vec{u}$ is obviously in the same direction as $\overrightarrow{QP}$ (since $\vec{u}$ is a positive scalar multiple of $\overrightarrow{QP}$), and notice that

$$\begin{aligned}|\vec{u}| &= \sqrt{\left(\frac{9}{41}\right)^2 + \left(\frac{40}{41}\right)^2}\\ &= \sqrt{\frac{81 + 1600}{1681}}\\ &= 1\end{aligned}$$

17. a. O, P, and R can be thought of as the vertices of a triangle.

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{OR} - \overrightarrow{OP}\\ &= (-8, -1) - (-7, 24)\\ &= (-1, -25)\\ |\overrightarrow{PR}|^2 &= (-1)^2 + (-25)^2\\ &= 626\\ |\overrightarrow{OR}|^2 &= (-8)^2 + (-1)^2\\ &= 65\\ |\overrightarrow{OP}|^2 &= (-7)^2 + 24^2\\ &= 625\end{aligned}$$

By the cosine law, the angle, θ, between $\overrightarrow{OR}$ and $\overrightarrow{OP}$ satisfies

$$\begin{aligned}\cos\theta &= \frac{|\overrightarrow{OR}|^2 + |\overrightarrow{OP}|^2 - |\overrightarrow{PR}|^2}{2|\overrightarrow{OR}| \cdot |\overrightarrow{OP}|}\\ &= \frac{65 + 625 - 626}{2\sqrt{65} \cdot \sqrt{625}}\\ \theta &= \cos^{-1}\left(\frac{65 + 625 - 626}{2\sqrt{65} \cdot \sqrt{625}}\right)\\ &\doteq 80.9°\end{aligned}$$

So the angle between $\overrightarrow{OR}$ and $\overrightarrow{OP}$ is about 80.86°.

b. We found the vector $\overrightarrow{PR} = (-1, -25)$ in part a., so $\overrightarrow{RP} = -\overrightarrow{PR} = (1, 25)$ and

$$\begin{aligned}|\overrightarrow{RP}|^2 &= |\overrightarrow{PR}|^2\\ &= 626\end{aligned}$$

Also, by the parallelogram law of vector addition,

$$\begin{aligned}\overrightarrow{OQ} &= \overrightarrow{OR} + \overrightarrow{OP}\\ &= (-8, -1) + (-7, 24)\\ &= (-15, 23)\\ |\overrightarrow{OQ}|^2 &= (-15)^2 + 23^2\\ &= 754\end{aligned}$$

Placing $\overrightarrow{RP} = (1, 25)$ and $\overrightarrow{OQ} = (-15, 23)$ with their tails at the origin, a triangle is formed by joining the heads of these two vectors. The third side of this triangle is the vector

$$\begin{aligned}\vec{v} &= \overrightarrow{RP} - \overrightarrow{OQ}\\ &= (1, 25) - (-15, 23)\\ &= (16, 2)\\ |\vec{v}|^2 &= 16^2 + 2^2\\ &= 260\end{aligned}$$

Now by reasoning similar to part a., the cosine law implies that the angle, θ, between $\overrightarrow{RP}$ and $\overrightarrow{OQ}$ satisfies

$$\begin{aligned}\cos\theta &= \frac{|\overrightarrow{RP}|^2 + |\overrightarrow{OQ}|^2 - |\vec{v}|^2}{2|\overrightarrow{RP}| \cdot |\overrightarrow{OQ}|}\\ &= \frac{626 + 754 - 260}{2\sqrt{626} \cdot \sqrt{754}}\\ \theta &= \cos^{-1}\left(\frac{626 + 754 - 260}{2\sqrt{626} \cdot \sqrt{754}}\right)\\ &\doteq 35.4°\end{aligned}$$

So the angle between $\overrightarrow{RP}$ and $\overrightarrow{OQ}$ is about 35.40°. However, since we are discussing the diagonals of parallelogram $OPQR$ here, it would also have been appropriate to report the supplement of this angle, or about $180° - 35.40° = 144.60°$, as the angle between these vectors.

6.7 Operations with Vectors in R^3, pp. 332–333

1. a. $\overrightarrow{OA} = -1\vec{i} + 2\vec{j} + 4\vec{k}$

b. $|\overrightarrow{OA}| = \sqrt{(-1)^2 + 2^2 + 4^2} = \sqrt{21} \doteq 4.58$

2. $\overrightarrow{OB} = (3, 4, -4)$

$|\overrightarrow{OB}| = \sqrt{3^2 + 4^2 + (-4)^2} = \sqrt{41} \doteq 6.40$

3.
$$\begin{aligned}\vec{a} + \frac{1}{3}\vec{b} - \vec{c} &= (1, 3, -3) + (-1, 2, 4)\\ &\quad - (0, 8, 1)\\ &= (1 + (-1) - 0, 3 + 2 - 8,\\ &\quad (-3) + 4 - 1)\\ &= (0, -3, 0)\\ \left|\vec{a} + \frac{1}{3}\vec{b} - \vec{c}\right| &= \sqrt{0^2 + (-3)^2 + 0^2}\\ &= 3\end{aligned}$$

16. $\overrightarrow{QP}$ is in the direction opposite to $\overrightarrow{PQ}$, and

$$\begin{aligned}\overrightarrow{QP} &= \overrightarrow{OP} - \overrightarrow{OQ}\\ &= (11, 19) - (2, -21)\\ &= (9, 40)\\ |\overrightarrow{QP}| &= \sqrt{9^2 + 40^2}\\ &= \sqrt{1681}\\ &= 41\end{aligned}$$

A unit vector in the direction of $\overrightarrow{QP}$ is

$$\begin{aligned}\vec{u} &= \frac{1}{41}\overrightarrow{QP}\\ &= \left(\frac{9}{41}, \frac{40}{41}\right)\end{aligned}$$

Indeed, $\vec{u}$ is obviously in the same direction as $\overrightarrow{QP}$ (since $\vec{u}$ is a positive scalar multiple of $\overrightarrow{QP}$), and notice that

$$\begin{aligned}|\vec{u}| &= \sqrt{\left(\frac{9}{41}\right)^2 + \left(\frac{40}{41}\right)^2}\\ &= \sqrt{\frac{81 + 1600}{1681}}\\ &= 1\end{aligned}$$

17. a. O, P, and R can be thought of as the vertices of a triangle.

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{OR} - \overrightarrow{OP}\\ &= (-8, -1) - (-7, 24)\\ &= (-1, -25)\\ |\overrightarrow{PR}|^2 &= (-1)^2 + (-25)^2\\ &= 626\\ |\overrightarrow{OR}|^2 &= (-8)^2 + (-1)^2\\ &= 65\\ |\overrightarrow{OP}|^2 &= (-7)^2 + 24^2\\ &= 625\end{aligned}$$

By the cosine law, the angle, θ, between $\overrightarrow{OR}$ and $\overrightarrow{OP}$ satisfies

$$\begin{aligned}\cos\theta &= \frac{|\overrightarrow{OR}|^2 + |\overrightarrow{OP}|^2 - |\overrightarrow{PR}|^2}{2|\overrightarrow{OR}| \cdot |\overrightarrow{OP}|}\\ &= \frac{65 + 625 - 626}{2\sqrt{65} \cdot \sqrt{625}}\\ \theta &= \cos^{-1}\left(\frac{65 + 625 - 626}{2\sqrt{65} \cdot \sqrt{625}}\right)\\ &\doteq 80.9°\end{aligned}$$

So the angle between $\overrightarrow{OR}$ and $\overrightarrow{OP}$ is about 80.86°.

b. We found the vector $\overrightarrow{PR} = (-1, -25)$ in part a., so $\overrightarrow{RP} = -\overrightarrow{PR} = (1, 25)$ and

$$\begin{aligned}|\overrightarrow{RP}|^2 &= |\overrightarrow{PR}|^2\\ &= 626\end{aligned}$$

Also, by the parallelogram law of vector addition,

$$\begin{aligned}\overrightarrow{OQ} &= \overrightarrow{OR} + \overrightarrow{OP}\\ &= (-8, -1) + (-7, 24)\\ &= (-15, 23)\\ |\overrightarrow{OQ}|^2 &= (-15)^2 + 23^2\\ &= 754\end{aligned}$$

Placing $\overrightarrow{RP} = (1, 25)$ and $\overrightarrow{OQ} = (-15, 23)$ with their tails at the origin, a triangle is formed by joining the heads of these two vectors. The third side of this triangle is the vector

$$\begin{aligned}\vec{v} &= \overrightarrow{RP} - \overrightarrow{OQ}\\ &= (1, 25) - (-15, 23)\\ &= (16, 2)\\ |\vec{v}|^2 &= 16^2 + 2^2\\ &= 260\end{aligned}$$

Now by reasoning similar to part a., the cosine law implies that the angle, θ, between $\overrightarrow{RP}$ and $\overrightarrow{OQ}$ satisfies

$$\begin{aligned}\cos\theta &= \frac{|\overrightarrow{RP}|^2 + |\overrightarrow{OQ}|^2 - |\vec{v}|^2}{2|\overrightarrow{RP}| \cdot |\overrightarrow{OQ}|}\\ &= \frac{626 + 754 - 260}{2\sqrt{626} \cdot \sqrt{754}}\\ \theta &= \cos^{-1}\left(\frac{626 + 754 - 260}{2\sqrt{626} \cdot \sqrt{754}}\right)\\ &\doteq 35.4°\end{aligned}$$

So the angle between $\overrightarrow{RP}$ and $\overrightarrow{OQ}$ is about 35.40°. However, since we are discussing the diagonals of parallelogram $OPQR$ here, it would also have been appropriate to report the supplement of this angle, or about $180° - 35.40° = 144.60°$, as the angle between these vectors.

6.7 Operations with Vectors in R^3, pp. 332–333

1. a. $\overrightarrow{OA} = -1\vec{i} + 2\vec{j} + 4\vec{k}$

b. $|\overrightarrow{OA}| = \sqrt{(-1)^2 + 2^2 + 4^2} = \sqrt{21} \doteq 4.58$

2. $\overrightarrow{OB} = (3, 4, -4)$

$|\overrightarrow{OB}| = \sqrt{3^2 + 4^2 + (-4)^2} = \sqrt{41} \doteq 6.40$

3.

$$\begin{aligned}\vec{a} + \frac{1}{3}\vec{b} - \vec{c} &= (1, 3, -3) + (-1, 2, 4)\\ &\quad - (0, 8, 1)\\ &= (1 + (-1) - 0, 3 + 2 - 8,\\ &\quad (-3) + 4 - 1)\\ &= (0, -3, 0)\\ \left|\vec{a} + \frac{1}{3}\vec{b} - \vec{c}\right| &= \sqrt{0^2 + (-3)^2 + 0^2}\\ &= 3\end{aligned}$$

4. a. $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$
$= ((-3) + 2, 4 + 2, 12 + (-1))$
$= (-1, 6, 11)$

b. $|\overrightarrow{OA}| = \sqrt{(-3)^2 + 4^2 + 12^2} = 13$
$|\overrightarrow{OB}| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$
$|\overrightarrow{OP}| = \sqrt{(-1)^2 + 6^2 + 11^2} \doteq 12.57$

c. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= (2, 2, -1) - (-3, 4, 12)$
$= (2 - (-3), 2 - 4, (-1) - 12)$
$= (5, -2, -13)$
$|\overrightarrow{AB}| = \sqrt{5^2 + (-2)^2 + (-13)^2} = \sqrt{198} \doteq 14.07$
$\overrightarrow{AB}$ represents the vector from the tip of $\overrightarrow{OA}$ to the tip of $\overrightarrow{OB}$. It is the difference between the two vectors.

