CHAPTER 3
Polynomial Functions

Getting Started, p. 122

1. a) $2x^2(3x - 11)$

$6x^3 - 22x^2$

b) $(x - 4)(x + 6)$

$x^2 + 6x - 4x - 24$

$x^2 + 2x - 24$

c) $4x(2x - 5)(3x + 2)$

$(8x^2 - 20x)(3x + 2)$

$24x^3 + 16x^2 - 60x^2 - 40x$

$24x^3 - 44x^2 - 40x$

d) $(5x - 4)(x^2 + 7x - 8)$

$5x^3 + 35x^2 - 40x - 4x^2 - 28x + 32$

$5x^3 + 31x^2 - 68x + 32$

2. a) $x^2 + 3x - 28$

$(x + 7)(x - 4)$

b) $2x^2 - 18x + 28$

$2(x^2 - 9x + 14)$

$2(x - 2)(x - 7)$

3. a) $3x + 7 = x - 5$

$3x + 7 - x - 7 = x - 5 - x - 7$

$2x = -12$

$x = -6$

b) $(x + 3)(2x - 9) = 0$

$x + 3 = 0$ and $2x - 9 = 0$

$x = -3$ and $x = \frac{9}{2}$

$x = -3, 4.5$

c) $x^2 + 11x + 24 = 0$

$(x + 3)(x + 8) = 0$

$x + 3 = 0$ and $x + 8 = 0$

$x = -3 \qquad x = -8$

d) $6x^2 + 22x = 8$

$6x^2 + 22x - 8 = 0$

$3x^2 + 11x - 4 = 0$

$(3x - 1)(x + 4) = 0$

$3x - 1 = 0$ and $x + 4 = 0$

$x = \frac{1}{3} \qquad x = -4$

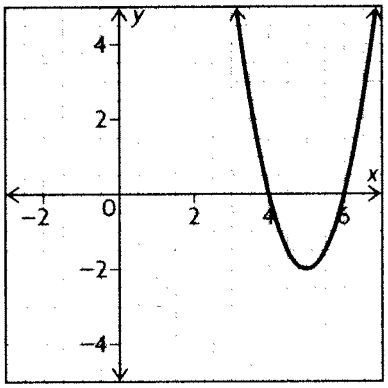
4. a) $y = \frac{1}{4}(x - 3)^2 + 9$

vertical compression by a factor of $\frac{1}{4}$, horizontal translation 3 units to the right; vertical translation 9 units up

b) $y = \left(\frac{1}{2}x\right)^2 - 7$

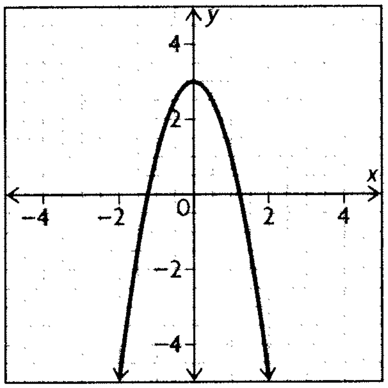
vertical compression by a factor of $\frac{1}{4}$, vertical translation 7 units down

5. a)



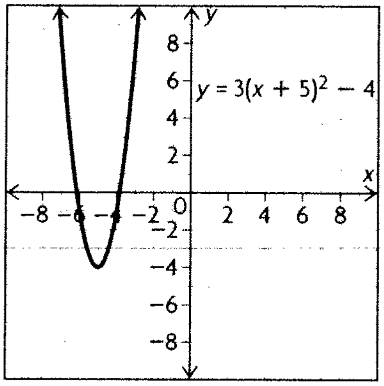
Working from the parent function $y = x^2$, the vertex is at $(5, -2)$. It opens upward. It is vertically stretched by a factor of 2. The equation is $y = 2(x - 5)^2 - 2$.

b)

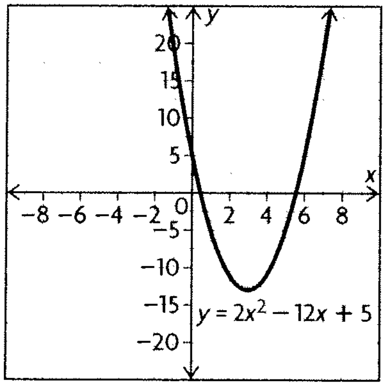


Working from the parent function $y = x^2$, the vertex is at $(0, 3)$. It opens downward. It is vertically stretched by a factor of 2. The equation is $y = -2x^2 + 3$.

6. a)



b)



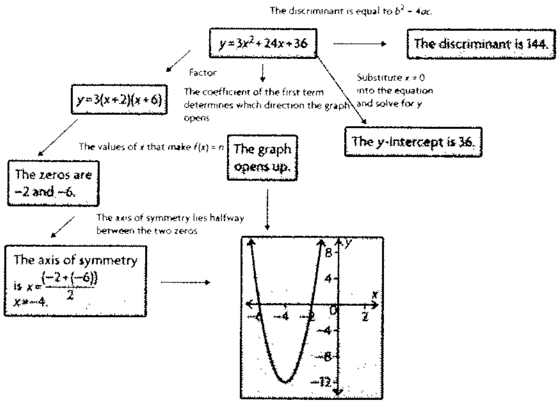
7. a) The first differences are -5.8, -5.6, -5.4, and -5.2. The second differences are all 0.2, so the function is quadratic.

b) The first differences are -6, -3, 5, and 6. The second differences are 3, 8, and 1. The function is not linear or quadratic.

c) The first differences are 4, 12, 36, and 108. The second differences are 8, 24, and 72. The function is not linear or quadratic.

d) The first differences are -0.5, -0.5, -0.5, and -0.5. The function is linear.

8.



3.1 Exploring Polynomial Functions, pp. 127–128

1. a) This represents a polynomial function because the domain is the set of all real numbers, the range does not have a lower bound, and the graph does not have horizontal or vertical asymptotes.

b) This represents a polynomial function because the domain is the set of all real numbers, the range is the set of all real numbers, and the graph does not have horizontal or vertical asymptotes.

c) This is not a polynomial function because it has a horizontal asymptote.

d) This represents a polynomial function because the domain is the set of all real numbers, the range does not have an upper bound, and the graph does not have horizontal or vertical asymptotes.

e) This is not a polynomial function because its domain is not all real numbers.

f) This is not a polynomial function because it is a periodic function.

2. a) polynomial; the exponents of the variables are all natural numbers

b) polynomial; the exponents of the variables are all natural numbers

c) polynomial; the exponents of the variables are all natural numbers

d) other; the variable is under a radical sign

e) other; the function contains another function in the denominator

f) polynomial; the exponents of the variables are all natural numbers

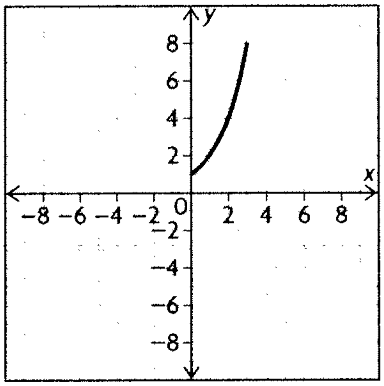
3. a) The first differences are all 25, so the function is linear.

b) The first differences are 15, 5, -5, and -15. The second differences are all -10, so the function is quadratic.

c) The first differences are all 25, so the function is linear.

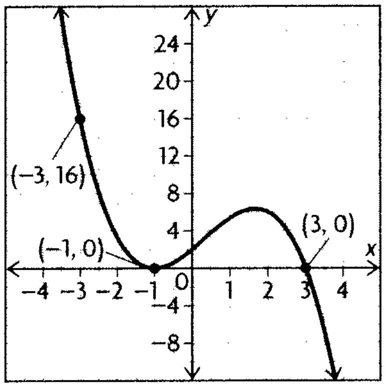
d) The first differences are 4, 28, 76, 148, 244, and 364. The second differences are 24, 48, 72, 96, and 120. The third differences are all 24, so the function is cubic.

4.



a) The graph looks like one half of a parabola, which is the graph of a quadratic equation.
b) There is a variable in the exponent.

5.



6. Answers may vary. For example: Any equation of the form $y = a(-\frac{4}{3}x^2 + \frac{8}{3}x + 4)$ will have the same zeros, but have a different y-intercept and a different value for $f(-3)$. Any equation of the form $y = x(-\frac{4}{3}x^2 + \frac{8}{3}x + 4)$ would have two of the same zeros, but a different value for $f(-3)$ and different positive/negative intervals.

7. $y = x + 5, y = x^2 + 5, y = x^3 + 5, y = x^4 + 5$ all have a y-intercept of 5.

8. Answers may vary. For example:

Definition A polynomial is an expression of the form $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number.	**Characteristics** The domain of the function is all real numbers, but the range can have restrictions; except for polynomial functions of degree zero (whose graphs are horizontal lines), the graphs of polynomials do not have horizontal or vertical asymptotes. The shape of the graph depends on its degree.
Polynomials	
Examples $x^2 + 4x + 6$	**Non-Examples** $\sqrt{x + 1}$

3.2 Characteristics of Polynomial Functions, pp. 136–138

1. a) $f(x) = -4x^4 + 3x^2 - 15x + 5$
The function is of degree 4 and -4 is the leading coefficient.
Since the function is of even degree and the leading coefficient is negative, the end behaviour is:
as $x \to +/-\infty, y \to -\infty$

b) $g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$
The function is of degree 5 and 2 is the leading coefficient.
Since the function is of odd degree and the leading coefficient is positive, the end behaviour is:
as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$

c) $p(x) = 4 - 5x + 4x^2 - 3x^3$
$= -3x^3 + 4x^2 - 5x + 4$
The function is of degree 3 and -3 is the leading coefficient.
Since the function is of odd degree and the leading coefficient is negative, the end behaviour is:
as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$

d) $h(x) = 2x(x - 5)(3x + 2)(4x - 3)$
$= (2x^2 - 10x)(12x^2 - x - 6)$
$= 24x^4 - 2x^3 - 12x^2 - 120x^3 + 10x^2 + 60x$
$= 24x^4 - 122x^3 - 2x^2 + 60x$
The function is of degree 4 and 24 is the leading coefficient.
Since the function is of even degree and the leading coefficient is positive, the end behaviour is:
as $x \to +/-\infty, y \to \infty$

2. $f(x) = -4x^4 + 3x^2 - 15x + 5$
a) Turning points
a) Since the function is of degree 4, it will have at least 1 turning point and at most 4 − 1 or 3 turning points.
b) Since the degree of the function is even, the minimum number of zeros is 0 and the maximum number of zeros is equal to the degree of the function, 4.
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$
c) Since the function is of degree 5, it will have at least 0 turning points and at most 5 − 1 or 4 turning points.
d) Since the degree of the function is odd, the minimum number of zeros is 1 and the maximum number of zeros is equal to the degree of the function, 5.
$p(x) = 4 - 5x + 4x^2 - 3x^3$

b) Zeros

a) Since the function is of degree 3, it will have at least 0 turning points and at most $3 - 1$ or 2 turning points.

b) Since the degree of the function is odd, the minimum number of zeros is 1 and the maximum number of zeros is equal to the degree of the function, 3.

$h(x) = 24x^4 - 122x^3 - 2x^2 + 60$

c) Since the function is of degree 4, it will have at least 1 turning point and at most $4 - 1$ or 3 turning points.

d) Since the degree of the function is even, the minimum number of zeros is 0 and the maximum number of zeros is equal to the degree of the function, 4.

3. i. a) There are 3 turning points which means the degree is $3 + 1 = 4$. The degree is even.

b) The leading coefficient is negative because as $x \rightarrow +/-\infty, y \rightarrow -\infty$.

ii. a) There are 5 turning points which means the degree is $5 + 1 = 6$. The degree is even.

b) The leading coefficient is negative because as $x \rightarrow +/-\infty, y \rightarrow -\infty$.

iii. a) There are 4 turning points which means the degree is $4 + 1 = 5$. The degree is odd.

b) The leading coefficient is negative because as $x \rightarrow -\infty, y \rightarrow \infty$ and as $x \rightarrow \infty, y \rightarrow -\infty$.

iv. a) There is 1 turning point which means the degree is $1 + 1 = 2$. The degree is even.

b) The leading coefficient is positive because as $x \rightarrow +/-\infty,\ y \rightarrow \infty$.

v. a) There are 2 turning points which means the degree is $2 + 1 = 3$. The degree is odd.

b) The leading coefficient is negative because as $x \rightarrow -\infty, y \rightarrow \infty$ and as $x \rightarrow \infty, y \rightarrow -\infty$.

vi. a) There are 2 turning points which means the degree is $2 + 1 = 3$. The degree is odd.

b) The leading coefficient is positive because as $x \rightarrow -\infty, y \rightarrow -\infty$ and as $x \rightarrow \infty, y \rightarrow \infty$.