5. a. $\vec{x} - 2\vec{y} - \vec{z}$
$= (1, 4, -1) - 2(1, 3, -2) - (-2, 1, 0)$
$= (1, 4, -1) - (2, 6, -4) - (-2, 1, 0)$
$= (1 - 2 - (-2), 4 - 6 - 1, -1 - (-4) - 0)$
$= (1, -3, 3)$

b. $-2\vec{x} - 3\vec{y} + \vec{z}$
$= -2(1, 4, -1) - 3(1, 3, -2) + (-2, 1, 0)$
$= (-2, -8, 2) - (3, 9, -6) + (-2, 1, 0)$
$= (-2 - 3 - 2, -8 - 9 + 1, 2 + 6 + 0)$
$= (-7, -16, 8)$

c. $\frac{1}{2}\vec{x} - \vec{y} + 3\vec{z}$
$= \frac{1}{2}(1, 4, -1) - (1, 3, -2) + 3(-2, 1, 0)$
$= \left(\frac{1}{2}, 2, -\frac{1}{2}\right) - (1, 3, -2) + (-6, 3, 0)$
$= \left(\frac{1}{2} - 1 + (-6), 2 - 3 + 3, -\frac{1}{2} - (-2) + 0\right)$
$= \left(-\frac{13}{2}, 2, \frac{3}{2}\right)$

d. $3\vec{x} + 5\vec{y} + 3\vec{z}$
$= 3(1, 4, -1) + 5(1, 3, -2) + 3(-2, 1, 0)$
$= (3, 12, -3) + (5, 15, -10) + (-6, 3, 0)$
$= (3 + 5 - 6, 12 + 15 + 3, -3 - 10 + 0)$
$= (2, 30, -13)$

6. a. $\vec{p} + \vec{q} = (2\vec{i} - \vec{j} + \vec{k}) + (-\vec{i} - \vec{j} + \vec{k})$
$= (2 - 1)\vec{i} + (-1 - 1)\vec{j} + (1 + 1)\vec{k}$
$= \vec{i} - 2\vec{j} + 2\vec{k}$

b. $\vec{p} - \vec{q} = (2\vec{i} - \vec{j} + \vec{k}) - (-\vec{i} - \vec{j} + \vec{k})$
$= (2 + 1)\vec{i} + (-1 + 1)\vec{j} + (1 - 1)\vec{k}$
$= 3\vec{i} + 0\vec{j} + 0\vec{k}$

c. $2\vec{p} - 5\vec{q} = 2(2\vec{i} - \vec{j} + \vec{k}) - 5(-\vec{i} - \vec{j} + \vec{k})$
$= (4\vec{i} - 2\vec{j} + 2\vec{k}) - (-5\vec{i} - 5\vec{j} + 5\vec{k})$
$= (4 + 5)\vec{i} + (-2 + 5)\vec{j} + (2 - 5)\vec{k}$
$= 9\vec{i} + 3\vec{j} - 3\vec{k}$

d. $-2\vec{p} + 5\vec{q} = -2(2\vec{i} - \vec{j} + \vec{k}) + 5(-\vec{i} - \vec{j} + \vec{k})$
$= (-4\vec{i} + 2\vec{j} - 2\vec{k}) + (-5\vec{i} - 5\vec{j} + 5\vec{k})$
$= (-4 - 5)\vec{i} + (2 - 5)\vec{j} + (-2 + 5)\vec{k}$
$= -9\vec{i} - 3\vec{j} + 3\vec{k}$

7. a. $\vec{m} - \vec{n} = (2\vec{i} - \vec{k}) - (-2\vec{i} + \vec{j} + 2\vec{k})$
$= (2 - (-2))\vec{i} + (-1)\vec{j} + (-1 - 2)\vec{k}$
$= 4\vec{i} - \vec{j} - 3\vec{k}$
$|\vec{m} - \vec{n}| = \sqrt{4^2 + (-1)^2 + (-3)^2} = \sqrt{26} \doteq 5.10$

b. $\vec{m} + \vec{n} = (2\vec{i} - \vec{k}) + (-2\vec{i} + \vec{j} + 2\vec{k})$
$= (2 + (-2))\vec{i} + \vec{j} + (-1 + 2)\vec{k}$
$= 0\vec{i} + \vec{j} + \vec{k}$
$|\vec{m} + \vec{n}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2} \doteq 1.41$

c. $2\vec{m} + 3\vec{n} = 2(2\vec{i} - \vec{k}) + 3(-2\vec{i} + \vec{j} + 2\vec{k})$
$= (4\vec{i} - 2\vec{k}) + (-6\vec{i} + 3\vec{j} + 6\vec{k})$
$= (4 + (-6))\vec{i} + 3\vec{j} + (-2 + 6)\vec{k}$
$= -2\vec{i} + 3\vec{j} + 4\vec{k}$
$|2\vec{m} + 3\vec{n}| = \sqrt{(-2)^2 + 3^2 + 4^2} = \sqrt{29} \doteq 5.39$

d. $-5\vec{m} = -5(2\vec{i} - \vec{k}) = -10\vec{i} + 5\vec{k}$
$|-5\vec{m}| = \sqrt{(-10)^2 + (5)^2} = \sqrt{125} \doteq 11.18$

8.
$$\begin{aligned} \vec{x} + \vec{y} &= -\vec{i} + 2\vec{j} + 5\vec{k} \\ + \vec{x} - \vec{y} &= 3\vec{i} + 6\vec{j} - 7\vec{k} \\ 2\vec{x} &= 2\vec{i} + 8\vec{j} - 2\vec{k} \end{aligned}$$
Divide by 2 on both sides to get:
$\vec{x} = \vec{i} + 4\vec{j} - \vec{k}$
Plug this equation into the first given equation:
$\vec{i} + 4\vec{j} - \vec{k} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$
$\vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k} - (\vec{i} + 4\vec{j} - \vec{k})$
$\vec{y} = (-1 - 1)\vec{i} + (2 - 4)\vec{j} + (5 + 1)\vec{k}$
$\vec{y} = -2\vec{i} - 2\vec{j} + 6\vec{k}$

9. a. The vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ represent the xy-plane, xz-plane, and yz-plane, respectively. They are also the vector from the origin to points $(a, b, 0)$, $(a, 0, c)$, and $(0, b, c)$, respectively.

b. $\overrightarrow{OA} = a\vec{i} + b\vec{j} + 0\vec{k}$
$\overrightarrow{OB} = a\vec{i} + 0\vec{j} + c\vec{k}$
$\overrightarrow{OC} = 0\vec{i} + b\vec{j} + c\vec{k}$

c. $|\overrightarrow{OA}| = \sqrt{a^2 + b^2}$
$|\overrightarrow{OB}| = \sqrt{a^2 + c^2}$
$|\overrightarrow{OB}| = \sqrt{b^2 + c^2}$

d. $\overrightarrow{AB} = (a, 0, c) - (a, b, 0) = (0, -b, c)$

$\overrightarrow{AB}$ is a direction vector from A to B.

10. a. $|\overrightarrow{OA}| = \sqrt{(-2)^2 + (-6)^2 + 3^2} = \sqrt{49} = 7$

b. $|\overrightarrow{OB}| = \sqrt{(3)^2 + (-4)^2 + 12^2} = \sqrt{169} = 13$

c. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (3, -4, 12) - (-2, -6, 3)$

$= (3 - (-2), -4 - (-6), 12 - 3)$

$= (5, 2, 9)$

d. $|\overrightarrow{AB}| = \sqrt{5^2 + 2^2 + 9^2} = \sqrt{110} \doteq 10.49$

e. $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

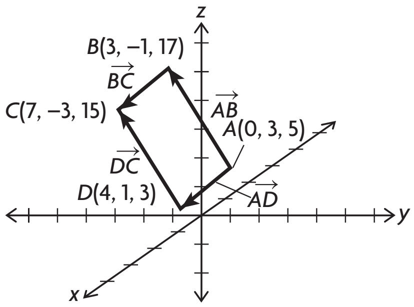
$= (-2, -6, 3) - (3, -4, 12)$

$= (-5, -2, -9)$

f. $|\overrightarrow{BA}| = \sqrt{(-5)^2 + (-2)^2 + (-9)^2}$

$= \sqrt{110} \doteq 10.49$

11.



In order to show that $ABCD$ is a parallelogram, we must show that $\overrightarrow{AB} = \overrightarrow{DC}$ or $\overrightarrow{BC} = \overrightarrow{AD}$. This will show they have the same direction, thus the opposite sides are parallel. By showing the vectors are equal they will have the same magnitude, implying the opposite sides having congruency.

$\overrightarrow{AB} = (3, -1, 17) - (0, 3, 5)$

$= (3 - 0, -1 - 3, 17 - 5)$

$= (3, -4, 12)$

$\overrightarrow{DC} = (7, -3, 15) - (4, 1, 3)$

$= (7 - 4, -3 - 1, 15 - 3)$

$= (3, -4, 12)$

Thus $\overrightarrow{AB} = \overrightarrow{DC}$. Do the calculations for the other pair as a check.

$\overrightarrow{BC} = (7, -3, 15) - (3, -1, 17)$

$= (7 - 3, -3 - (-1), 15 - 17)$

$= (4, -2, -2)$

$\overrightarrow{AD} = (4, 1, 3) - (0, 3, 5)$

$= (4 - 0, 1 - 3, 3 - 5)$

$= (4, -2, -2)$

So $\overrightarrow{BC} = \overrightarrow{AD}$.

We have shown $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{BC} = \overrightarrow{AD}$, so $ABCD$ is a parallelogram.