4. a) $f(x) = 2x^2 - 3x + 5$

The function is of degree 2 and 2 is the leading coefficient.

Since the function is of even degree and the leading coefficient is positive, the end behaviour is:

as $x \rightarrow +/-\infty, y \rightarrow \infty$

b) $f(x) = -3x^3 + 2x^2 + 5x + 1$

The function is of degree 3 and -3 is the leading coefficient.

Since the function is of odd degree and the leading coefficient is negative, the end behaviour is:

as $x \rightarrow -\infty, y \rightarrow \infty$ and as $x \rightarrow \infty, y \rightarrow -\infty$

c) $f(x) = 5x^3 - 2x^2 - 2x + 6$

The function is of degree 5 and 2 is the leading coefficient.

Since the function is of odd degree and the leading coefficient is positive, the end behaviour is:

as $x \rightarrow -\infty, y \rightarrow -\infty$ and as $x \rightarrow \infty, y \rightarrow \infty$

d) $f(x) = -2x^4 + 5x^3 - 2x^2 + 3x - 1$

The function is of degree 4 and -4 is the leading coefficient.

Since the function is of even degree and the leading coefficient is negative, the end behaviour is:

as $x \rightarrow +/-\infty, y \rightarrow -\infty$

e) $f(x) = 0.5x^4 + 2x^2 - 6$

The function is of degree 4 and 0.5 is the leading coefficient.

Since the function is of even degree and the leading coefficient is positive, the end behaviour is:

as $x \rightarrow +/-\infty, y \rightarrow +\infty$.

f) $f(x) = -3x^5 + 2x^3 - 4x$

The function is of degree 5 and -3 is the leading coefficient.

Since the function is of odd degree and the leading coefficient is negative, the end behaviour is:

as $x \rightarrow -\infty, y \rightarrow \infty$ and as $x \rightarrow \infty, y \rightarrow -\infty$

5. a) D: the graph extends from quadrant III to quadrant I and the y-intercept is 2

b) A: the graph extends from quadrant III to quadrant IV

c) E: the graph extends from quadrant II to quadrant I and the y-intercept is -5

d) C: the graph extends from quadrant II to quadrant I and the y-intercept is 0

e) F: the graph extends from quadrant II to quadrant IV

f) B: the graph extends from quadrant III to quadrant I and the y-intercept is 1

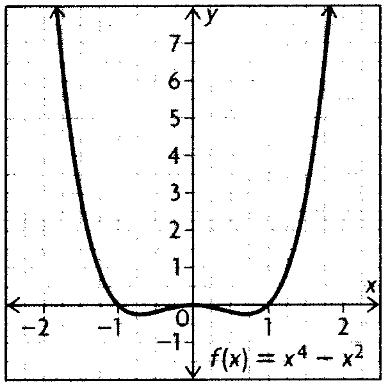
6. a) Answers may vary, but the function should be an odd degree with a positive leading coefficient. One example is $f(x) = 2x^3 + 5$.

b) Answers may vary, but the function should be an even degree with a positive leading coefficient. One example is $f(x) = 6x^2 + x - 4$.

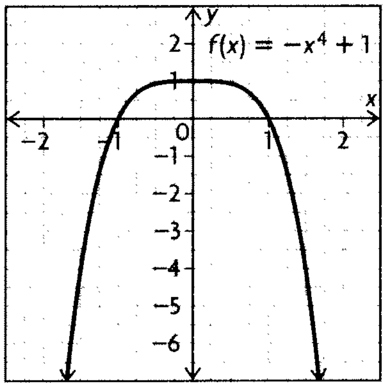
c) Answers may vary, but the function should be an even degree with a negative leading coefficient. One example is $f(x) = -x^4 - x^3 + 7$.

d) Answers may vary, but the function should be an odd degree with a negative leading coefficient. One example is $f(x) = -9x^5 + x^4 - x^3 - 2$.

7. a) Answers may vary. For example:

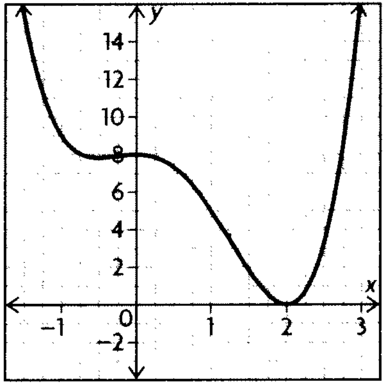


b) Answers may vary. For example:

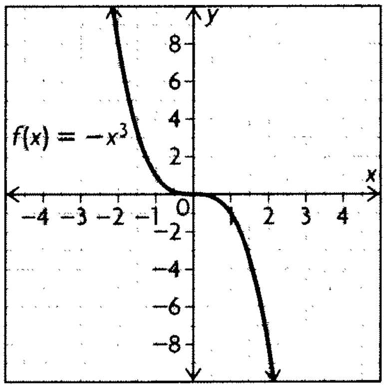


c) Answers may vary. For example:

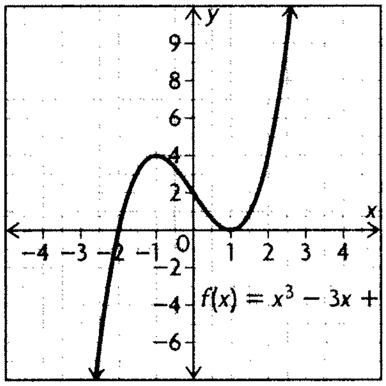
$f(x) = x^4 - 2x^3 - 2x^2 + 8$



d) Answers may vary. For example:

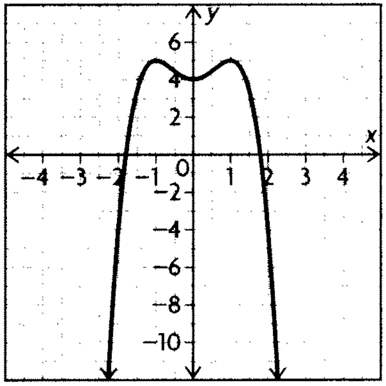


e) Answers may vary. For example:



f) Answers may vary. For example:

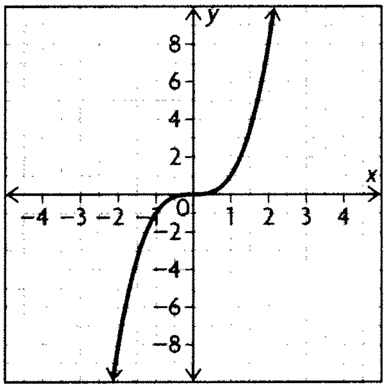
$f(x) = -x^4 + 2x^2 + 4$



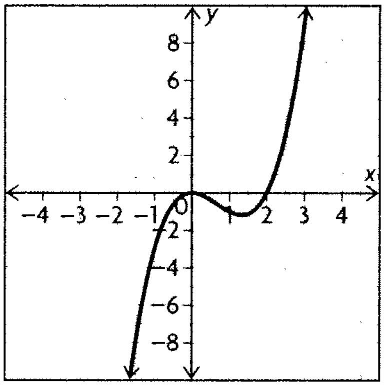
8. An odd-degree polynomial can have only local maximums and minimums because the *y*-value goes to $-\infty$ and ∞ at each end of the function. Whereas an even-degree polynomial can have absolute maximums and minimums because it will go to either $-\infty$ at both ends or ∞ at both ends of the function.

9. A polynomial of even degree cannot be symmetrical with respect to the origin, since the end behaviours must be the same. Therefore, $f(x)$ is of odd degree, meaning that the number of turning points is even.

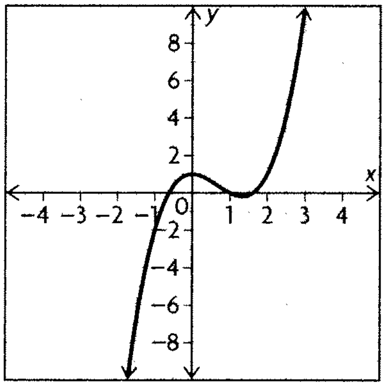
10. a) Answers may vary. For example:
$f(x) = x^3$



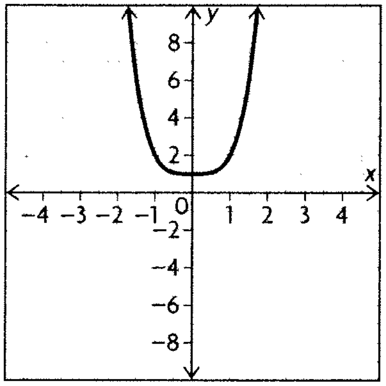
b) Answers may vary. For example:
$f(x) = x^3 - 2x^2$



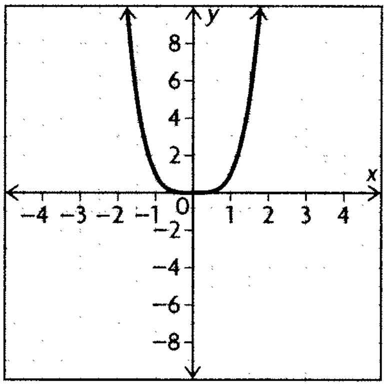
c) Answers may vary. For example:
$f(x) = x^3 - 2x^2 + 1$



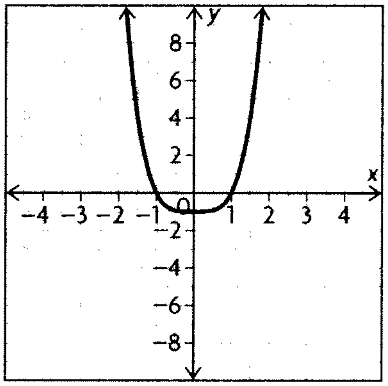
11. a) Answers may vary. For example:
$f(x) = x^4 + 1$



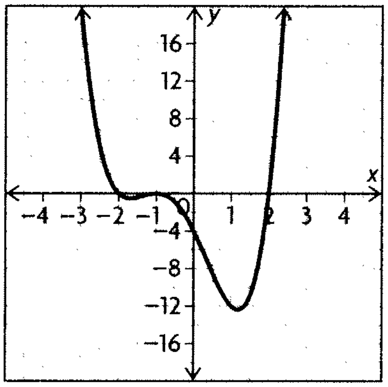
b) Answers may vary. For example: $f(x) = x^4$



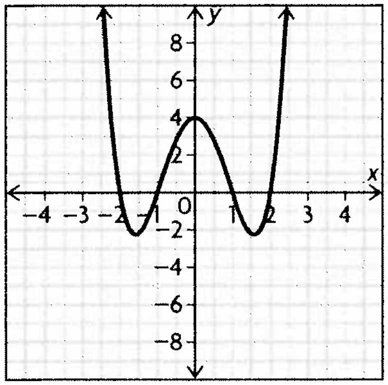
c) Answers may vary. For example:
$f(x) = x^4 - 1$



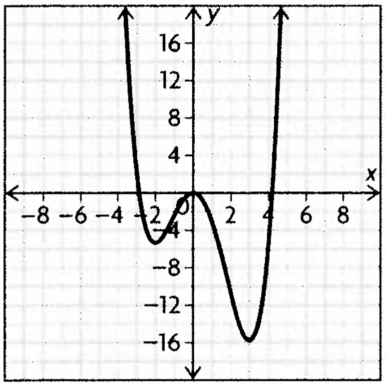
d) Answers may vary. For example:
$f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$



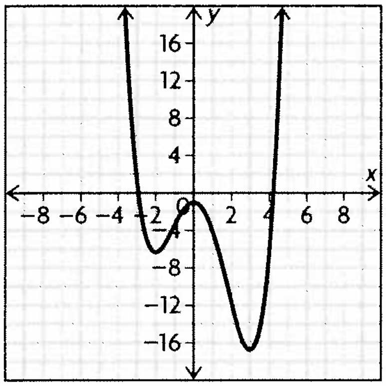
e) Answers may vary. For example:
$f(x) = x^4 - 5x^2 + 4$



12. a) Answers may vary. For example:
$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$



and $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 - 1$



b) the zeros of the function and the leading coefficient of the function

13. a) In 1900, $x = 0$.
$y = -0.1(0)^4 + 0.5(0)^3 + 0.4(0)^2 + 10(0) + 7$
The population was 700.

b) Though the population will grow from the original number, it will end up decreasing because the leading coefficient is negative.

14. a) False; answers may vary; for example, $f(x) = x^2 + x$ is not an even function.

b) true

c) False; answers may vary; for example, $f(x) = x^2 + 1$ has no zeros.

d) False; answers may vary; for example, $f(x) = -x^2$ has end behaviour opposite the behaviour stated.

15. Answers may vary. For example: "What are the turning points of the function?", "What is the leading coefficient of the function?", and "What are the zeros of the function?" If the function has 0 turning points or an even number of turning points, then it must extend to the opposite side of the x-axis. If it has an odd number of turning points, it must extend to the same side of the x-axis. If the leading coefficient is known, it can be determined exactly which quadrants the function extends to/from and if the function has been vertically stretched. If the zeros are known, it can be determined if the function has been vertically translated up or down.

16. a) Since f is an even function, $f(x) = f(-x)$.

$$ax^2 + bx + c = a(-x)^2 + b(-x) + c$$
$$ax^2 + bx + c = ax^2 - bx + c$$
$$2bx = 0$$
So, $b = 0$.

b) Since g is an odd function, $-g(x) = g(-x)$ for all real x.

$$-ax^3 - bx^2 - cx - d$$
$$= a(-x)^3 + b(-x)^2 + c(-x) + d$$
$$-ax^3 - bx^2 - cx - d = -ax^3 + bx^2 - cx + d$$
$$-bx^2 - d = bx^2 + d$$

Since the coefficient of x^2 and the constant on each side must be equal, $-b = b$ and $-d = d$. So $b = 0$ and $d = 0$.

3.3 Characteristics of Polynomial Functions in Factored Form, pp. 146–148

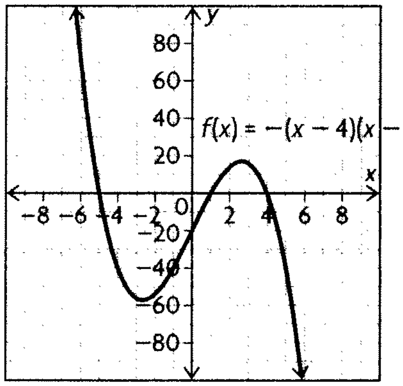
1. a) C: the graph has zeros of -1 and 3, and it extends from quadrant III to quadrant I

b) A: the graph has zeros of -1 and 3, and it extends from quadrant II to quadrant I

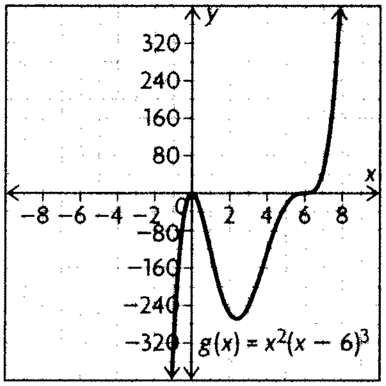
c) B: the graph has zeros of -1 and 3, and it extends from quadrant II to quadrant IV

d) D: the graph has zeros of -1, 0, 3, and 5, and it extends from quadrant II to quadrant I

2. a)

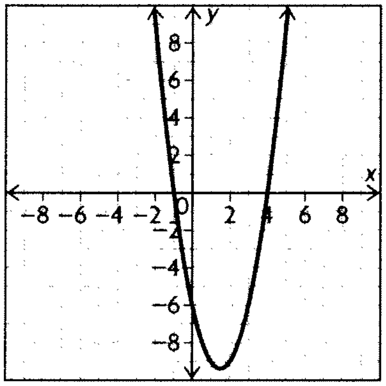


b)



3. a) $f(x) = k(x + 1)(x - 4)$
$f(x) = 4(x + 1)(x - 4)$
$f(x) = -2(x + 1)(x - 4)$

b) $9 = k(5 + 1)(5 - 4)$
$9 = k(6)(1)$
$\frac{3}{2} = k$
$f(x) = \frac{3}{2}(x + 1)(x - 4)$

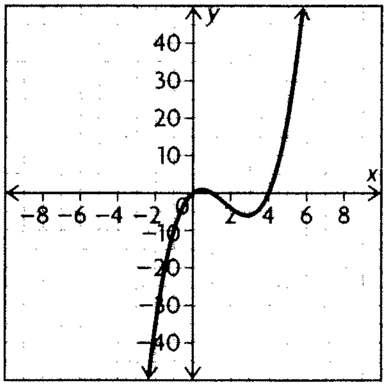


4. a) The zeros of the function are located at −3, 2, and 5.
$y = a(x + 3)(x - 2)(x - 5)$
$8 = a(1 + 3)(1 - 2)(1 - 5)$
$8 = a(4)(-1)(-4)$
$8 = 16a$
$\frac{1}{2} = a$
$y = 0.5(x + 3)(x - 2)(x - 5)$

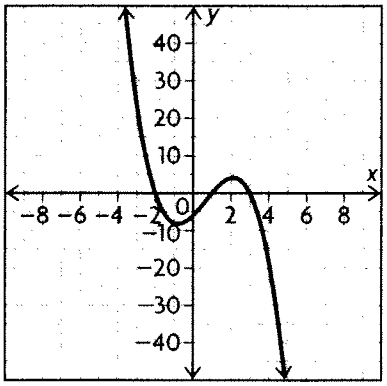
b) The zeros of the function are located at −1, 2, and 4. Since $x = -1$ is a turning point, this factor is squared.
$y = a(x + 1)^2(x - 2)(x - 4)$
$-12 = a(1 + 1)^2(1 - 2)(1 - 4)$
$-12 = a(2)^2(-1)(-3)$
$-12 = 12a$
$-1 = a$
$y = -(x + 1)^2(x - 2)(x - 4)$

5. Family 1: A, G, I
These functions have single zeros at 3 and −5.
Family 2: B, E
These functions have double zeros at 3 and single zeros at −5.
Family 3: C, F, H, K
These functions have single zeros at −6 and −8.
Family 4: D, J, L
These functions have double zeros at −3 and single zeros at −5.

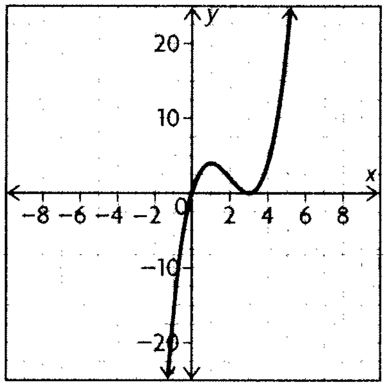
6. a)



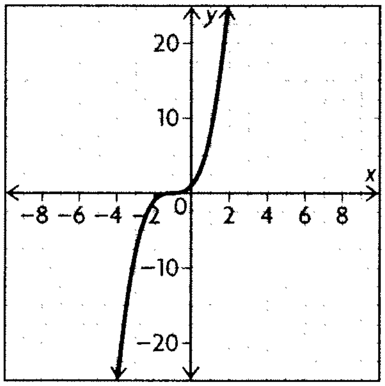
b)



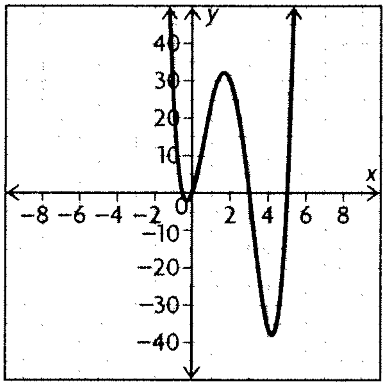
c)



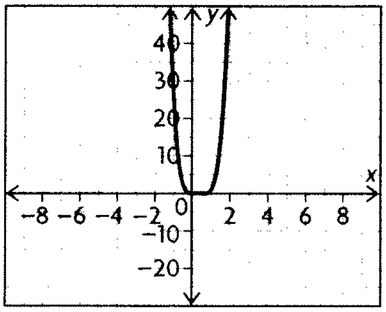
d)



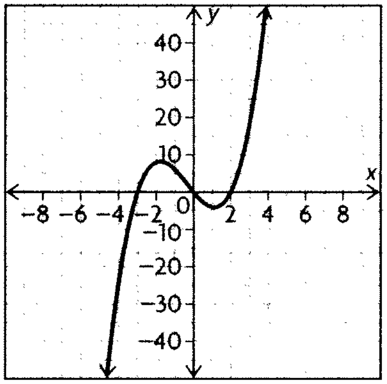
e)



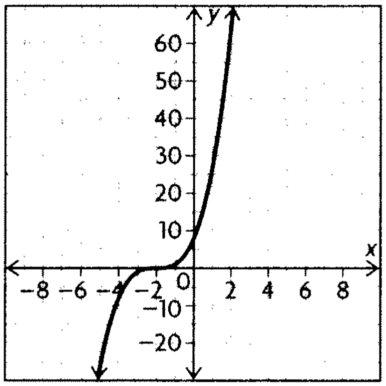
f)



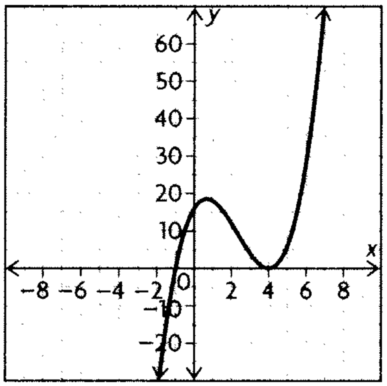
7. Answers may vary. For example:
a) i) $y = x(x + 3)(x - 2)$



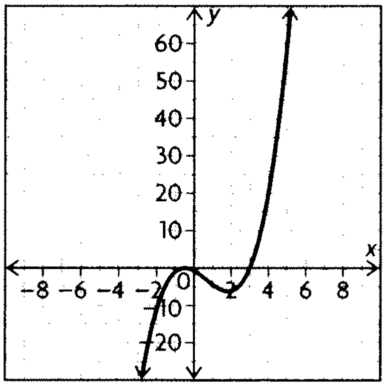
ii) $y = (x + 2)^3$



iii) $y = (x + 1)(x - 4)^2$



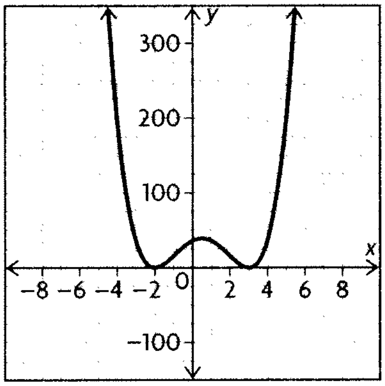
iv) $y = (x - 3)\left(x + \frac{1}{2}\right)^2$



b) No, all of the characteristics of the graphs are not unique because each equation belongs to a family of equations.