12. $2\vec{x} + \vec{y} - 2\vec{z}$

$= 2(-1, b, c) + (a, -2, c) - 2(-a, 6, c)$

$= (-2, 2b, 2c) + (a, -2, c) - (-2a, 12, 2c)$

$= (-2 + a + 2a, 2b - 2 - 12, 2c + c - 2c)$

$= (-2 + 3a, 2b - 14, c)$

$= (0, 0, 0)$

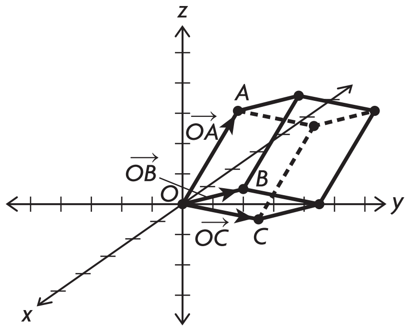
$-2 + 3a = 0; 2b - 14 = 0; c = 0$

$3a = 2; a = \dfrac{2}{3}$

$2b = 14; b = 7$

$c = 0$

13. a.



b. $V_1 = (0, 0, 0)$, the origin

$V_2 =$ end point of $\overrightarrow{OA} = (-2, 2, 5)$

$V_3 =$ end point of $\overrightarrow{OB} = (0, 4, 1)$

$V_4 =$ end point of $\overrightarrow{OC} = (0, 5, -1)$

$V_5 = \overrightarrow{OA} + \overrightarrow{OB} = (-2, 2, 5) + (0, 4, 1)$

$= (-2 + 0, 2 + 4, 5 + 1)$

$= (-2, 6, 6)$

$V_6 = \overrightarrow{OA} + \overrightarrow{OC} = (-2, 2, 5) + (0, 5, -1)$

$= (-2 + 0, 2 + 5, 5 - 1)$

$= (-2, 7, 4)$

$V_7 = \overrightarrow{OB} + \overrightarrow{OC} = (0, 4, 1) + (0, 5, -1)$

$= (0 + 0, 4 + 5, 1 - 1)$

$= (0, 9, 0)$

$V_8 = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$

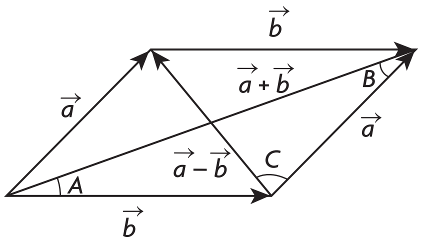
$= (-2, 2, 5) + (0, 9, 0)$ (by V_7)

$= (-2 + 0, 2 + 9, 5 + 0)$

$= (-2, 11, 5)$

14. Any point on the x-axis has y-coordinate 0 and z-coordinate 0. The z-coordinate of each of A and B is 3, so the z-component of the distance from the desired point is the same for each of A and B. The y-component of the distance from the desired point will be 1 for each of A and B, $12 = (-1)^2$. So, the x-coordinate of the desired point has to be halfway between the x-coordinates of A and B. The desired point is (1, 0, 0).

15.



To solve this problem, we must first consider the triangle formed by $\vec{a}$, $\vec{b}$, and $\vec{a}+\vec{b}$. We will use their magnitudes to solve for angle A, which will be used to solve for $\frac{1}{2}\vec{a}-\vec{b}$ in the triangle formed by $\vec{b}$, $\frac{1}{2}\vec{a}+\vec{b}$, and $\frac{1}{2}\vec{a}-\vec{b}$.

Using the cosine law, we see that:

$$\cos(A) = \frac{|\vec{b}|^2 + |\vec{a}+\vec{b}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{a}+\vec{b}|}$$
$$= \frac{25 + 49 - 9}{70}$$
$$= \frac{13}{14}$$

Now, consider the triangle formed by $\vec{b}$, $\frac{1}{2}\vec{a}+\vec{b}$, and $\frac{1}{2}\vec{a}-\vec{b}$. Using the cosine law again:

$$\cos(A) = \frac{|\vec{b}|^2 + \left(\frac{1}{2}|\vec{a}+\vec{b}|\right)^2 - \left(\frac{1}{2}|\vec{a}-\vec{b}|\right)^2}{|\vec{b}||\vec{a}+\vec{b}|}$$
$$\frac{13}{14} = \frac{\frac{149}{4} - \left(\frac{1}{2}|\vec{a}-\vec{b}|\right)^2}{35}$$
$$|\vec{a}-\vec{b}|^2 = -4\left(\frac{65}{2} - \frac{149}{4}\right)$$
$$|\vec{a}-\vec{b}|^2 = 19$$
$$|\vec{a}-\vec{b}| = \sqrt{19} \text{ or } 4.36$$

6.8 Linear Combinations and Spanning Sets, pp. 340–341

1. They are collinear, thus a linear combination is not applicable.
2. It is not possible to use $\vec{0}$ in a spanning set. Therefore, the remaining vectors only span R^2.
3. The set of vectors spanned by (0, 1) is $m(0, 1)$. If we let $m = -1$, then $m(0, 1) = (0, -1)$.
4. $\vec{i}$ spans the set $m(1, 0, 0)$. This is any vector along the x-axis. Examples: (2, 0, 0), $(-21, 0, 0)$
5. As in question 2, it is not possible to use $\vec{0}$ in a spanning set.
6. $\{(-1, 2), (-1, 1)\}, \{(2, -4), (-1, 1)\}, \{(-1, 1), (-3, 6)\}$ are all the possible spanning sets for R^2 with 2 vectors.
7. a. $2(2\vec{a} - 3\vec{b} + \vec{c}) = 4\vec{a} - 6\vec{b} + 2\vec{c}$
$= 4\vec{i} - 8\vec{j} - 6\vec{j} + 18\vec{k} + 2\vec{i} - 6\vec{j} + 4k$
$= 6\vec{i} - 20\vec{j} + 22\vec{k}$
$4(-\vec{a} + \vec{b} - \vec{c}) = -4\vec{a} + 4\vec{b} - 4\vec{c}$
$= -4\vec{i} + 8\vec{j} + 4\vec{j} - 12\vec{k} - 4\vec{i} + 12\vec{j} - 8k$
$= -8\vec{i} + 24\vec{j} - 20\vec{k}$
$(\vec{a} - \vec{c}) = \vec{i} - 2\vec{j} - \vec{i} + 3\vec{j} - 2\vec{k}$
$= \vec{j} - 2\vec{k}$
$2(2\vec{a} - 3\vec{b} + \vec{c}) - 4(-\vec{a} + \vec{b} - \vec{c}) + (\vec{a} - \vec{c})$
$= 6\vec{i} - 20\vec{j} + 22\vec{k} + 8\vec{i} - 24\vec{j} + 20\vec{k} + \vec{j} - 2\vec{k}$
$= 14\vec{i} - 43\vec{j} + 40\vec{k}$
b. $\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) = \vec{a} - 2\vec{b} - 4\vec{c}$
$= \vec{i} - 2\vec{j} - 2\vec{j} + 6\vec{k} - 4\vec{i} + 12\vec{j} - 8\vec{k}$
$= -3\vec{i} + 8\vec{j} - 2\vec{k}$
$\frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c}) = \vec{a} - 2\vec{b} + 3\vec{c}$
$= \vec{i} - 2\vec{j} - 2\vec{j} + 6\vec{k} + 3\vec{i} - 9\vec{j} + 6\vec{k}$
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$\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) - \frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c})$
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$= -7\vec{i} + 23\vec{j} - 14\vec{k}$
8. $\{(1, 0, 0), (0, 1, 0)\}$:
$(-1, 2, 0) = -1(1, 0, 0) + 2(0, 1, 0)$
$(3, 4, 0) = 3(1, 0, 0) + 4(0, 1, 0)$
$\{(1, 1, 0), (0, 1, 0)\}$
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9. a. It is the set of vectors in the xy-plane.
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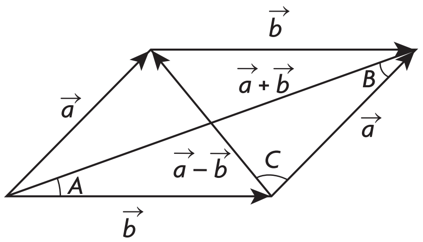
c. By part a. the vector is not in the xy-plane. There is no combination that would produce a number other than 0 for the z-component.
d. It would still only span the xy-plane. There would be no need for that vector.
10. Looking at the x-component:
$2a + 3c = 5$
The y-component:
$6 + 21 = b + c$
The z-component:
$2c + 3c = 15$
$5c = 15$
$c = 3$

15.



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$$= \frac{25 + 49 - 9}{70}$$
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Now, consider the triangle formed by $\vec{b}$, $\frac{1}{2}\vec{a} + \vec{b}$, and $\frac{1}{2}\vec{a} - \vec{b}$. Using the cosine law again:

$$\cos(A) = \frac{|\vec{b}|^2 + \left(\frac{1}{2}|\vec{a} + \vec{b}|\right)^2 - \left(\frac{1}{2}|\vec{a} - \vec{b}|\right)^2}{|\vec{b}||\vec{a} + \vec{b}|}$$
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b. $\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) = \vec{a} - 2\vec{b} - 4\vec{c}$
$= \vec{i} - 2\vec{j} - 2\vec{j} + 6\vec{k} - 4\vec{i} + 12\vec{j} - 8\vec{k}$
$= -3\vec{i} + 8\vec{j} - 2\vec{k}$
$\frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c}) = \vec{a} - 2\vec{b} + 3\vec{c}$
$= \vec{i} - 2\vec{j} - 2\vec{j} + 6\vec{k} + 3\vec{i} - 9\vec{j} + 6\vec{k}$
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$\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) - \frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c})$
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The z-component:
$2c + 3c = 15$
$5c = 15$
$c = 3$

Substituting this into the first and second equation:
$2a + 9 = 5$
$a = -2$
$27 = b + 3$
$b = 24$
11. $(-10, -34) = a(-1, 3) + b(1, 5)$
Looking at the x-component:
$-10 = -a + b \quad a = 10 + b$
Looking at the y-component:
$-34 = 3a + 5b$
Substituting in a:
$-34 = 30 + 3b + 5b$
$b = -8$
Substituting b into x-component equation:
$-10 = -a + (-8)$
$a = -2$
$(-10, -34) = -2(-1, 3) - 8(1, 5)$
12. a. $a(2, -1) + b(-1, 1) = (x, y)$
$x = 2a - b$
$b = 2a - x$
$y = -a + b$
Substitute in b:
$y = -a + 2a - x$
$a = x + y$
Substitute this back into the first equation:
$b = 2x + 2y - x$
$b = x + 2y$
b. Using the formulas in part a:
For $(2, -3)$:
$a = x + y = 2 - 3 = -1$
$b = x + 2y = 2 - 6 = -4$
$(2, -3) = -1(2, -1) - 4(-1, 1)$
For $(124, -5)$:
$a = 124 - 5 = 119$
$b = 124 - 10 = 114$
$(124, -5) = 119(2, -1) + 114(-1, 1)$
For $(4, -11)$
$a = 4 - 11 = -7$
$b = 4 - 22 = -18$
$(4, -11) = -7(2, -1) - 18(-1, 1)$
13. Try: $a(-1, 2, 3) + b(4, 1, -2)$
$= (-14, -1, 16)$
x components:
$-a + 4b = -14$
$a = 14 + 4b$
y components:
$2a + b = -1$
Substitute in a:
$28 + 8b + b = -1$
$b = -3$
Substitute this result into the x-components:
$a = 14 - 3 = 11$
Check by substituting into z-components:
$3a - 2b = 16$
$33 + 5 = 16$
Therefore:
$a(-1, 2, 3) + b(4, 1, -2) \neq (-14, -1, 16)$ for any a and b. They do not lie on the same plane.
b. $a(-1, 3, 4) + b(0, -1, 1) = (-3, 14, 7)$
x components:
$-a = -3$
$a = 3$
y components:
$3a - b = 14$
Substitute in a:
$9 - b = 14$
$b = -5$
Check with z components:
$4a + b = 7$
$12 - 5 = 7$
Since there exists an a and b to form a linear combination of 2 of the vectors to form the third, they lie on the same plane.
$3(-1, 3, 4) - 5(0, -1, 1) = (-3, 14, 7)$
14. Let vector $\vec{a} = (-1, 3, 4)$ and $\vec{b} = (-2, 3, -1)$ (vectors from the origin to points A and B, respectively). To determine x, we let $\vec{c}$ (vector from origin to C) be a linear combination of $\vec{a}$ and $\vec{b}$.
$a(-1, 3, 4) + b(-2, 3, -1) = (-5, 6, x)$
x components:
$-a - 2b = -5$
$a = 5 - 2b$
y components:
$3a + 3b = 6$
Substitute in a:
$15 - 6b + 3b = 6$
$b = 3$
Substitute in b in x component equation:
$a = 5 - 6 = -1$
z components:
$4a - b = x$
Substitute in a and b:
$x = -4 - 3 = -7$
15. $m = 2$, $n = 3$. Non-parallel vectors cannot be equal, unless their magnitudes equal 0.
16. Answers may vary. For example:
Try linear combinations of the 2 vectors such that