8. Answers may vary. For example:

a) $y = (x + 5)(x + 3)(x - 2)(x - 4)$

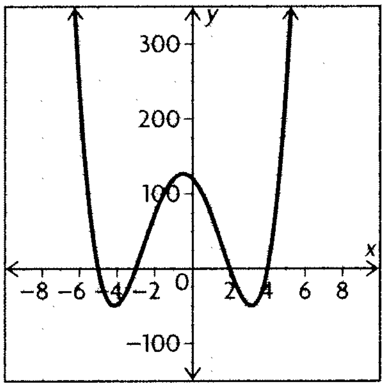


$y = 2(x + 5)(x + 3)(x - 2)(x - 4)$

$y = -5(x + 5)(x + 3)(x - 2)(x - 4)$

b) Answers may vary. For example:

$y = (x + 2)^2(x - 3)^2$

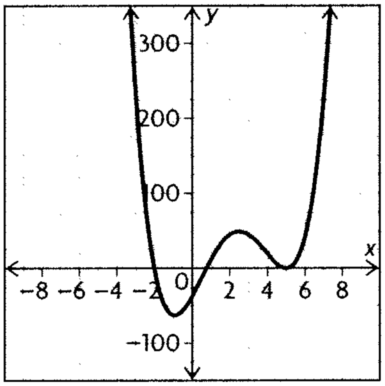


$y = 10(x + 2)^2(x - 3)^2$

$y = 7(x + 2)^2(x - 3)^2$

c) Answers may vary. For example:

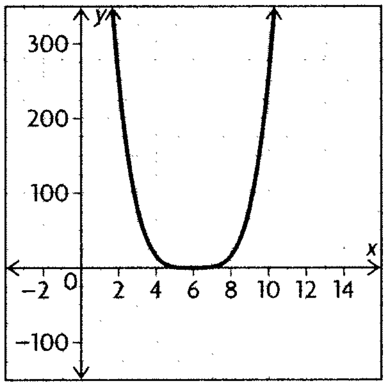
$y = (x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$



$y = -(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

$y = \frac{2}{5}(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

d) Answers may vary. For example: $y = (x - 6)^4$



$y = 15(x - 6)^4$

$y = -3(x - 6)^4$

9. a) $y = 3x^3 - 48x$

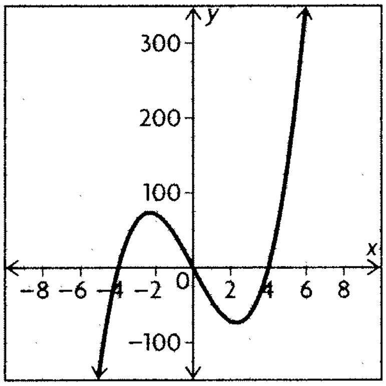
$y = 3x(x^2 - 16)$

$y = 3x(x + 4)(x - 4)$

The zeros are $x = 0, -4$, and 4.

The y-intercept is $f(0) = 3(0)^3 - 48(0) = 0$.

The function is an odd degree, and the leading coefficient is positive. So, the end behaviour is $x \rightarrow -\infty, y \rightarrow -\infty$ and as $x \rightarrow \infty, y \rightarrow \infty$.



b) $y = x^4 + 4x^3 + 4x^2$

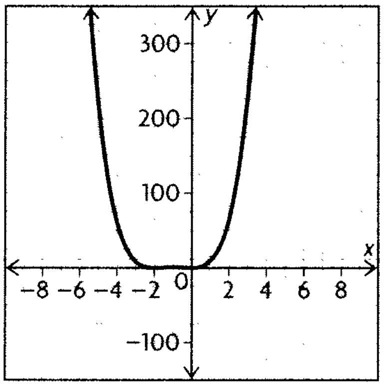
$y = x^2(x^2 + 4x + 4)$

$y = x^2(x + 2)^2$

The zeros are $x = 0$, and -2.

The y-intercept is $f(0) = 0^4 + 4(0)^3 + 4(0)^2 = 0$.

The function is an even degree, and the leading coefficient is positive. So, the end behaviour is $x \rightarrow +/-\infty, y \rightarrow \infty$.



c) $y = x^3 - 9x^2 + 27x - 27$

$y = (x - 3)(x^2 - 6x + 9)$

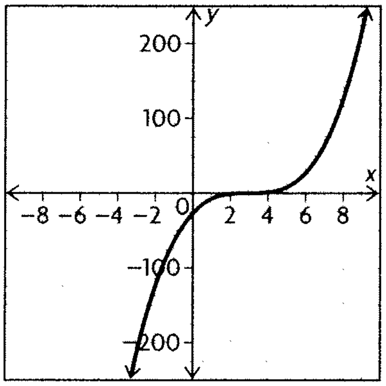
$y = (x - 3)(x - 3)(x - 3)$ or $(x - 3)^3$

$y = (0)^3 - 9(0)^2 + 27(0) - 27$

$y = -27$

The y-intercept is -27.

The function is an odd degree, and the leading coefficient is positive. So, the end behaviour is $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$.



d) $y = -x^4 - 15x^3 - 75x^2 - 125x$

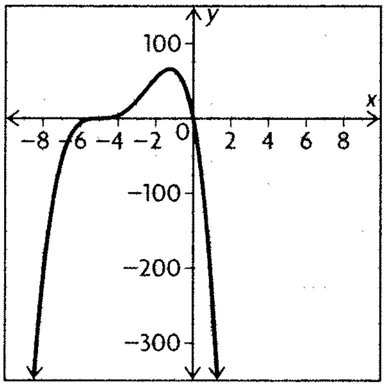
$y = -x(x^3 + 15x^2 + 75x + 125)$

$y = -x(x + 5)(x^2 + 10x + 25)$

$y = -x(x + 5)^3$

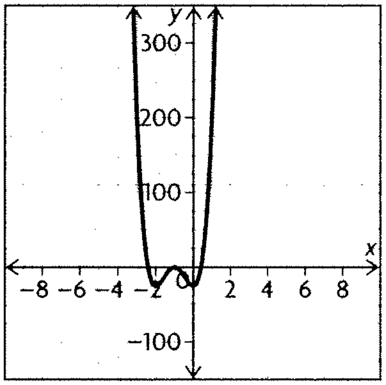
The y-intercept is 0.

The function is an even degree, and the leading coefficient is negative. So, the end behaviour is $x \to +/-\infty, y \to -\infty$.

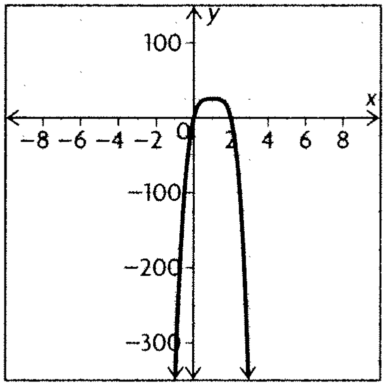


10. Answers may vary. For example:

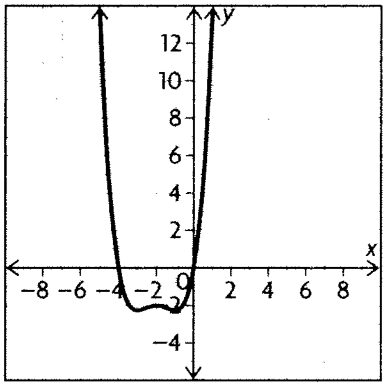
a) $y = 25x^4 + 100x^3 + 100x^2 - 25$



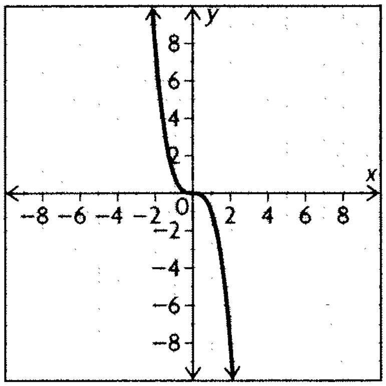
b) $y = -25x^4 - 100x^3 - 150x^2 + 100x$



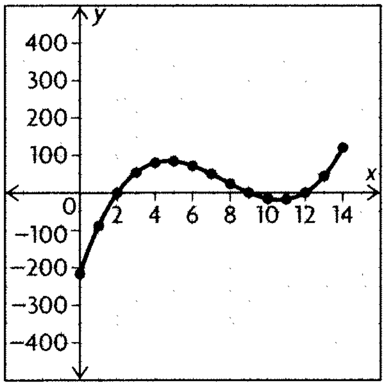
c) $y = \frac{1}{4}x^4 + 2x^3 + \frac{11}{2}x^2 + 6x$



d) $y = -x^3$



11. a)



b) The zeros are at $x = 2, 9, 12$.
$y = (x - 2)(x - 9)(x - 12)$

c) No, this is not likely to continue. The equation $y = (x - 2)(x - 9)(x - 12)$ only has 3 zeros, so after year 12, the company's profit will increase forever. The function's domain should be $\{x \in \mathbf{R} | 0 \leq x \leq 14\}$.

12. a) The zeros are at -2, -1, and 1.
$y = (x + 2)(x + 1)(x - 1)$
$y = x^3 + 2x^2 - x - 2$

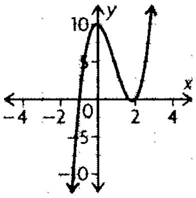
b) The zeros are at -4, -2, and 1.

$$y = a(x + 4)(x + 2)(x - 1)$$
$$-1.6 = a(-3 + 4)(-3 + 2)(-3 - 1)$$
$$-1.6 = a(1)(-1)(-4)$$
$$-1.6 = 4a$$
$$-0.4 = -\frac{2}{5} = a$$
$$y = -\frac{2}{5}(x - 1)(x + 2)(x + 4)$$

13. a)
$$f(x) = a(x + 3)(x + 5)$$
$$f(7) = a(7 + 3)(7 + 5)$$
$$-720 = 120a$$
$$-6 = a$$
$$f(x) = -6(x + 3)(x + 5)$$

b)
$$f(x) = a(x + 2)(x - 3)(x - 4)$$
$$f(5) = a(5 + 2)(5 - 3)(5 - 4)$$
$$28 = 14a$$
$$2 = a$$
$$f(x) = 2(x + 2)(x - 3)(x - 4)$$

14. $f(x) = kx^3 - 8x^2 - x + 3k + 1$
$f(2) = 0$, so
$$8k - 32 - 2 + 3k + 1 = 0$$
$$11k = 33$$
$$k = 3$$



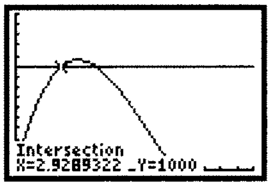
The zeros are $\frac{5}{3}$, -1, and 2.
$f(x) = (3x - 5)(x + 1)(x - 2)$

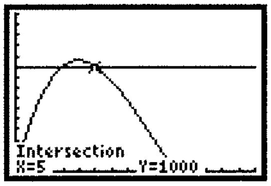
15. a) It has zeros at 2 and 4, and it has turning points at 2, 3, and 4. It extends from quadrant II to quadrant I.

b) It has zeros at -4 and 3, and it has turning points at $-\frac{5}{3}$ and 3. It extends from quadrant III to quadrant I.

16. a)
$$V(2) = 2(30 - 4)(20 - 4)$$
$$= 2(26)(16)$$
$$= 832 \text{ cm}^3$$

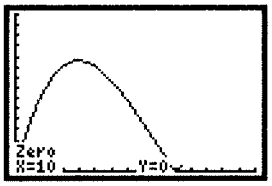
b) $1000 = x(30 - 2x)(20 - 2x)$
Graph the left and right sides as separate functions on a graphing calculator and solve.


Intersection
X=2.9289322 Y=1000


Intersection
X=5 Y=1000

The volume is 1000 when $x = 2.93$ or $x = 5$. So the dimensions are 2.93 cm by 24.14 cm by 14.14 cm or 5 cm by 20 cm by 10 cm.

c)


Zero
X=10 Y=0

$V(x) > 0$ for $0 < x < 10$. The values of x are the side lengths of squares that can be cut from the sheet of cardboard to produce a box with positive volume. Since the sheet of cardboard is 30 cm by 20 cm, the side lengths of a square cut from each corner have to be less than 10 cm, or an entire edge would be cut away, leaving nothing to fold up.
d) The square that is cut from each corner must be larger than 0 cm by 0 cm but smaller than 10 cm by 10 cm.