the z component equals 5. Then calculate what p and q would equal.
$-1(4, 1, 7) + 2(-1, 1, 6) = (-6, 1, 5)$
So $p = -6$ and $q = 1$
$5(4, 1, 5) - 5(-1, 1, 6) = (25, 0, 5)$
So $p = 25$ and $q = 0$
$(4, 1, 7) - \frac{1}{3}(-1, 1, 6) = \left(\frac{13}{3}, \frac{2}{3}, 5\right)$
So $p = \frac{13}{3}$ and $q = \frac{2}{3}$
17. As in question 15, non-parallel vectors. Their magnitudes must be 0 again to make the equality true.
$m^2 + 2m - 3 = (m - 1)(m + 3)$
$m = 1, -3$
$m^2 + m - 6 = (m - 2)(m + 3)$
$m = 2, -3$
So, when $m = -3$, their sum will be 0.

Review Exercise, pp. 344–347

1. a. false; Let $\vec{b} = -\vec{a} \neq 0$ then:
$$\begin{aligned} |\vec{a} + \vec{b}| &= |\vec{a} + (-\vec{a})| \\ &= |0| \\ &= 0 < |\vec{a}| \end{aligned}$$
b. true; $|\vec{a} + \vec{b}|$ and $|\vec{a} + \vec{c}|$ both represent the lengths of the diagonal of a parallelogram, the first with sides $\vec{a}$ and $\vec{b}$ and the second with sides $\vec{a}$ and $\vec{c}$; since both parallelograms have $\vec{a}$ as a side and diagonals of equal length $|\vec{b}| = |\vec{c}|$.
c. true; Subtracting $\vec{a}$ from both sides shows that $\vec{b} = \vec{c}$
d. true; Draw the parallelogram formed by $\overrightarrow{RF}$ and $\overrightarrow{SW}$. $\overrightarrow{FW}$ and $\overrightarrow{RS}$ are the opposite sides of a parallelogram and must be equal.
e. true; The distributive law for scalars
f. false; Let $\vec{b} = -\vec{a}$ and let $\vec{c} = \vec{d} \neq 0$. Then,
$|\vec{a}| = |-\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$
but $|\vec{a} + \vec{b}| = |\vec{a} + (-\vec{a})| = 0$
$|\vec{c} + \vec{d}| = |\vec{c} + \vec{c}| = |2\vec{c}|$
so $|\vec{a} + \vec{b}| \neq |\vec{c} + \vec{d}|$
2. a. Substitute the given values of $\vec{x}$, $\vec{y}$, and $\vec{z}$ into the expression $2\vec{x} - 3\vec{y} + 5\vec{z}$
$2\vec{x} - 3\vec{y} + 5\vec{z}$
$= 2(2\vec{a} - 3\vec{b} - 4\vec{c}) - 3(-2\vec{a} + 3\vec{b} + 3\vec{c}) + 5(2\vec{a} - 3\vec{b} + 5\vec{c})$
$= 4\vec{a} - 6\vec{b} - 8\vec{c} + 6\vec{a} - 9\vec{b} - 9\vec{c} + 10\vec{a} - 15\vec{b} + 25\vec{c}$
$= 4\vec{a} + 6\vec{a} + 10\vec{a} - 6\vec{b} - 9\vec{b} - 15\vec{b} - 8\vec{c} - 9\vec{c} + 25\vec{c}$
$= 20\vec{a} - 30\vec{b} + 8\vec{c}$
b. Simplify the expression before substituting the given values of $\vec{x}$, $\vec{y}$, and $\vec{z}$
$3(-2\vec{x} - 4\vec{y} + \vec{z}) - (2\vec{x} - \vec{y} + \vec{z}) - 2(-4\vec{x} - 5\vec{y} + \vec{z})$
$= -6\vec{x} - 12\vec{y} + 3\vec{z} - 2\vec{x} + \vec{y} - \vec{z} + 8\vec{x} + 10\vec{y} - 2\vec{z}$
$= -6\vec{x} - 2\vec{x} + 8\vec{x} - 12\vec{y} + \vec{y} + 10\vec{y} + 3\vec{z} - \vec{z} - 2\vec{z}$
$= 0\vec{x} - \vec{y} + 0\vec{z}$
$= -\vec{y}$
$= 2\vec{a} - 3\vec{b} - 3\vec{c}$
3. a. $\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$
$$\begin{aligned} &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (-4 - (-2), 4 - 1, 8 - 2) \\ &= (-2, 3, 6) \end{aligned}$$
$$\begin{aligned} |\overrightarrow{XY}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(-2)^2 + (3)^2 + (6)^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$
b. The components of a unit vector in the same direction as $\overrightarrow{XY}$ are $\frac{1}{7}(-2, 3, 6) = \left(-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$.
4. a. The position vector $\overrightarrow{OP}$ is equivalent to $\overrightarrow{YX}$.
$$\begin{aligned} \overrightarrow{OP} &= \overrightarrow{YX} \\ &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (-1 - 5, 2 - 5, 6 - 12) \\ &= (-6, -3, -6) \end{aligned}$$
b. $$\begin{aligned} |\overrightarrow{YX}| &= \sqrt{(-6)^2 + (-3)^2 + (-6)^2} \\ &= \sqrt{81} \\ &= 9 \end{aligned}$$
The components of a unit vector in the same direction as $\overrightarrow{YX}$ are $\frac{1}{9}(-6, -3, -6) = \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$
5. $-\overrightarrow{MN} = \overrightarrow{NM}$
$$\begin{aligned} &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (2 - 8, 3 - 1, 5 - 2) \\ &= (-6, 2, 3) \end{aligned}$$

the z component equals 5. Then calculate what p and q would equal.
$-1(4, 1, 7) + 2(-1, 1, 6) = (-6, 1, 5)$
So $p = -6$ and $q = 1$
$5(4, 1, 5) - 5(-1, 1, 6) = (25, 0, 5)$
So $p = 25$ and $q = 0$
$(4, 1, 7) - \frac{1}{3}(-1, 1, 6) = \left(\frac{13}{3}, \frac{2}{3}, 5\right)$
So $p = \frac{13}{3}$ and $q = \frac{2}{3}$
17. As in question 15, non-parallel vectors. Their magnitudes must be 0 again to make the equality true.
$m^2 + 2m - 3 = (m - 1)(m + 3)$
$m = 1, -3$
$m^2 + m - 6 = (m - 2)(m + 3)$
$m = 2, -3$
So, when $m = -3$, their sum will be 0.

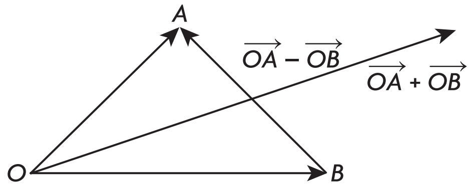
Review Exercise, pp. 344–347

1. a. false; Let $\vec{b} = -\vec{a} \neq 0$ then:
$$\begin{aligned}|\vec{a} + \vec{b}| &= |\vec{a} + (-\vec{a})| \\ &= |0| \\ &= 0 < |\vec{a}|\end{aligned}$$
b. true; $|\vec{a} + \vec{b}|$ and $|\vec{a} + \vec{c}|$ both represent the lengths of the diagonal of a parallelogram, the first with sides $\vec{a}$ and $\vec{b}$ and the second with sides $\vec{a}$ and $\vec{c}$; since both parallelograms have $\vec{a}$ as a side and diagonals of equal length $|\vec{b}| = |\vec{c}|$.
c. true; Subtracting $\vec{a}$ from both sides shows that $\vec{b} = \vec{c}$
d. true; Draw the parallelogram formed by $\overrightarrow{RF}$ and $\overrightarrow{SW}$. $\overrightarrow{FW}$ and $\overrightarrow{RS}$ are the opposite sides of a parallelogram and must be equal.
e. true; The distributive law for scalars
f. false; Let $\vec{b} = -\vec{a}$ and let $\vec{c} = \vec{d} \neq 0$. Then,
$|\vec{a}| = |-\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$
but $|\vec{a} + \vec{b}| = |\vec{a} + (-\vec{a})| = 0$
$|\vec{c} + \vec{d}| = |\vec{c} + \vec{c}| = |2\vec{c}|$
so $|\vec{a} + \vec{b}| \neq |\vec{c} + \vec{d}|$
2. a. Substitute the given values of $\vec{x}$, $\vec{y}$, and $\vec{z}$ into the expression $2\vec{x} - 3\vec{y} + 5\vec{z}$
$2\vec{x} - 3\vec{y} + 5\vec{z}$
$= 2(2\vec{a} - 3\vec{b} - 4\vec{c}) - 3(-2\vec{a} + 3\vec{b} + 3\vec{c}) + 5(2\vec{a} - 3\vec{b} + 5\vec{c})$
$= 4\vec{a} - 6\vec{b} - 8\vec{c} + 6\vec{a} - 9\vec{b} - 9\vec{c} + 10\vec{a} - 15\vec{b} + 25\vec{c}$
$= 4\vec{a} + 6\vec{a} + 10\vec{a} - 6\vec{b} - 9\vec{b} - 15\vec{b} - 8\vec{c} - 9\vec{c} + 25\vec{c}$
$= 20\vec{a} - 30\vec{b} + 8\vec{c}$
b. Simplify the expression before substituting the given values of $\vec{x}$, $\vec{y}$, and $\vec{z}$
$3(-2\vec{x} - 4\vec{y} + \vec{z}) - (2\vec{x} - \vec{y} + \vec{z}) - 2(-4\vec{x} - 5\vec{y} + \vec{z})$
$= -6\vec{x} - 12\vec{y} + 3\vec{z} - 2\vec{x} + \vec{y} - \vec{z} + 8\vec{x} + 10\vec{y} - 2\vec{z}$
$= -6\vec{x} - 2\vec{x} + 8\vec{x} - 12\vec{y} + \vec{y} + 10\vec{y} + 3\vec{z} - \vec{z} - 2\vec{z}$
$= 0\vec{x} - \vec{y} + 0\vec{z}$
$= -\vec{y}$
$= 2\vec{a} - 3\vec{b} - 3\vec{c}$
3. a.
$$\begin{aligned}\overrightarrow{XY} &= \overrightarrow{OY} - \overrightarrow{OX} \\ &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (-4 - (-2), 4 - 1, 8 - 2) \\ &= (-2, 3, 6)\end{aligned}$$
$$\begin{aligned}|\overrightarrow{XY}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(-2)^2 + (3)^2 + (6)^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} \\ &= 7\end{aligned}$$
b. The components of a unit vector in the same direction as $\overrightarrow{XY}$ are $\frac{1}{7}(-2, 3, 6) = \left(-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$.
4. a. The position vector $\overrightarrow{OP}$ is equivalent to $\overrightarrow{YX}$.
$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{YX} \\ &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (-1 - 5, 2 - 5, 6 - 12) \\ &= (-6, -3, -6)\end{aligned}$$
b.
$$\begin{aligned}|\overrightarrow{YX}| &= \sqrt{(-6)^2 + (-3)^2 + (-6)^2} \\ &= \sqrt{81} \\ &= 9\end{aligned}$$
The components of a unit vector in the same direction as $\overrightarrow{YX}$ are $\frac{1}{9}(-6, -3, -6) = \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$
5.
$$\begin{aligned}-\overrightarrow{MN} &= \overrightarrow{NM} \\ &= (x_2, y_2, z_2) - (x_1, y_1, z_1) \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ &= (2 - 8, 3 - 1, 5 - 2) \\ &= (-6, 2, 3)\end{aligned}$$

$$|\overrightarrow{NM}| = \sqrt{(-6)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{49}$$
$$= 7$$

The components of the unit vector with the opposite direction to $\overrightarrow{MN}$ are $\frac{1}{7}(-6, 2, 3) = \left(-\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

6. a. The two diagonals can be found by calculating $\overrightarrow{OA} + \overrightarrow{OB}$ and $\overrightarrow{OA} - \overrightarrow{OB}$.



$$\overrightarrow{OA} + \overrightarrow{OB} = (3, 2, -6) + (-6, 6, -2)$$
$$= (3 + -6, 2 + 6, -6 + -2)$$
$$= (-3, 8, -8)$$
$$\overrightarrow{OA} - \overrightarrow{OB} = (3, 2, -6) + (-6, 6, -2)$$
$$= (3 - (-6), 2 - 6, -6 - (-2))$$
$$= (9, -4, -4)$$

b. To determine the angle between the sides of the parallelogram, calculate $|\overrightarrow{OA}|$, $|\overrightarrow{OB}|$, and $|\overrightarrow{OA} - \overrightarrow{OB}|$ and apply the cosine law.

$$|\overrightarrow{OA}| = \sqrt{(3)^2 + (2)^2 + (-6)^2}$$
$$= \sqrt{49}$$
$$= 7$$
$$|\overrightarrow{OB}| = \sqrt{(-6)^2 + (6)^2 + (-2)^2}$$
$$= \sqrt{76}$$
$$= 2\sqrt{19}$$
$$|\overrightarrow{OA} - \overrightarrow{OB}| = \sqrt{(9)^2 + (-4)^2 + (-4)^2}$$
$$= \sqrt{113}$$
$$\cos\theta = \frac{|\overrightarrow{OA}|^2 + |\overrightarrow{OB}|^2 - |\overrightarrow{OA} - \overrightarrow{OB}|^2}{2|\overrightarrow{OA}||\overrightarrow{OB}|}$$
$$\cos\theta = \frac{(7)^2 + (2\sqrt{19})^2 - (\sqrt{113})^2}{2(7)(2\sqrt{19})}$$
$$\cos\theta \doteq 0.098$$
$$\theta \doteq 84.4°$$

7. a. $|\overrightarrow{AB}| = \sqrt{(2 - (-1))^2 + (0 - 1)^2 + (3 - 1)^2}$
$$= \sqrt{(3)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{9 + 1 + 4}$$
$$= \sqrt{14}$$

$$|\overrightarrow{BC}| = \sqrt{(3 - 2)^2 + (3 - 0)^2 + (-4 - 3)^2}$$
$$= \sqrt{(1)^2 + (3)^2 + (-7)^2}$$
$$= \sqrt{1 + 9 + 49}$$
$$= \sqrt{59}$$
$$|\overrightarrow{CA}| = \sqrt{(-1 - 3)^2 + (1 - 3)^2 + (1 - (-4))^2}$$
$$= \sqrt{(-4)^2 + (-2)^2 + (5)^2}$$
$$= \sqrt{16 + 4 + 25}$$
$$= \sqrt{45}$$

Triangle ABC is a right triangle if and only if $|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = |\overrightarrow{BC}|^2$.

$$|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = (\sqrt{14})^2 + (\sqrt{45})^2$$
$$= 14 + 45$$
$$= 59$$
$$|\overrightarrow{BC}|^2 = (\sqrt{59})^2$$
$$= 59$$

So triangle ABC is a right triangle.

b. Area of a triangle $= \frac{1}{2}bh$. For triangle ABC the longest side $\overrightarrow{BC}$ is the hypotenuse, so $\overrightarrow{AB}$ and $\overrightarrow{CA}$ are the base and height of the triangle.

$$\text{Area} = \frac{1}{2}(|\overrightarrow{AB}|)(|\overrightarrow{CA}|)$$
$$= \frac{1}{2}\sqrt{14}\sqrt{45}$$
$$= \frac{1}{2}\sqrt{630}$$
$$= \frac{3}{2}\sqrt{70} \text{ or } 12.5$$

c. Perimeter of a triangle equals the sum of the sides.

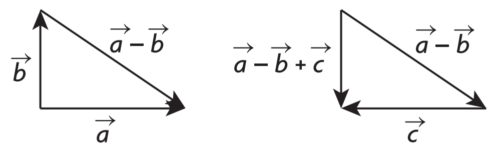
$$\text{Perimeter} = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CA}|$$
$$= \sqrt{14} + \sqrt{59} + \sqrt{45}$$
$$\doteq 18.13$$

d. The fourth vertex D is the head of the diagonal vector from A. To find $\overrightarrow{AD}$ take $\overrightarrow{AB} + \overrightarrow{AC}$.

$$\overrightarrow{AB} = (2 - (-1), 0 - 1, 3 - 1) = (3, -1, 2)$$
$$\overrightarrow{AC} = (3 - (-1), 3 - 1, -4 - 1) = (4, 2, -5)$$
$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$$
$$= (3 + 4, -1 + 2, 2 + (-5))$$
$$= (7, 1, -3)$$

So the fourth vertex is $D(-1 + 7, 1 + 1, 1 + (-3))$ or $D(6, 2, -2)$.

8. a.



b. Since the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$. So,

$$\begin{aligned}|\vec{a}+\vec{b}|^2 &= (4)^2 + (3)^2\\ &= 16+9\\ &= 25\\ |\vec{a}+\vec{b}| &= \sqrt{25} = 5\end{aligned}$$

9. Express $\vec{r}$ as a linear combination of $\vec{p}$ and $\vec{q}$:
Solve for a and b:

$$\begin{aligned}\vec{r} &= a\vec{p} + b\vec{q}\\ (-1,2) &= a(-11,7) + b(-3,1)\\ (-1,2) &= (-11a,7a) + (-3b,b)\\ (-1,2) &= (-11a-3b, 7a+b)\end{aligned}$$

Solve the system of equations:

$$\begin{aligned}-1 &= -11a - 3b\\ 2 &= 7a + b\end{aligned}$$

Use the method of elimination:

$$\begin{aligned}3(2) &= 3(7a+b)\\ 6 &= 21a + 3b\\ +\ -1 &= -11a - 3b\\ \hline 5 &= 10a\\ \frac{1}{2} &= a\end{aligned}$$

By substitution, $b = -\frac{3}{2}$

Therefore $\frac{1}{2}(-11,7) + -\frac{3}{2}(-3,1) = (-1,2)$

Express $\vec{q}$ as a linear combination of $\vec{p}$ and $\vec{r}$.
Solve for a and b:

$$\begin{aligned}\vec{q} &= a\vec{p} + b\vec{r}\\ (-3,1) &= a(-11,7) + b(-1,2)\\ (-3,1) &= (-11a,7a) + (-b,2b)\\ (-3,1) &= (-11a-b, 7a+2b)\end{aligned}$$

Solve the system of equations:

$$\begin{aligned}-3 &= -11a - b\\ 1 &= 7a + 2b\end{aligned}$$

Use the method of elimination:

$$\begin{aligned}2(-3) &= 2(-11a-b)\\ -6 &= -22a - 2b\\ +1 &= 7a + 2b\\ \hline -5 &= -15a\\ \frac{1}{3} &= a\end{aligned}$$

By substitution, $-\frac{2}{3} = b$

Therefore $\frac{1}{3}(-11,7) + -\frac{2}{3}(-1,2) = (-3,1)$

Express $\vec{p}$ as a linear combination of $\vec{q}$ and $\vec{r}$.
Solve for a and b:

$$\begin{aligned}\vec{p} &= a\vec{q} + b\vec{r}\\ (-11,7) &= a(-3,1) + b(-1,2)\\ (-11,7) &= (-3a,a) + (-b,2b)\\ (-11,7) &= (-3a-b, a+2b)\end{aligned}$$

Solve the system of equations:

$$\begin{aligned}-11 &= -3a - b\\ 7 &= a + 2b\end{aligned}$$

Use the method of elimination:

$$\begin{aligned}2(-11) &= 2(-3a-b)\\ -22 &= -6a - 2b\\ +\quad 7 &= a + 2b\\ \hline -15 &= -5a\\ 3 &= a\end{aligned}$$

By substitution, $2 = b$

Therefore $3(-3,1) + 2(-1,2) = (-11,7)$

10. a. Let $P(x,y,z)$ be a point equidistant from A and B. Then $|\overrightarrow{PA}| = |\overrightarrow{PB}|$.

$$\begin{aligned}&(x-2)^2 + (y-(-1))^2 + (z-3)^2\\ &= (x-1)^2 + (y-2)^2 + (z-(-3))^2\\ &x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 6z + 9\\ &= x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + 6z + 9\\ &-2x + 6y - 12z = 0\\ &x - 3y + 6z = 0\end{aligned}$$

b. (0, 0, 0) and $\left(1, \frac{1}{3}, 0\right)$ clearly satisfy the equation and are equidistant from A and B.