3.4 Transformations of Cubic and Quartic Functions, pp. 155–158

1. a) B: $y = x^3$ has been vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 1 unit up.
b) C: $y = x^3$ has been reflected in the x-axis, vertically compressed by a factor of $\frac{1}{3}$, horizontally translated 1 unit to the left, and vertically translated 1 unit down.
c) A: $y = x^4$ has been vertically compressed by a factor of 0.2, horizontally translated 4 units to the right, and vertically translated 3 units down.
d) D: $y = x^4$ has been reflected in the x-axis, vertically stretched by a factor of 1.5, horizontally translated 3 units to the left, and vertically translated 4 units up.
2. a) $y = x^4$, vertical stretch by a factor of $\frac{5}{4}$ and vertical translation of 3 units up
b) $y = x$, vertical stretch by a factor of 3 and vertical translation of 4 units down
c) $y = x^3$, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation of $\frac{4}{3}$ units to the left, and vertical translation of 7 units down
d) $y = x^4$, reflection in the x-axis and horizontal translation of 8 units to the left
e) $y = x^2$, reflection in the x-axis, vertical stretch by a factor of 4.8, and horizontal translation 3 units right
f) $y = x^3$, vertical stretch by a factor of 2, horizontal stretch by a factor of 5, horizontal translation of 7 units to the left, and vertical translation of 4 units down
3. a) $y = x^3$ has been translated 3 units to the left and 4 units down.
$y = (x + 3)^3 - 4$
b) $y = x^4$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 4 units to the left, and vertically translated 5 units up.
$y = -2(x + 4)^4 + 5$
c) $y = x^4$ has been vertically compressed by a factor of $\frac{1}{4}$, horizontally translated 1 unit to the right, and vertically translated 2 units down.
$y = \frac{1}{4}(x - 1)^4 - 2$
d) $y = x^3$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 4 units down.
$y = -2(x - 3)^3 - 4$
4. a) $y = x^3$ has been vertically stretched by a factor of 12, horizontally translated 9 units to the right, and vertically translated 7 units down.
b) $y = x^3$ has been horizontally stretched by a factor of $\frac{8}{7}$, horizontally translated 1 unit to the left, and vertically translated 3 units up.
c) $y = x^3$ has been vertically stretched by a factor of 2, reflected in the x-axis, horizontally translated 6 units to the right, and vertically translated 8 units down.
d) $y = x^3$ has been horizontally translated 9 units to the left.
e) $y = x^3$ has been reflected in the x-axis, vertically stretched by a factor of 2, reflected in the y-axis, horizontally compressed by a factor of $\frac{1}{3}$, horizontally translated 4 units to the right, and vertically translated 5 units down.
f) $y = x^3$ has been horizontally stretched by a factor of $\frac{4}{3}$ and horizontally translated 10 units to the right.
5. a) Since the vertex is at $(0, -1)$, $k = -11$, $h = 0$.
$y = a(x - 0)^2 - 11$
$y = ax^2 - 11$
Substitute $(1, -3)$ to determine a.
$-3 = a(1)^2 - 11$
$8 = a$
$y = 8x^2 - 11$
$y = x^2$ was vertically stretched by a factor of 8 and vertically translated 11 units down.
b) Since the vertex is at $(0, 1.25)$, $k = 1.25$, $h = 0$. Since the parabola opens downward, a is negative.
$y = -a(x - 0)^2 + 1.25$
$y = -ax^2 + 1.25$
Substitute $(5, -5)$ to determine a.
$-5 = a(5)^2 + 1.25$
$-6.25 = 25a$
$-0.25 = a$

$$y = -\frac{1}{4}x^2 + 1.25$$

$y = x^2$ was reflected in the x-axis, vertically compressed by a factor of $\frac{1}{4}$, and vertically translated 1.25 units up.

6. a) $y = \frac{1}{2}(5(x + 6)^3)$

$$(x, y) \rightarrow \left(\frac{x}{5} + (-6), \frac{1}{2}y + 0\right)$$

$$(-1, -1) \rightarrow \left(\frac{-1}{5} + (-6), \frac{1}{2}(-1) + 0\right)$$

$$(-1, -1) \rightarrow \left(-6\frac{1}{5}, -\frac{1}{2}\right)$$

$$(0, 0) \rightarrow \left(\frac{0}{5} + (-6), \frac{1}{2}(0) + 0\right)$$

$$(0, 0) \rightarrow (-6, 0)$$

$$(2, 8) \rightarrow \left(\frac{2}{5} + (-6), \frac{1}{2}(8) + 0\right)$$

$$(2, 8) \rightarrow \left(-5\frac{3}{5}, 4\right)$$

b) $y = \left(-\frac{1}{2}x\right)^3 + 3$

$$(x, y) \rightarrow (-2x + 0, y + 3)$$

$$(-1, -1) \rightarrow (-2(-1), -1 + 3)$$

$$(-1, -1) \rightarrow (2, 2)$$

$$(0, 0) \rightarrow (-2(0), 0 + 3)$$

$$(0, 0) \rightarrow (0, 3)$$

$$(2, 8) \rightarrow (-2(2), 8 + 3)$$

$$(2, 8) \rightarrow (-4, 11)$$

c) $y = -3(x - 4)^3 - \frac{1}{2}$

$$(x, y) \rightarrow \left(x + 4, -3y - \frac{1}{2}\right)$$

$$(-1, -1) \rightarrow \left(-1 + 4, -3(-1) - \frac{1}{2}\right)$$

$$(-1, -1) \rightarrow \left(3, 2\frac{1}{2}\right)$$

$$(0, 0) \rightarrow \left(0 + 4, -3(0) - \frac{1}{2}\right)$$

$$(0, 0) \rightarrow \left(4, -\frac{1}{2}\right)$$

$$(2, 8) \rightarrow \left(2 + 4, -3(8) - \frac{1}{2}\right)$$

$$(2, 8) \rightarrow \left(6, -24\frac{1}{2}\right)$$

d) $y = \frac{1}{10}\left(\frac{1}{7}x\right)^3 - 2$

$$(x, y) \rightarrow \left(7x + 0, \frac{1}{10}y - 2\right)$$

$$(-1, -1) \rightarrow \left(7(-1), \frac{1}{10}(-1) - 2\right)$$

$$(-1, -1) \rightarrow \left(-7, -2\frac{1}{10}\right)$$

$$(0, 0) \rightarrow \left(7(0), \frac{1}{10}(0) - 2\right)$$

$$(0, 0) \rightarrow (0, -2)$$

$$(2, 8) \rightarrow \left(7(2), \frac{1}{10}(8) - 2\right)$$

$$(2, 8) \rightarrow \left(14, -1\frac{1}{5}\right)$$

e) $y = -(-x)^3 + \frac{9}{10}$

$$(x, y) \rightarrow \left(-x + 0, -y + \frac{9}{10}\right)$$

$$(-1, -1) \rightarrow \left(-(-1), -(-1) + \frac{9}{10}\right)$$

$$(-1, -1) \rightarrow \left(1, 1\frac{9}{10}\right)$$

$$(0, 0) \rightarrow \left(-(0), -(0) + \frac{9}{10}\right)$$

$$(0, 0) \rightarrow \left(0, \frac{9}{10}\right)$$

$$(2, 8) \rightarrow \left(-(2), -(8) + \frac{9}{10}\right)$$

$$(2, 8) \rightarrow \left(-2, -7\frac{1}{10}\right)$$

f) $y = \left(\frac{1}{7}(x + 4)^3\right) - 2$

$$(x, y) \rightarrow (7x - 4, y - 2)$$

$$(-1, -1) \rightarrow (7(-1) - 4, -1 - 2)$$

$$(-1, -1) \rightarrow (-11, -8)$$

$$(0, 0) \rightarrow (7(0) - 4, 0 - 2)$$

$$(0, 0) \rightarrow (-4, -7)$$

$$(2, 8) \rightarrow (7(2) - 4, 8 - 2)$$

$$(2, 8) \rightarrow (10, 1)$$

7. Since the vertex is at (1, 3), $k = 3$, $h = 1$. Since the parabola opens downward, a is negative.

$y = -a(x - 1)^4 + 3$

$y = -a(x - 1)^4 + 3$

Substitute $(5, -61)$ to determine a.

$-61 = a(5 - 1)^4 + 3$

$-61 = 256a + 3$

$-64 = 256a$

$-\frac{1}{4} = a$

$y = -\frac{1}{4}(x - 1)^4 + 3$

8. $\left(11, -\frac{23}{3}\right) = \left(11 - 13, \frac{3}{2}\left(-\frac{23}{3} + 13\right)\right)$

$= (-2, 8)$

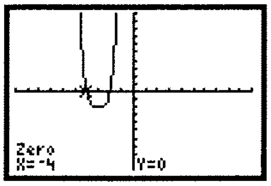
$(13, -13) = (13 - 13,) = (0, 0)$

$\left(15, -\frac{55}{3}\right) = \left(15 - 3, \frac{3}{2}\left(-\frac{55}{3} + 13\right)\right)$

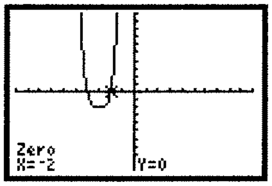
$= (2, -8)$

9. The x-intercepts are the zeros of the function:

a)



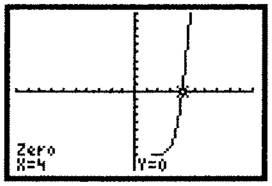
Zero
X=-4 Y=0



Zero
X=-2 Y=0

The x-intercepts are at -2 and -4.

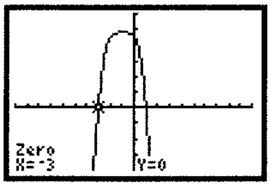
b)



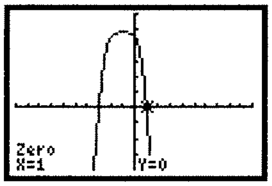
Zero
X=4 Y=0

The x-intercept is at 4.

c)



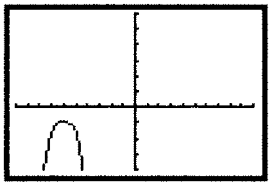
Zero
X=-3 Y=0



Zero
X=1 Y=0

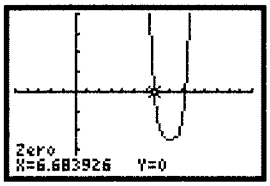
The x-intercepts are at -3 and 1.

d)

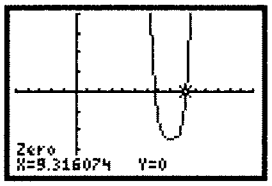


There are no x-intercepts.

e)



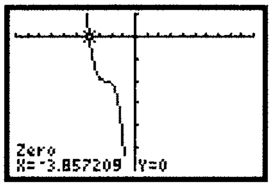
Zero
X=6.683926 Y=0



Zero
X=9.316074 Y=0

The x-intercepts are at 6.68 and 9.32.

f)



Zero
X=-3.857209 Y=0

The x-intercept is at -3.86.

10. a) It will have one zero. If the function is set equal to 0, there is only one solution for x.

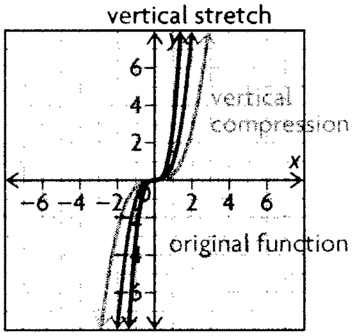
b) It will have no zeros. If the function is set equal to 0, there are no solutions for x because there is no real number that can be raised to the fourth power to produce a negative number.

c) The function will have one zero if n is odd, and it will have no zeros if n is even. This is because an odd root of a negative number is one negative number, and a negative number does not have any even roots.

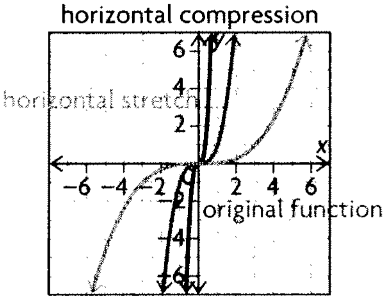
11. a) The reflection of the function $y = x^n$ in the x-axis will be the same as its reflection in the y-axis for odd values of n.

b) The reflections will be different for even values of n. The reflection in the x-axis will be $y = -x^n$, and the reflection in the y-axis will be $y = (-x)^n$. For odd values of n, $-x^n$ equals $(-x)^n$. For even values of n, $-x^n$ does not equal $(-x)^n$.

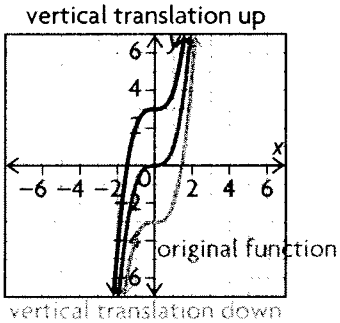
12. a) Vertical stretch and compression: $y = ax^3$



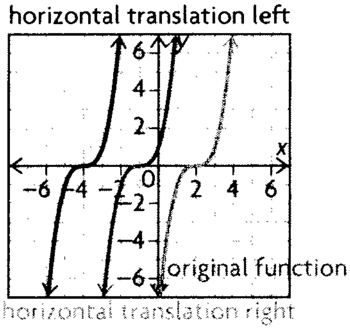
Horizontal stretch and compression: $y = (kx)^3$



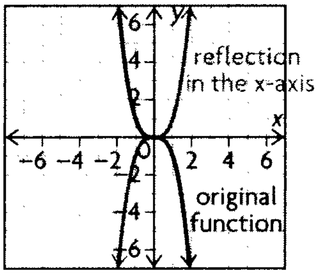
Vertical translation up or down: $y = x^3 + c$



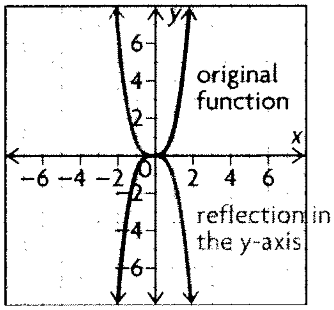
Horizontal translation left or right: $y = (x - d)^3$



Reflection in the x-axis: $y = -x^3$



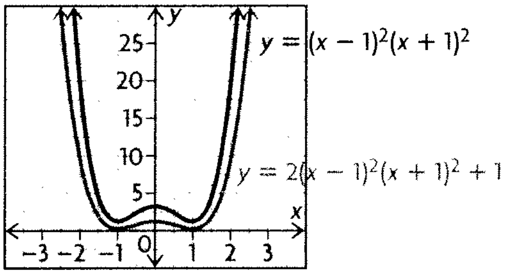
Reflection in the y-axis: $y = (-x)^3$



b) When using a table of values to sketch the graph of a function, you may not select a large enough range of values for the domain to produce an accurate representation of the function.

13. Yes, you can. The zeros of the first function have the same spacing between them as the zeros of the second function. Also, the ratio of the distances of the two curves above or below the x-axis at similar distances between the zeros is always the same. Therefore, the two curves have the same general shape, and one can be transformed into the other.

14. The graph of $y = (x - 1)^2(x + 1)^2$ must be vertically stretched by a factor of 2 and vertically translated 1 unit up to produce the graph of $y = 2(x - 1)^2(x + 1)^2 + 1$. The two graphs are shown below.



From the two graphs, it is apparent that the function $y = 2(x - 1)^2(x + 1)^2 + 1$ does not intersect the x-axis and, therefore, does not have any roots.

15. $f(x) = -4(4(x + 3))^2 - 5$

$$= \left(-\frac{5}{4}\right)(-4)\left(\left(\frac{1}{2}\right)(2)(x + 3)\right)^2 - 5 + 6$$

$$f(x) = 5(2(x + 3))^2 + 1$$

Mid-Chapter Review, p. 161

1. a) Yes.

b) No; it contains a rational exponent.

c) Yes.

d) No; it is a rational function.

2. a) Answers may vary. For example, $f(x) = x^3 + 2x^2 - 8x + 1$.

b) Answers may vary. For example, $f(x) = 5x^4 - x^2 - 7$.

c) Answers may vary. For example, $f(x) = 7x^6 + 3$.

d) Answers may vary. For example, $f(x) = -2x^5 - 4x^4 + 3x^3 - 2x^2 + 9$.

3. a) Since it is an odd degree and has a negative leading coefficient, the end behaviour is: as $x \rightarrow -\infty, y \rightarrow \infty$ and as $x \rightarrow \infty, y \rightarrow -\infty$.

b) Since it is an even degree and has a positive leading coefficient, the end behaviour is: as $x \rightarrow \pm\infty, y \rightarrow \infty$.

c) Since it is an odd degree and has a positive leading coefficient, the end behaviour is: as $x \rightarrow -\infty, y \rightarrow -\infty$ and as $x \rightarrow \infty, y \rightarrow \infty$.

d) Since it is an even degree and has a negative leading coefficient, the end behaviour is: as $x \rightarrow \pm\infty, y \rightarrow -\infty$.

4. a) $3 - 1 = 2 \rightarrow$ even

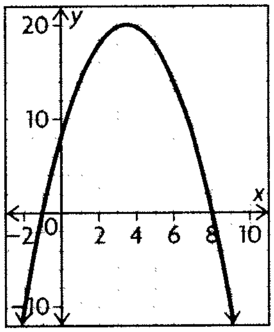
b) $6 - 1 = 5 \rightarrow$ odd

c) $4 - 1 = 3 \rightarrow$ odd

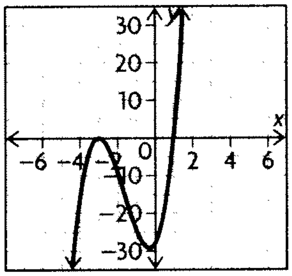
d) $5 - 1 = 4 \rightarrow$ even

5. Answers may vary. For example:

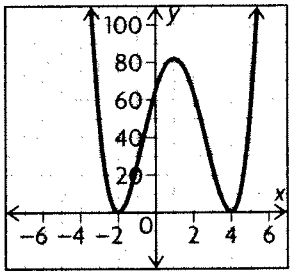
a)



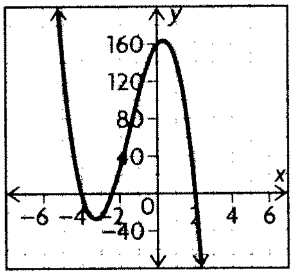
b)



c)



d)



6. The end behaviours cannot be determined if k is unknown because we must know the sign of the leading coefficient to determine the end behaviour.

7.
$$\begin{aligned}
y &= a(x-2)(x+3)^2(x-5) \\
5000 &= a(7-2)(7+3)^2(7-5) \\
5000 &= a(5)(10)^2(2) \\
5000 &= 1000a \\
5 &= a \\
y &= 5(x-2)(x+3)^2(x-5)
\end{aligned}$$

8. a) reflection in the x-axis, vertical stretch by a factor of 25, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 4 units to the left, vertical translation 60 units down

b) vertical stretch by a factor of 8, horizontal stretch by a factor of $\frac{4}{3}$, vertical translation 43 units up

c) reflection in the y-axis, horizontal compression by a factor of $\frac{1}{13}$, horizontal translation 2 units to the right, vertical translation 13 units up

d) vertical compression by a factor of $\frac{8}{11}$, reflection in the y-axis, vertical translation 1 unit down

9. $y = x^3$ was reflected in the x-axis, vertically stretched by a factor of 5, horizontally translated 4 units to the left, and vertically translated 2 units down.

3.5 Dividing Polynomials, pp. 168–170

1. a) i)

$$\begin{array}{rrrrrr}
 & & x^3 & -\,14x^2 & -\,24x & -\,38 \\ \cline{2-6}
x-2 & \big)\,x^4 & -\,16x^3 & +\,4x^2 & +\,10x & -\,11 \\
x^3(x-2)\rightarrow & \underline{x^4} & \underline{+\,2x^3} & \downarrow & \downarrow & \downarrow \\
 & & -\,14x^3 & +\,4x^2 & & \\
-14x^2(x-2)\rightarrow & & \underline{-\,14x^3} & \underline{+\,28x^2} & & \\
 & & & -\,24x^2 & +\,10x & \\
-24x(x-2)\rightarrow & & & \underline{-\,24x^2} & \underline{+\,48x} & \\
 & & & & -\,38x & -\,11 \\
-38(x-2)\rightarrow & & & & \underline{-\,38x} & \underline{+\,76} \\
 & & & & & -\,87
\end{array}$$

$x^3 - 14x^2 - 24x - 38$ remainder -87

ii)

$$\begin{array}{rr}
 & x^3 - 20x^2 - 84x - 326 \\
 & x + 4\overline{)x^4 - 16x^3 + 4x^2 + 10x - 11} \\
x^3(x + 4) \rightarrow & \underline{-x^4 + 4x^3} \quad \downarrow \quad \downarrow \quad \downarrow \\
 & -20x^3 + 4x^2 \\
-20x^2(x + 4) \rightarrow & \underline{-20x^3 + 80x^2} \\
 & 84x^2 + 10x \\
84x(x + 4) \rightarrow & \underline{84x^2 + 336x} \\
 & -326x - 11 \\
-326(x + 4) \rightarrow & \underline{-326x - 1304} \\
 & 1293
\end{array}$$

$x^3 - 20x^2 + 84x - 326$ remainder 1293

iii)

$$\begin{array}{rr}
 & x^3 - 15x^2 - 11x - 1 \\
 & x - 1\overline{)x^4 - 16x^3 + 4x^2 + 10x - 11} \\
x^3(x - 1) \rightarrow & \underline{x^4 - 1x^3} \quad \downarrow \quad \downarrow \quad \downarrow \\
 & -15x^3 + 4x^2 \\
-15x^2(x - 1) \rightarrow & \underline{-15x^3 + 15x^2} \\
 & -11x^2 + 10x \\
-11x(x - 1) \rightarrow & \underline{-11x^2 + 118x} \\
 & -1x - 11 \\
-1(x - 1) \rightarrow & \underline{-1x + 1} \\
 & -12
\end{array}$$

$x^3 - 15x^2 - 11x - 1$ remainder -12

b) No, they are not, because for each division problem there is a remainder.

a) degree of quotient: 2; degree of remainder: 1;

b) degree of quotient: 2; degree of remainder: 0;

c) degree of quotient: 1; degree of remainder: 3;

d) not possible

2. a) The degree of the quotient is 2 because $x^4 \div x^2 = x^2$.

b) The degree of the quotient is 2 because $x^3 \div x = x^2$.

c) The degree of the quotient is 1 because $x^4 \div x^3 = x$.

d) This is not possible because the degree of the divisor is greater than the degree of the dividend.

3. a)

$$\begin{array}{rr}
 & x^2 - 15x + 6 \\
 & x^2 - 4\overline{)x^4 - 15x^3 + 2x^2 + 12x - 10} \\
x^2(x^2 - 4) \rightarrow & \underline{x^4 \qquad - 4x^2} \quad \downarrow \quad \downarrow \\
 & -15x^3 + 6x^2 + 12x \\
-15x(x^2 - 4) \rightarrow & \underline{-15x^3 \qquad + 60x} \\
 & 6x^2 - 48x - 10 \\
6(x^2 - 4) \rightarrow & \underline{6x^2 \qquad - 24} \\
 & -48x + 14
\end{array}$$

$x^2 - 15x + 6$ remainder $-48x + 14$

b)

$$\begin{array}{r|rrrr}
-3 & 5 & -4 & 3 & -4 \\
 & \downarrow & -15 & 57 & -180 \\
\hline
 & 5 & -19 & 60 & -184
\end{array}$$

$5x^2 - 19x + 60$ remainder -184

c)

$$\begin{array}{rr}
 & x - 6 \\
 & x^3 - x^2 + 2x + 1\overline{)x^4 - 7x^3 + 2x^2 + 9x + 0} \\
x(x^3 - x^2 + 2x + 1) \rightarrow & \underline{x^4 - \ x^3 + 2x^2 - 1x \quad \downarrow} \\
 & - 6x^3 \qquad + 10x + 0 \\
-6(x^3 - x^2 + 2x + 1) \rightarrow & \underline{-6x^3 + 6x^2 - 12x - 6} \\
 & -6x^2 + 22x + 6
\end{array}$$

$x - 6$ remainder $-6x^2 + 22x + 6$

d) Not possible

4.

$$\begin{array}{rr}
 & 2x^2 - 11x + 41 \\
 & x + 3\overline{)2x^3 - 5x^2 + 8x + 4} \\
2x^2(x + 3) & \underline{2x^3 + 6x^2} \quad \downarrow \quad \downarrow \\
 & -11x^2 + 8x \\
-11x(x + 3) & \underline{-11x^2 - 33x} \\
 & 41x + \ \ 4 \\
41(x + 3) & \underline{41x + 123} \\
 & -119
\end{array}$$

$$(2x + 4)(3x^3 - 5x + 8) - 3 = 6x^4 - 10x^2 + 16x$$
$$+ 12x^3 - 20x + 32 - 3$$
$$= 6x^4 + 12x^3 - 10x^2 - 4x + 29$$

$$\begin{array}{rr}
 & 3x + 1 \\
 & 2x^3 + 0x^2 + x - 4\overline{)6x^4 + 2x^3 + 3x^2 - 11x - 9} \\
3x(2x^3 + x - 4) \rightarrow & \underline{6x^4 + 0x^3 + 3x^2 - 12x} \quad \downarrow \\
 & 2x^3 + 0x^2 + \ \ x - 9 \\
1(2x^3 + x - 4) \ \ \rightarrow & \underline{2x^3 + 0x^2 + \ \ x - 4} \\
 & -5
\end{array}$$

$$\begin{array}{rr}
 & 3x^2 - 5x + 4 \\
 & x + 2\overline{)3x^3 + x^2 - 6x + 16} \\
3x^2(x + 2) \rightarrow & \underline{3x^3 + 6x^2} \quad \downarrow \quad \downarrow \\
 & -5x^2 + 6x \\
-5x(x + 2) \rightarrow & \underline{-5x^2 - 10x} \\
 & 4x + 16 \\
4(x + 2) & \underline{4x + 8} \\
 & 8
\end{array}$$