11. a.

$$\begin{aligned}(-24,3,25) &= 2(a,b,4) + \frac{1}{2}(6,8,c) - 3(7,c,-4)\\ (-24,3,25) &= (2a,2b,8) + \left(3,4,\frac{c}{2}\right)\\ &\quad - (21,3c,-12)\\ (-24,3,25) &= \left(2a-18, 2b+4-3c, \frac{c}{2}+20\right)\end{aligned}$$

Solve the equations:

i. $-24 = 2a - 18$
$-6 = 2a$
$-3 = a$

ii. $25 = \frac{c}{2} + 20$
$5 = \frac{c}{2}$
$10 = c$

iii. $3 = 2b + 4 - 3c$
$3 = 2b + 4 - 3(18)$
$3 = 2b - 50$
$53 = 2b$
$26.5 = b$

b. $(3, -22, 54)$

$= 2\left(a, a, \frac{1}{2}a\right) + (3b, 0, -5c) + 2\left(c, \frac{3}{2}c, 0\right)$

$(3, -22, 54)$
$= (2a, 2a, a) + (3b, 0, -5c) + (2c, 3c, 0)$
$(3, -22, 54) = (2a + 3b + 2c, 2a + 3c, a - 5c)$

Solve the system of equations:

$-22 = 2a + 3c$
$54 = a - 5c$

Use the method of elimination:

$$\begin{aligned} -2(54) &= -2(a - 5c) \\ -108 &= -2a + 10c \\ + -22 &= 2a + 3c \\ \hline -130 &= 13c \\ -10 &= c \end{aligned}$$

By substitution, $8 = a$

Solve the equation:

$3 = 2a + 3b + 2c$
$3 = 2(8) + 3b + 2(-10)$
$3 = 16 + 3b - 20$
$3 = 3b - 4$
$7 = 3b$
$\frac{7}{3} = b$

12. a. Find $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$, $|\overrightarrow{CA}|$

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(2-1)^2 + (2-(-1))^2 + (2-1)^2} \\ &= \sqrt{(1)^2 + (3)^2 + (1)^2} \\ &= \sqrt{11} \end{aligned}$$

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{(4-2)^2 + (-2-2)^2 + (1-2)^2} \\ &= \sqrt{(2)^2 + (-4)^2 + (-1)^2} \\ &= \sqrt{21} \end{aligned}$$

$$\begin{aligned} |\overrightarrow{CA}| &= \sqrt{(4-1)^2 + (-2-(-1))^2 + (1-1)^2} \\ &= \sqrt{(3)^2 + (-1)^2} \\ &= \sqrt{10} \end{aligned}$$

Test $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$, $|\overrightarrow{CA}|$ in the Pythagorean theorem:

$$\begin{aligned} |\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 &= (\sqrt{11})^2 + (\sqrt{10})^2 \\ &= 11 + 10 \\ &= 21 \\ |\overrightarrow{BC}|^2 &= (\sqrt{21})^2 \\ &= 21 \end{aligned}$$

So triangle ABC is a right triangle.

b. Yes, $P(1, 2, 3)$, $Q(2, 4, 6)$, and $R(-1, -2, -3)$ are collinear because:

$2P = (2, 4, 6)$
$1Q = (2, 4, 6)$
$-2R = (2, 4, 6)$

13. a. Find $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$, $|\overrightarrow{CA}|$

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(1-3)^2 + (2-0)^2 + (5-4)^2} \\ &= \sqrt{(-2)^2 + (2)^2 + (1)^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} |\overrightarrow{BC}| &= \sqrt{(2-1)^2 + (1-2)^2 + (3-5)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (-2)^2} \\ &= \sqrt{6} \end{aligned}$$

$$\begin{aligned} |\overrightarrow{CA}| &= \sqrt{(2-3)^2 + (1-0)^2 + (3-4)^2} \\ &= \sqrt{(-1)^2 + (1)^2 + (-1)^2} \\ &= \sqrt{3} \end{aligned}$$

Test $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$, $|\overrightarrow{CA}|$ in the Pythagorean theorem:

$$\begin{aligned} |\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2 &= (\sqrt{6})^2 + (\sqrt{3})^2 \\ &= 6 + 3 \\ &= 9 \\ |\overrightarrow{AB}|^2 &= (3)^2 \\ &= 9 \end{aligned}$$

So triangle ABC is a right triangle.

b. Since triangle ABC is a right triangle,

$\cos\angle ABC = \sqrt{\frac{6}{3}}$

14. a. $\overrightarrow{DA}$, $\overrightarrow{BC}$ and $\overrightarrow{EB}$, $\overrightarrow{ED}$

b. $\overrightarrow{DC}$, $\overrightarrow{AB}$ and $\overrightarrow{CE}$, $\overrightarrow{EA}$

c. $|\overrightarrow{AD}|^2 + |\overrightarrow{DC}|^2 = |\overrightarrow{AC}|^2$,

But $|\overrightarrow{AC}|^2 = |\overrightarrow{DB}|^2$

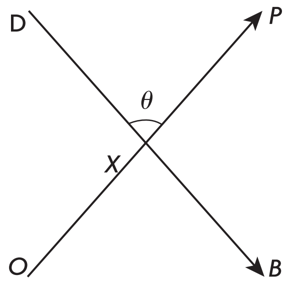
Therefore, $|\overrightarrow{AD}|^2 + |\overrightarrow{DC}|^2 = |\overrightarrow{DB}|^2$

15. a. $C(3, 0, 5)$; $P(3, 4, 5)$; $E(0, 4, 5)$; $F(0, 4, 0)$

b. $\overrightarrow{DB} = (3 - 0, 4 - 0, 0 - 5)$
$= (3, 4, -5)$

$\overrightarrow{CF} = (0 - 3, 4 - 0, 0 - 5)$
$= (-3, 4, -5)$

c.

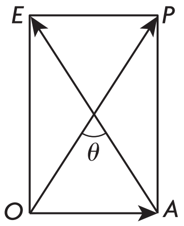


$|\overrightarrow{OD}| = 5$

$|\overrightarrow{DP}| = 5$ by the Pythagorean theorem

Thus $ODPB$ is a square and $\cos\theta = 0$, so the angle between the vectors is 90°.

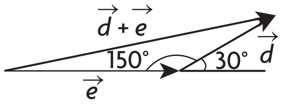
d.



$|\overrightarrow{OA}| = 3$, $|\overrightarrow{OP}| = \sqrt{5}$

$$\begin{aligned}\theta &= 180° - 2(m\angle POA)\\ &= 180° - 2\left(\cos^{-1}\left(\frac{3}{\sqrt{50}}\right)\right)\\ &\doteq 50.2°\end{aligned}$$

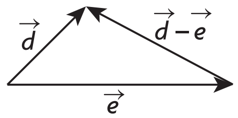
16. a.



Use the cosine law to evaluate $|\vec{d} + \vec{e}|$

$$\begin{aligned}|\vec{d} + \vec{e}|^2 &= |\vec{d}|^2 + |\vec{e}|^2 - 2|\vec{d}||\vec{e}|\cos\theta\\ &= (3)^2 + (5)^3 - 2(3)(5)\cos 150°\\ &= 9 + 25 - 30\frac{-\sqrt{3}}{2}\\ &\doteq 59.98\\ |\vec{d} + \vec{e}| &\doteq \sqrt{59.98}\\ &\doteq 7.74\end{aligned}$$

b.

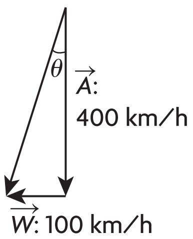


Use the cosine law to evaluate $|\vec{d} - \vec{e}|$

$$\begin{aligned}|\vec{d} - \vec{e}|^2 &= |\vec{d}|^2 + |\vec{e}|^2 - 2|\vec{d}||\vec{e}|\cos\theta\\ &= (3)^2 + (5)^3 - 2(3)(5)\cos 30°\\ &= 9 + 25 - 30\frac{\sqrt{3}}{2}\\ &\doteq 8.02\\ |\vec{d} - \vec{e}| &\doteq \sqrt{8.02}\\ &\doteq 2.83\end{aligned}$$

c. $|\vec{e} - \vec{d}| = |-(\vec{d} - \vec{e})| = |\vec{d} - \vec{e}| \doteq 2.83$

17. a.



Let $\vec{A}$ represent the air speed of the airplane and let $\vec{W}$ represent the velocity of the wind. In one hour, the plane will travel $|\vec{A} + \vec{W}|$ kilometers. Because $\vec{A}$ and $\vec{W}$ make a right angle, use the Pythagorean theorem:

$$\begin{aligned}|\vec{A} + \vec{W}|^2 &= |\vec{A}|^2 + |\vec{W}|^2\\ &= (400)^2 + (100)^2\\ &= 170000\\ |\vec{A} + \vec{W}| &= \sqrt{170000}\\ &\doteq 412.3 \text{ km}\end{aligned}$$

So in 3 hours, the plane will travel

$3(412.3)\text{km} \doteq 1236.9$ km

b. $$\begin{aligned}\tan\theta &= \frac{|\vec{W}|}{|\vec{A}|}\\ &= \frac{100}{400}\\ \theta &= \tan^{-1}\left(\frac{1}{4}\right)\\ &\doteq 14.0°\end{aligned}$$

The direction of the airplane is S14.0°W.

18. a. Any pair of nonzero, noncollinear vectors will span R^2. To show that (2, 3) and (3, 5) are noncollinear, show that there does not exist any number k such that $k(2, 3) = (3, 5)$. Solve the system of equations:

$2k = 3$

$3k = 5$

Solving both equations gives two different values for k, $\frac{3}{2}$ and $\frac{5}{3}$, so (2, 3) and (3, 5) are noncollinear and thus span R^2

b. $$\begin{aligned}(323, 795) &= m(2, 3) + n(3, 5)\\ (323, 795) &= (2m, 3m) + (3n, 5n)\\ (323, 795) &= (2m + 3n, 3m + 5n)\end{aligned}$$

Solve the system of equations:

$323 = 2m + 3n$

$795 = 3m + 5n$

Use the method of elimination:

$$\begin{aligned}-3(323) &= -3(2m + 3n)\\ 2(795) &= 2(3m + 5n)\\ -969 &= -6m - 9n\\ +\quad 1590 &= 6m + 10n\\ \hline 621 &= n\end{aligned}$$

By substitution, $m = -770$.

19. a. Find a and b such that

$(5, 9, 14) = a(-2, 3, 1) + b(3, 1, 4)$

$(5, 9, 14) = (-2a, 3a, a) + (3b, b, 4b)$

$(5, 9, 14) = (-2a + 3b, 3a + b, a + 4b)$

i. $5 = -2a + 3b$

ii. $9 = 3a + b$

iii. $14 = a + 4b$

Use the method of elimination with **i.** and **iii.**

$$\begin{aligned} 2(14) &= 2(a + 4b) \\ 28 &= 2a + 8b \\ +\quad 5 &= -2a + 3b \\ \hline 33 &= 11b \\ 3 &= b \end{aligned}$$

By substitution, $a = 2$.

$\vec{a}$ lies in the plane determined by $\vec{b}$ and $\vec{c}$ because it can be written as a linear combination of $\vec{b}$ and $\vec{c}$.

b. If vector $\vec{a}$ is in the span of $\vec{b}$ and $\vec{c}$, then $\vec{a}$ can be written as a linear combination of $\vec{b}$ and $\vec{c}$. Find m and n such that

$$\begin{aligned} (-13, 36, 23) &= m(-2, 3, 1) + n(3, 1, 4) \\ &= (-2m, 3m, m) + (3n, n, 4n) \\ &= (-2m + 3n, 3m + n, m + 4n) \end{aligned}$$

Solve the system of equations:

$$\begin{aligned} -13 &= -2m + 3n \\ 36 &= 3m + n \\ 23 &= m + 4n \end{aligned}$$

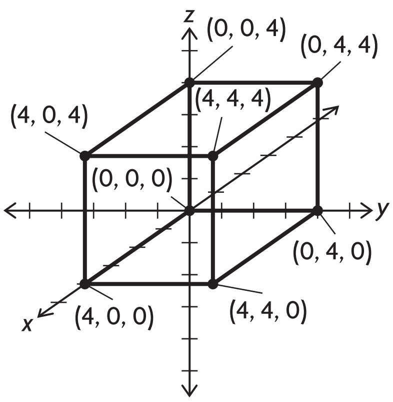
Use the method of elimination:

$$\begin{aligned} 2(23) &= 2(m + 4n) \\ 46 &= 2m + 8n \\ + -13 &= -2m + 3n \\ \hline 33 &= 11n \\ 3 &= n \end{aligned}$$

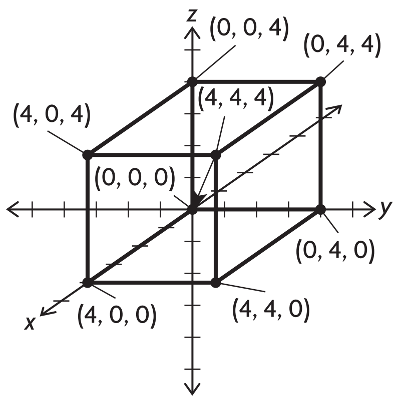
By substitution, $m = 11$.

So, vector $\vec{a}$ is in the span of $\vec{b}$ and $\vec{c}$.

20. a.



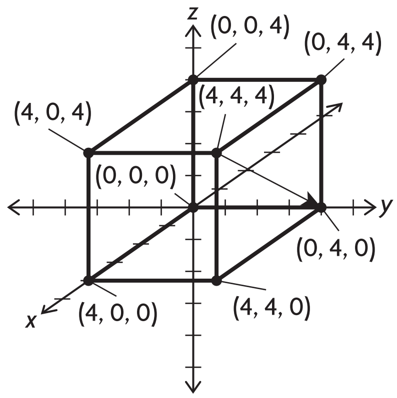
b.



$\overrightarrow{PO} = (4, 4, 4)$ so,

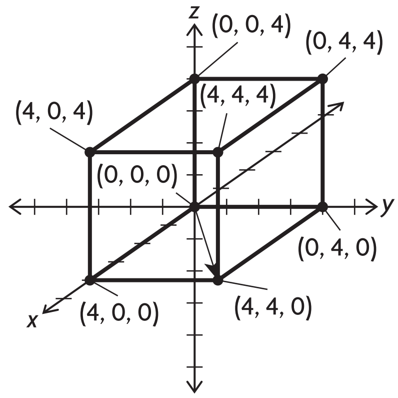
$\overrightarrow{OP} = -\overrightarrow{PO} = -(4, 4, 4) = (-4, -4, -4)$

c.



The vector $\overrightarrow{PQ}$ from $P(4, 4, 4)$ to $Q(0, 4, 0)$ can be written as $\overrightarrow{PQ} = (-4, 0, -4)$.

d.



The vector with the coordinates (4, 4, 0).

21. $|2(\vec{a} + \vec{b} - \vec{c}) - (\vec{a} + 2\vec{b}) + 3(\vec{a} - \vec{b} + \vec{c})|$

$= |2\vec{a} + 2\vec{b} - 2\vec{c} - \vec{a} - 2\vec{b} + 3\vec{a} - 3\vec{b} + 3\vec{c}|$

$= |4\vec{a} - 3\vec{b} + \vec{c}|$

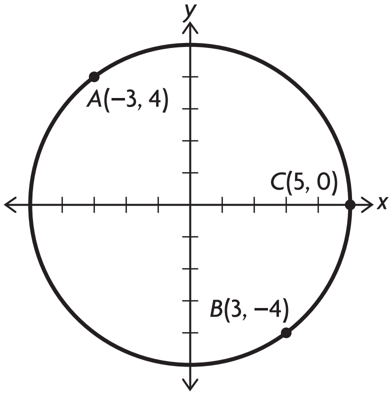
$= |4(1, 1, -1) - 3(2, -1, 3) + (2, 0, 13)|$

$= |(4, 4, -4) + (-6, 3, -9) + (2, 0, 13)|$

$= |(0, 7, 0)|$

$= 7$

22.



a. $|\overrightarrow{AB}| = 10$ because it is the diameter of the circle.

$$|\overrightarrow{BC}| = \sqrt{(5-3)^2 + (0-(-4))^2}$$
$$= \sqrt{(2)^2 + (4)^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5} \text{ or } 4.47$$
$$|\overrightarrow{CA}| = \sqrt{(5-(-3))^2 + (0-4)^2}$$
$$= \sqrt{(8)^2 + (-4)^2}$$
$$= \sqrt{80} \text{ or } 8.94$$

b. If *A, B,* and *C* are vertices of a right triangle, then

$$|\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2 = |\overrightarrow{AB}|^2$$
$$|\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2 = (2\sqrt{5})^2 + (\sqrt{80})^2$$
$$= 20 + 80$$
$$= 100$$
$$|\overrightarrow{AB}|^2 = 10^2$$
$$= 100$$

So, triangle *ABC* is a right triangle.

23. a. $\overrightarrow{FL} = \overrightarrow{FG} + \overrightarrow{GH} + \overrightarrow{HL} = \vec{a} + \vec{b} + \vec{c}$

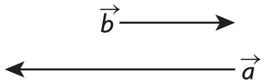
b. $\overrightarrow{MK} = \overrightarrow{JK} - \overrightarrow{JM} = \vec{a} - \vec{b}$

c. $\overrightarrow{HJ} = \overrightarrow{HG} + \overrightarrow{GF} + \overrightarrow{FJ} = -\vec{b} - \vec{a} + \vec{c}$

d. $\overrightarrow{IH} + \overrightarrow{KJ} = \overrightarrow{FG} + \overrightarrow{GF} = 0$

e. $\overrightarrow{IK} - \overrightarrow{IH} = \overrightarrow{HK} = \overrightarrow{IJ} = \vec{b} - \vec{c}$

24.



25. a. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$ by the Pythagorean theorem

b. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$ by the Pythagorean theorem

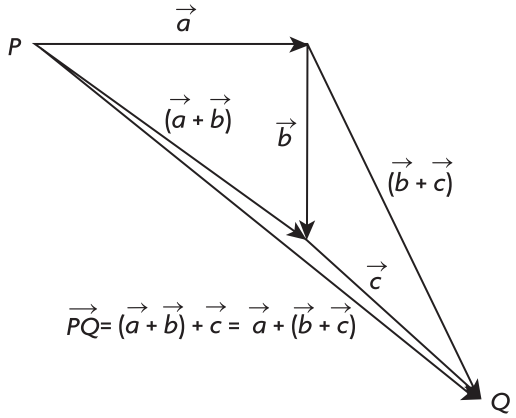
c. $\sqrt{4|\vec{a}|^2 + 9|\vec{b}|^2}$ by the Pythagorean theorem

26. Case 1 If $\vec{b}$ and $\vec{c}$ are collinear, then $2\vec{b} + 4\vec{c}$ is also collinear with both $\vec{b}$ and $\vec{c}$. But $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, so $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Case 2 If $\vec{b}$ and $\vec{c}$ are not collinear, then by spanning sets, $\vec{b}$ and $\vec{c}$ span a plane in R^3, and $\overline{2\vec{b}} + 4\vec{c}$ is in that plane. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then it is perpendicular to the plane and all vectors in the plane. So, $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Chapter 6 Test, p. 348

1. Let *P* be the tail of $\vec{a}$ and let *Q* be the head of $\vec{c}$. The vector sums $[\vec{a} + (\vec{b} + \vec{c})]$ and $[(\vec{a} + \vec{b}) + \vec{c}]$ can be depicted as in the diagram below, using the triangle law of addition. We see that $\overrightarrow{PQ} = \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$. This is the associative property for vector addition.



2. a. $\overrightarrow{AB} = (6-(-2), 7-3, 3-(-5)) = (8, 4, 8)$

b. $|\overrightarrow{AB}| = \sqrt{8^2 + 4^2 + 8^2} = 12$

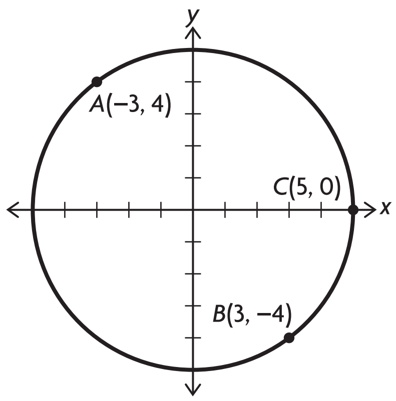
c. $\overrightarrow{BA} = (-1)\overrightarrow{AB} = (-8, -4, -8)$;

$|\overrightarrow{BA}| = |\overrightarrow{AB}| = 12$; unit vector in direction of

$$|\overrightarrow{BA}| = \frac{1}{|\overrightarrow{BA}|}\overrightarrow{BA}$$
$$= \frac{1}{12}(-8, -4, -8)$$
$$= \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$$

3. Let $\vec{x} = \overrightarrow{PQ}$, $\vec{y} = \overrightarrow{QR}$, and $-\vec{y} = \overrightarrow{QS}$, as in the diagram below. Note that $|\overrightarrow{RS}| = |2\vec{y}| = 6$ and that triangle *PQR* and triangle *PRS* share angle θ.

22.



a. $|\vec{AB}| = 10$ because it is the diameter of the circle.

$$\begin{aligned}|\vec{BC}| &= \sqrt{(5-3)^2 + (0-(-4))^2}\\ &= \sqrt{(2)^2 + (4)^2}\\ &= \sqrt{20}\\ &= 2\sqrt{5} \text{ or } 4.47\end{aligned}$$

$$\begin{aligned}|\vec{CA}| &= \sqrt{(5-(-3))^2 + (0-4)^2}\\ &= \sqrt{(8)^2 + (-4)^2}\\ &= \sqrt{80} \text{ or } 8.94\end{aligned}$$

b. If *A, B,* and *C* are vertices of a right triangle, then

$$|\vec{BC}|^2 + |\vec{CA}|^2 = |\vec{AB}|^2$$

$$\begin{aligned}|\vec{BC}|^2 + |\vec{CA}|^2 &= (2\sqrt{5})^2 + (\sqrt{80})^2\\ &= 20 + 80\\ &= 100\\ |\vec{AB}|^2 &= 10^2\\ &= 100\end{aligned}$$

So, triangle *ABC* is a right triangle.