Dividend	**Divisor**	**Quotient**	**Remainder**
$2x^3 - 5x^2 + 8x + 4$	$x + 3$	$2x^2 - 11x - 41$	-119
$6x^4 + 12x^3 - 10x^2 - 4x + 29$	$2x + 4$	$3x^3 - 5x + 8$	-3
$6x^4 + 2x^3 + 3x^2 - 11x - 9$	$3x + 1$	$2x^3 + x - 4$	-5
$3x^3 + x^2 - 6x + 16$	$x + 2$	$3x^2 - 5x + 4$	8

5. a)

$$\begin{array}{rl}
 & x^2+4x+14 \\
x-4 & \overline{)x^3+0x^2-2x+1} \\
x^2(x-4)\rightarrow & \underline{x^3-4x^2} \quad \downarrow \quad \downarrow \\
 & \quad 4x^2-2x \\
4x(x-4)\rightarrow & \quad \underline{4x^2-16x} \\
 & \qquad 14x+1 \\
14(x-4)\rightarrow & \qquad \underline{14x-56} \\
 & \qquad\qquad +57
\end{array}$$

$x^2 + 4x + 14$ remainder 57

b)

$$\begin{array}{rl}
 & x^2-6 \\
x+2 & \overline{)x^3+2x^2-6x+1} \\
x^2(x+2)\rightarrow & \underline{x^3+2x^2} \quad \downarrow \quad \downarrow \\
 & \qquad -6x+1 \\
-6(x+2)\rightarrow & \qquad \underline{-6x-12} \\
 & \qquad\quad +13
\end{array}$$

$x^2 - 6$ remainder 13

c)

$$\begin{array}{rl}
 & x^2+2x-3 \\
2x+1 & \overline{)2x^3+5x^2-4x-5} \\
x^2(2x+1)\rightarrow & \underline{2x^3+1x^2} \quad \downarrow \quad \downarrow \\
 & \quad 4x^2-4x \\
2x(2x+1)\rightarrow & \quad \underline{4x^2+2x} \\
 & \qquad -6x-5 \\
-3(2x+1)\rightarrow & \qquad \underline{-6x-3} \\
 & \qquad\quad -2
\end{array}$$

$x^2 + 2x - 3$ remainder -2

d)

$$\begin{array}{rl}
 & x^2+3x-9 \\
x^2+7 & \overline{)x^4+3x^3-2x^2+5x-1} \\
x^2(x^2+7)\rightarrow & \underline{x^4 \qquad +7x^2} \quad \downarrow \quad \downarrow \\
 & \quad 3x^3-9x^2+5x \\
3x(x^2+7)\rightarrow & \quad \underline{3x^3 \qquad +21x} \\
 & \qquad -9x^2-16x-1 \\
-9(x^2+7)\rightarrow & \qquad \underline{-9x^2 \qquad -63} \\
 & \qquad\qquad -16x+62
\end{array}$$

$x^2 + 3x - 9$ remainder $-16x + 62$

e)

$$\begin{array}{rl}
 & x+1 \\
x^3-x^2-x+1 & \overline{)x^4+0x^3+6x^2-8x+12} \\
x(x^3-x^2-x+1)\rightarrow & \underline{x^4-x^3-x^2+x} \quad \downarrow \\
 & \quad 1x^3+7x^2-9x+12 \\
1(x^3-x^2-x+1)\rightarrow & \quad \underline{x^3-x^2-x+1} \\
 & \qquad 8x^2-8x+11
\end{array}$$

$x + 1$ remainder $8x^2 - 8x + 11$

f)

$$\begin{array}{rl}
 & x+3 \\
x^4+x^3+x^2+x-2 & \overline{)x^5+4x^4+0x^3+0x^2+9x+8} \\
x(x^4+x^3+x^2+x-2)\rightarrow & \underline{x^5+x^4+x^3+x^2-2x} \quad \downarrow \\
 & \quad 3x^4-x^3-x^2+11x+8 \\
3(x^4+x^3+x^2+x-2)\rightarrow & \quad \underline{3x^4+3x^3+3x^2+3x-6} \\
 & \qquad -4x^3-4x^2+8x+14
\end{array}$$

$x + 3$ remainder $-4x^3 - 4x^2 + 8x + 14$

6. a)

$$\begin{array}{r|rrrr}
3 & 1 & 0 & -7 & -6 \\
 & \downarrow & 3 & 9 & 6 \\
\hline
 & 1 & 3 & 2 & 0
\end{array}$$

$x^2 + 3x + 2$ no remainder

b)

$$\begin{array}{r|rrrr}
1 & 2 & -7 & -7 & 19 \\
 & \downarrow & 2 & -5 & -12 \\
\hline
 & 2 & -5 & -12 & 7
\end{array}$$

$2x^2 - 5x - 12$ remainder 7

c)

$$\begin{array}{r|rrrrr}
-3 & 6 & 13 & -34 & -47 & 28 \\
 & \downarrow & -18 & 15 & 57 & -30 \\
\hline
 & 6 & -5 & -19 & 10 & -2
\end{array}$$

$6x^3 - 5x^2 - 19x + 10$ remainder -2

d)

$$\begin{array}{r|rrrr}
\frac{3}{2} & 2 & 1 & -22 & 20 \\
 & \downarrow & 3 & 6 & -24 \\
\hline
 & 2 & 4 & -16 & -4 \\
 & \div 2 & \div 2 & \div 2 & \div 2 \\
\hline
 & 1 & 2 & -8 & -2
\end{array}$$

$x^2 + 2x - 8$ remainder -2

e)

$$\begin{array}{r|rrrrr}
-\frac{1}{2} & 12 & -56 & 59 & 9 & -18 \\
 & \downarrow & -6 & 31 & -45 & 18 \\
\hline
 & 12 & -62 & 90 & -36 & 0 \\
 & \div 2 & \div 2 & \div 2 & \div 2 & \\
\hline
 & 6 & -31 & 45 & -18 &
\end{array}$$

$6x^3 - 31x^2 + 45x - 18$ no remainder

f)

$$\begin{array}{r|rrrr}
\frac{5}{2} & 6 & -15 & -2 & 5 \\
 & \downarrow & 15 & 0 & -5 \\
\hline
 & 6 & 0 & -2 & 0 \\
 & \div 2 & & \div 2 & \\
\hline
 & 3 & & -1 &
\end{array}$$

$3x^2 - 1$ no remainder

7. a) Divisor: $x + 10$; Quotient: $x^2 - 6x + 9$;
Remainder: -1

$= (x + 10)(x^2 - 6x + 9) - 1$

$= x^3 - 6x^2 + 9x + 10x^2 - 60x + 90 - 1$

$= x^3 + 4x^2 - 51x + 89$

b) Divisor: $3x - 2$; Quotient: $x^3 + x - 12$; Remainder: 15

$= (3x - 2)(x^3 + x - 12) + 15$

$= 3x^4 + 3x^2 - 36x - 2x^3 - 2x + 24 + 15$

$= 3x^4 - 2x^3 + 3x^2 - 38x + 39$

c) Divisor: $5x + 2$; Quotient: $x^3 + 4x^2 - 5x + 6$; Remainder: $x - 2$

$= (5x + 2)(x^3 + 4x^2 - 5x + 6) + (x - 2)$

$= 5x^4 + 20x^3 - 25x^2 + 30x + 2x^3 + 8x^2 - 10x + 12 + x - 2$

$= 5x^4 + 22x^3 - 17x^2 + 21x + 10$

d) Divisor: $x^2 + 7x - 2$; Quotient: $x^4 + x^3 - 11x + 4$; Remainder: $x^2 - x + 5$

$= (x^2 + 7x - 2)(x^4 + x^3 - 11x + 4) + (x^2 - x + 5)$

$= x^6 + x^5 - 11x^3 + 4x^2 + 7x^5 + 7x^4 - 77x^2 + 28x - 2x^4 - 2x^3 + 22x - 8 + x^2 - x + 5$

$= x^6 + 8x^5 + 5x^4 - 13x^3 - 72x^2 + 49x - 3$

8. a) $(2x - 3)(3x + 5) + r = 6x^2 + x + 5$

$6x^2 + 10x - 9x - 15 + r = 6x^2 + x + 5$

$6x^2 + x - 15 + r = 6x^2 + x + 5$

$r = 6x^2 + x + 5 - (6x^2 + x - 15)$

$r = 20$

b) $(x + 3)(x + 5) + r = x^2 + 9x - 7$

$x^2 + 5x + 3x + 15 + r = x^2 + 9x - 7$

$x^2 + 8x + 15 + r = x^2 + 9x - 7$

$r = x^2 + 9x - 7 - (x^2 + 8x + 15)$

$r = x - 22$

c) $(x + 3)(x^2 - 1) + r = x^3 + 3x^2 - x - 3$

$x^3 - x + 3x^2 - 3 + r = x^3 + 3x^2 - x - 3$

$r = x^3 + 3x^2 - x - 3 - (x^3 + 3x^2 - x - 3)$

$r = 0$

d) $(x^2 + 1)(2x^3 - 1) + r = 2x^5 + 2x^3 + x^2 + 1$

$2x^5 - x^2 + 2x^3 - 1 + r = 2x^5 + 2x^3 + x^2 + 1$

$r = 2x^5 + 2x^3 + x^2 + 1 - (2x^5 + 2x^3 - x^2 - 1)$

$r = 2x^2 + 2$

9. a) Dividend: $5x^3 + x^2 + 3$; Quotient: $5x^2 - 14x + 42$; Remainder: -123

In order to determine the divisor, we need to determine:

$$\frac{5x^3 + x^2 + 3}{5x^2 - 14x + 42}.$$

$x + 3$ is the divisor.

b) Dividend: $10x^4 - x^2 + 20x - 2$; Quotient: $10x^3 - 100x^2 + 999x - 9970$; Remainder: 99 698

In order to determine the divisor, we need to determine:

$$\frac{10x^4 - x^2 + 20x - 2}{10x^3 - 100x^2 + 999x - 9970}.$$

$$\begin{array}{rr}
 & x + 10 \\
10x^3 - 100x^2 + 999x - 9970 & \overline{)10x^4 + 0x^3 - x^2 + 20x - 2} \\
x(10x^3 - 100x^2 + 999x - 9970) \rightarrow & \underline{10x^4 - 100x^3 + 999x^2 - 9970x} \quad \downarrow \\
 & 100x^3 - 1000x^2 + 9990x - 2 \\
10(10x^3 - 100x^2 + 999x - 9970) \rightarrow & \underline{100x^3 - 1000x^2 + 9990x - 99\,700} \\
 & 99\,698
\end{array}$$

$x + 10$ is the divisor.

c) Dividend: $x^4 + x^3 - 10x^2 - 1$; Quotient: $x^3 - 3x^2 + 2x - 8$

In order to determine the divisor, we need to determine:

$$\frac{x^4 + x^3 - 10x^2 - 1}{x^3 - 3x^2 + 2x - 8}.$$

$$\begin{array}{rr}
 & x + 4 \\
x^3 - 3x^2 + 2x - 8 & \overline{)x^4 + x^3 - 10x^2 + 0x - 1} \\
x(x^3 - 3x^2 + 2x - 8) \rightarrow & \underline{x^4 - 3x^3 + 2x^2 - 8x} \quad \downarrow \\
 & 4x^3 - 12x^2 + 8x - 1 \\
4(x^3 - 3x^2 + 2x - 8) \rightarrow & \underline{4x^3 - 12x^2 + 8x - 32} \\
 & 31
\end{array}$$

$x + 4$ is the divisor.

d) Dividend: $x^3 + x^2 + 7x - 7$; Quotient: $x^2 + 3x + 13$; Remainder: 19

In order to determine the divisor, we need to determine:

$$\frac{x^3 + x^2 + 7x - 7}{x^2 + 3x + 13}.$$

$$\begin{array}{rr}
 & x - 2 \\
x^2 + 3x + 13 & \overline{)x^3 + x^2 + 7x - 7} \\
(x^2 + 3x + 13) \rightarrow & \underline{x^3 + 3x^2 + 13x} \downarrow \\
 & -2x^2 - 6x - 7 \\
-2(x^2 + 3x + 13) \rightarrow & \underline{-2x^2 - 6x - 26} \\
 & 19
\end{array}$$

$x - 2$ is the divisor.