23. a. $\vec{FL} = \vec{FG} + \vec{GH} + \vec{HL} = \vec{a} + \vec{b} + \vec{c}$

b. $\vec{MK} = \vec{JK} - \vec{JM} = \vec{a} - \vec{b}$

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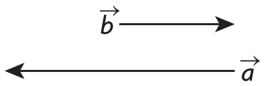
d. $\vec{IH} + \vec{KJ} = \vec{FG} + \vec{GF} = 0$

e. $\vec{IK} - \vec{IH} = \vec{HK} = \vec{IJ} = \vec{b} - \vec{c}$

24.



25. a. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$ by the Pythagorean theorem

b. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$ by the Pythagorean theorem

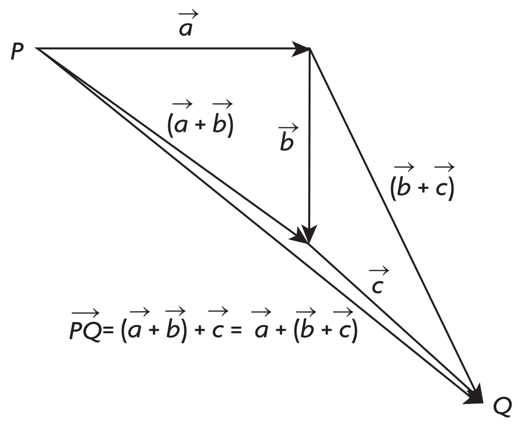
c. $\sqrt{4|\vec{a}|^2 + 9|\vec{b}|^2}$ by the Pythagorean theorem

26. Case 1 If $\vec{b}$ and $\vec{c}$ are collinear, then $2\vec{b} + 4\vec{c}$ is also collinear with both $\vec{b}$ and $\vec{c}$. But $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, so $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Case 2 If $\vec{b}$ and $\vec{c}$ are not collinear, then by spanning sets, $\vec{b}$ and $\vec{c}$ span a plane in R^3, and $2\vec{b} + 4\vec{c}$ is in that plane. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then it is perpendicular to the plane and all vectors in the plane. So, $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Chapter 6 Test, p. 348

1. Let *P* be the tail of $\vec{a}$ and let *Q* be the head of $\vec{c}$. The vector sums $[\vec{a} + (\vec{b} + \vec{c})]$ and $[(\vec{a} + \vec{b}) + \vec{c}]$ can be depicted as in the diagram below, using the triangle law of addition. We see that $\vec{PQ} = \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$. This is the associative property for vector addition.



2. a. $\vec{AB} = (6 - (-2), 7 - 3, 3 - (-5)) = (8, 4, 8)$

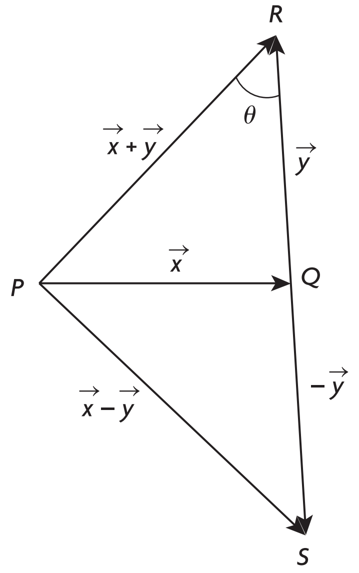
b. $|\vec{AB}| = \sqrt{8^2 + 4^2 + 8^2} = 12$

c. $\vec{BA} = (-1)\vec{AB} = (-8, -4, -8)$;

$|\vec{BA}| = |\vec{AB}| = 12$; unit vector in direction of

$$\begin{aligned}|\vec{BA}| &= \frac{1}{|\vec{BA}|}\vec{BA}\\ &= \frac{1}{12}(-8, -4, -8)\\ &= \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)\end{aligned}$$

3. Let $\vec{x} = \vec{PQ}$, $\vec{y} = \vec{QR}$, and $-\vec{y} = \vec{QS}$, as in the diagram below. Note that $|\vec{RS}| = |2\vec{y}| = 6$ and that triangle *PQR* and triangle *PRS* share angle θ.



By the cosine law:

$\cos\theta = \dfrac{|\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}|^2}{2|\vec{y}||\vec{x}+\vec{y}|}$ and

$\cos\theta = \dfrac{|2\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}-\vec{y}|^2}{2|2\vec{y}||\vec{x}+\vec{y}|}$.

Hence,

$$\frac{|\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}|^2}{2|\vec{y}||\vec{x}+\vec{y}|}$$
$$= \frac{|2\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}-\vec{y}|^2}{2|2\vec{y}||\vec{x}+\vec{y}|}$$
$$2(|\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}|^2)$$
$$= |2\vec{y}|^2 + |\vec{x}+\vec{y}|^2 - |\vec{x}-\vec{y}|^2$$
$$|\vec{x}-\vec{y}|^2 = 2|\vec{y}|^2 - |\vec{x}+\vec{y}|^2 + 2|\vec{x}|^2$$
$$|\vec{x}-\vec{y}| = \sqrt{2|\vec{y}|^2 - |\vec{x}+\vec{y}|^2} + \sqrt{2|\vec{x}|^2}$$
$$|\vec{x}-\vec{y}| = \sqrt{2(3)^2 - (\sqrt{17})^2} + \sqrt{2(3)^2}$$
$$|\vec{x}-\vec{y}| = \sqrt{19}$$

4. a. We have $3\vec{x} - 2\vec{y} = \vec{a}$ and $5\vec{x} - 3\vec{y} = \vec{b}$. Multiplying the first equation by -3 and the second equation by 2 yields: $-9\vec{x} + 6\vec{y} = -3\vec{a}$ and $10\vec{x} - 6\vec{y} = 2\vec{b}$. Adding these equations, we have: $\vec{x} = 2\vec{b} - 3\vec{a}$. Substituting this into the first equation yields: $3(2\vec{b} - 3\vec{a}) - 2\vec{y} = \vec{a}$. Simplifying, we have: $\vec{y} = 3\vec{b} - 5\vec{a}$.

b. First, conduct scalar multiplication on the third vector, yielding:
$(2, -1, c) + (a, b, 1) - (6, 3a, 12) = (-3, 1, 2c)$.
Now, each of the three components corresponds to an equation. First, $2 + a - 6 = -3$, which implies $a = 1$. Second, $-1 + b - 3a = 1$. Substituting $a = 1$ and simplifying yields $b = 5$. Third, $c + 1 - 12 = 2c$, so $c = -11$.

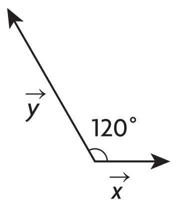
5. a. $\vec{a}$ and $\vec{b}$ span R^2, because any vector (x, y) in R^2 can be written as a linear combination of $\vec{a}$ and $\vec{b}$. These two vectors are not multiples of each other.

b. First, conduct scalar multiplication on the vectors, yielding: $(-2p, 3p) + (3q, -q) = (13, -9)$. Now, each component corresponds to an equation. First, $-2p + 3q = 13$. Second, $3p - q = -9$. Multiplying the second equation by 3 and adding the result to the first equation yields: $7p = -14$, which implies $p = -2$. Substituting this into the first equation and simplifying yields $q = 3$.

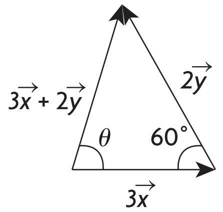
6. a. $\vec{a} = m\vec{b} + n\vec{c}$
$(1, 12, -29) = m(3, 1, 4) + n(1, 2, -3)$
$(1, 12, -29) = (3m, m, 4m) + (n, 2n, -3n)$
Each of the three components corresponds to an equation. First, $1 = 3m + n$. Second, $12 = m + 2n$. Third, $-29 = 4m - 3n$. Multiplying the first equation by -2 and adding the result to the second equation yields $m = -2$. Substituting $m = -2$ into the first equation yields $n = 7$. Since $m = -2$ and $n = 7$ also solves the third component's equation, $\vec{a} = m\vec{b} + n\vec{c}$ for $m = -2$ and $n = 7$. Hence, $\vec{a}$ can be written as a linear combination of $\vec{b}$ and $\vec{c}$.

b. $\vec{r} = m\vec{p} + n\vec{q}$
$(16, 11, -24) = m(-2, 3, 4) + n(4, 1, -6)$
$(16, 11, -24) = (-2m, 3m, 4m) + (4n, n, -6n)$
Each of the three components corresponds to an equation. First, $16 = -2m + 4n$. Second, $11 = 3m + n$. Third, $-24 = 4m - 6n$. Multiplying the first equation by 2 and adding the result to the third equation yields $n = 4$. Substituting $n = 4$ into the first equation yields $m = 0$. We have that $n = 4$ and $m = 0$ is the *unique* solution to the first and third equations, but $n = 4$ and $m = 0$ does not solve the second equation. Hence, this system of equations has no solution, and $\vec{r}$ cannot be written as a linear combination of $\vec{p}$ and $\vec{q}$. In other words, $\vec{r}$ does not lie in the plane determined by $\vec{p}$ and $\vec{q}$.

7. $\vec{x}$ and $\vec{y}$ have magnitudes of 1 and 2, respectively, and have an angle of 120° between them, as depicted in the picture below.



Since 60° is the complement of 120° $3\vec{x} + 2\vec{y}$ can be depicted as below.



By the cosine law:

$|3\vec{x} + 2\vec{y}|^2 = |3\vec{x}|^2 + |2\vec{y}|^2 - 2|3\vec{x}||2\vec{y}| \cos 60$

$|3\vec{x} + 2\vec{y}|^2 = 9|\vec{x}|^2 + 4|\vec{y}|^2 - 6|\vec{x}||\vec{y}|$

$|3\vec{x} + 2\vec{y}|^2 = 9 + 16 - 12$

$|3\vec{x} + 2\vec{y}| = \sqrt{13}$ or 3.61

The direction of $3\vec{x} + 2\vec{y}$ is θ, the angle from $\vec{x}$.
This can be computed from the sine law:

$$\frac{|3\vec{x} + 2\vec{y}|}{\sin 60} = \frac{|2\vec{y}|}{\sin \theta}$$

$$\sin \theta = \frac{|2\vec{y}| \sin 60}{|3\vec{x} + 2\vec{y}|}$$

$$\theta = \sin^{-1}\left(\frac{|2\vec{y}| \sin 60}{|3\vec{x} + 2\vec{y}|}\right)$$

$$\theta = \sin^{-1}\left(\frac{(4) \sin 60}{\sqrt{13}}\right)$$

$\theta \doteq 73.9°$ relative to x

8. $\overrightarrow{DE} = \overrightarrow{CE} - \overrightarrow{CD}$

$\overrightarrow{DE} = \vec{b} - \vec{a}$

Also,

$\overrightarrow{BA} = \overrightarrow{CA} - \overrightarrow{CB}$

$\overrightarrow{BA} = 2\vec{b} - 2\vec{a}$

Thus,

$\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BA}$