10. a)
$$\begin{array}{r|rrrr} -5 & 1 & 6 & -1 & -30 \\ & \downarrow & -5 & -5 & 30 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x + 5$ is a factor since there is no remainder.

b)
$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -5 & 0 & 4 \\ & \downarrow & -2 & 4 & 2 & -4 \\ \hline & 1 & -2 & -1 & 2 & 0 \end{array}$$

$x + 2$ is a factor since there is no remainder.

c)
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -5 & 0 & 6 \\ & \downarrow & 2 & 4 & -2 & -4 \\ \hline & 1 & 2 & -1 & -2 & 2 \end{array}$$

$x - 2$ is not a factor since there is a remainder of 2.

d)
$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -1 & -4 & 2 & 1 \\ & \downarrow & 1 & 0 & -2 & 0 \\ \hline & 2 & 0 & -4 & 0 & 1 \end{array}$$

$2x - 1$ is not a factor since there is a remainder of 1.

e)
$$\begin{array}{r|rrrrrrr} -\frac{5}{3} & 3 & 5 & 0 & 0 & 9 & 17 & -1 \\ & \downarrow & -5 & 0 & 0 & 0 & -15 & -\frac{10}{3} \\ \hline & 3 & 0 & 0 & 0 & 9 & 2 & -\frac{13}{3} \end{array}$$

$3x + 5$ is not a factor since there is a remainder of $-\frac{13}{3}$.

f)
$$\begin{array}{r|rrrrr} \frac{1}{5} & 5 & -1 & 0 & 10 & -10 \\ & \downarrow & 1 & 0 & 0 & 2 \\ \hline & 5 & 0 & 0 & 10 & -8 \end{array}$$

$5x - 1$ is not a factor since there is a remainder of -8.

11. $V = \ell wh$

$x^3 + 6x^2 + 11x + 6 = (x + 2)(x + 3)h$

$$\frac{x^3 + 6x^2 + 11x + 6}{(x + 2)(x + 3)} = h$$

$$\frac{x^3 + 6x^2 + 11x + 6}{x^2 + 5x + 6} = h$$

$$\begin{array}{r l} & \quad\quad\quad\quad\quad\quad\quad x + 1 \\ \cline{2-2} x^2 + 5x + 6 & \big)\, x^3 + 6x^2 + 11x + 6 \\ x(x^2 + 5x + 6) \rightarrow & \underline{x^3 + 5x^2 + 6x} \quad \downarrow \\ & \quad\quad 1x^2 + 5x + 6 \\ 1(x^2 + 5x + 6) \rightarrow & \quad\quad \underline{x^2 + 5x + 6} \\ & \quad\quad\quad\quad\quad\quad\quad 0 \end{array}$$

The height of the box is $(x + 1)$ cm.

12. a) Using synthetic division, the problem looks like:

$$\begin{array}{r|rrrr} -\frac{1}{2} & 8 & 10 & -p & -5 \\ & \downarrow & -4 & -3 & 5 \\ \hline & 8 & 6 & x & 0 \end{array}$$

In order for there to be no remainder, x must be -10 because $-\frac{1}{2}(-10) = 5$.

So, $-p - 3 = -10$.

$-p = -7$

$p = 7$

To check this:

$$\begin{array}{r|rrrr} -\frac{1}{2} & 8 & 10 & -7 & -5 \\ & \downarrow & -4 & -3 & 5 \\ \hline & 8 & 6 & -10 & 0 \end{array}$$

b) Using long division, the problem looks like:

$$\begin{array}{r l} & \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x^4 - 2 \\ \cline{2-2} x^2 + 1 & \big)\, x^6 - 0x^5 + x^4 + 0x^3 - 2x^2 + 0x + k \\ x^4(x^2 + 1) \rightarrow & \underline{x^6 + 0x^5 + 1x^4} \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ & \quad\quad\quad 0x^4 + 0x^3 - 2x^2 + 0x + k \\ -2(x^2 + 1) \rightarrow & \quad\quad\quad\quad\quad\quad \underline{-\ 2x^2 + 0x - 2} \\ & \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 5 \end{array}$$

So, $k + 2 = 5$

$k = 3$

13. Using synthetic division and filling in what we know or can simply figure out, the problem looks like:

$$\begin{array}{r|rrrr} 1 & 1 & p & -1 & -2 \\ & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

So, $p + 1 = 3$.

$p = 2$

To check this:

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -1 & -2 \\ & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

14. Yes, $f(x)$ is always divisible by $x - 1$. Regardless of the value of n, $f(x) = x^n - 1$ can always be written as

$f(x) = x^n + 0x^{n-1} + 0x^{n-2} + \ldots 0x - 1$.

Therefore, the same pattern continues when dividing $x^n - 1$ by $x - 1$, regardless of how large n is, and there is never a remainder.

15. a) $(x - 4)(x^2 + x - 6) + 7$

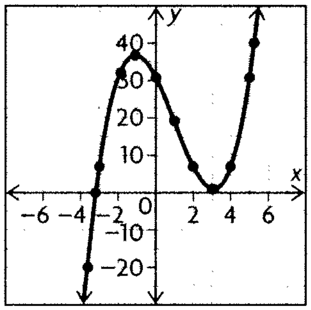
$x^3 + x^2 - 6x - 4x^2 - 4x + 24 + 7$

$x^3 - 3x^2 - 10x + 31$

So, $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)$ $(x^2 + x - 6)$ remainder 7

b) $x^2 + x - 6$
$(x + 3)(x - 2)$
So, $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)$
$(x + 3)(x - 2)$ remainder 7

c)



16.

$$\begin{array}{r|rrrrr} 3 & 2 & 3 & -25 & -7 & -14 \\ & \downarrow & 6 & 27 & 6 & -3 \\ \hline & \multicolumn{4}{c}{\underbrace{2 \quad 9 \quad 2 \quad -1}_{\text{Coefficients of quotient}}} & \boxed{-17}\ \text{Remainder} \end{array}$$

$$\overbrace{2x^3 + 9x^2 + 2x - 1}$$

$$\begin{array}{rl} & 2x^3 + 9x^2 + 2x - 1 \\ x - 3 & \overline{)2x^4 + 3x^3 - 25x^2 - 7x - 14} \\ 2x^3(x - 3) \rightarrow & \underline{2x^4 - 6x^3} \\ & \quad 9x^3 - 25x^2 \\ 9x^2(x - 3) \rightarrow & \quad \underline{9x^3 - 27x^2} \\ & \qquad 2x^2 - 7x \\ 2x(x - 3) \rightarrow & \qquad \underline{2x^2 - 6x} \\ & \qquad\quad -1x - 14 \\ -1(x - 3) \rightarrow & \qquad\quad \underline{-1x + 3} \\ & \qquad\qquad -17 \end{array}$$

17. $V = \pi r^2 h$
$4\pi x^3 + 28\pi x^2 + 65\pi x + 50\pi = \pi r^2(x + 2)$
$\cancel{\pi}(4x^3 + 28x^2 + 65x + 50) = \cancel{\pi} r^2(x + 2)$

$$\frac{4x^3 + 28x^2 + 65x + 50}{x + 2} = r^2$$

$$\begin{array}{r|rrrr} -2 & 4 & 28 & 65 & 50 \\ & \downarrow & -8 & -40 & -50 \\ \hline & 4 & 20 & 25 & 0 \end{array}$$

$4x^2 + 20x + 25 = r^2$
$\sqrt{4x^2 + 20x + 25} = r$
$r = 2x + 5$ cm

18. a)

$$\begin{array}{rl} & x^2 + xy + y^2 \\ x^2 - y^2 & \overline{)x^4 + x^3y + 0x^2y^2 - xy^3 - y^4} \\ x^2(x^2 - y^2) \rightarrow & \underline{x^4 \qquad - x^2y^2} \quad \downarrow \quad \downarrow \\ & \qquad x^3y + x^2y^2 - xy^3 \\ xy(x^2 - y^2) \rightarrow & \qquad \underline{x^3y \qquad - xy^3} \\ & \qquad\qquad x^2y^2 \qquad - y^4 \\ y^2(x^2 - y^2) \rightarrow & \qquad\qquad \underline{x^2y^2 \qquad - y^4} \\ & \qquad\qquad\qquad 0 \end{array}$$

b)

$$\begin{array}{rl} & x^2 - 2xy + y^2 \\ x^2 + y^2 & \overline{)x^4 - 2x^3y + 2x^2y^2 - 2xy^3 + y^4} \\ x^2(x^2 + y^2) \rightarrow & \underline{x^4 \qquad + x^2y^2} \quad \downarrow \quad \downarrow \\ & \qquad -2x^3y + x^2y^2 - 2xy^3 \\ -2xy(x^2 + y^2) \rightarrow & \qquad \underline{-2x^3y \qquad - 2xy^3} \\ & \qquad\qquad x^2y^2 \qquad + y^4 \\ y^2(x^2 + y^2) \rightarrow & \qquad\qquad \underline{x^2y^2 \qquad + y^4} \\ & \qquad\qquad\qquad 0 \end{array}$$

19.

$$\begin{array}{rl} & x^2 + xy + y^2 \\ x - y & \overline{)x^3 - 0x^2y + 0xy^2 + y^3} \\ x^2(x - y) \rightarrow & \underline{x^3 - x^2y} \quad \downarrow \quad \downarrow \\ & \quad x^2y + 0x^2y^2 \\ xy(x - y) \rightarrow & \quad \underline{x^2y - xy^2} \\ & \qquad xy^2 - y^3 \\ y^2(x - y) \rightarrow & \qquad \underline{x^2y^2 - y^3} \\ & \qquad\quad 0 \end{array}$$

$x - y$ is a factor because there is no remainder.

20. $q(x)(x + 5) < f(x)$
$[q(x) + 1](x + 5) = q(x)(x + 5) + (x + 5)$,
which is greater than
$f(x) = q(x)(x + 5) + (x + 3)$.
The first multiple of $(x + 5)$ that is greater than $f(x)$ is $[q(x) + 1](x + 5)$.

3.6 Factoring Polynomials, pp. 176–177

1. a) i) $f(2) = 2^4 + 5(2)^3 + 3(2)^2 - 7(2) + 10$
$= 16 + 40 + 12 - 14 + 10$
$= 64$

ii) $f(-4) = (-4)^4 + 5(-4)^3 + 3(-4)^2$
$- 7(-4) + 10$
$= 256 - 320 + 48 + 28 + 10$
$= 22$

iii) $f(1) = 1^4 + 5(1)^3 + 3(1)^2 - 7(1) + 10$
$= 1 + 5 + 3 - 7 + 10$
$= 12$

b) No, according to the factor theorem, $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

2. a) $f(1) = 1^4 - 15(1)^3 + 2(1)^2 + 12(1) - 10$
$= 1 - 15 + 2 + 12 - 10$
$= -10$

$x - 1$ is not a factor of $f(x)$ because there is a remainder.

b) $g(1) = 5(1)^3 - 4(1)^2 + 3(1) - 4$
$= 5 - 4 + 3 - 4$
$= 0$

$x - 1$ is a factor of $g(x)$ because there is not a remainder.

c) $h(1) = 1^4 - 7(1)^3 + 2(1)^3 + 9(1)$
$= 1 - 7 + 2 + 9$
$= 5$

$x - 1$ is not a factor of $h(x)$ because there is a remainder.

d) $j(1) = 1^3 - 1$
$= 1 - 1$
$= 0$

$x - 1$ is a factor of $j(x)$ because there is not a remainder.

3. $f(x) = x^3 + 2x^2 - 5x - 6$
$f(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$
$= -1 + 2 + 5 - 6$
$= 0$

$x + 1$ is a factor.

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$= (x + 1)(x^2 + x - 6)$
$= (x + 1)(x + 3)(x - 2)$

4. a) $f(-2) = (-2)^2 + 7(-2) + 9$
$= 4 - 14 + 9$
$= -1$

b) $f(-2) = 6(-2)^3 + 19(-2)^2 + 11(-2) - 11$
$= -48 + 76 - 22 - 11$
$= -5$

c) $f(-2) = (-2)^4 - 5(-2)^2 + 4$
$= 16 - 20 + 4$
$= 0$

d) $f(-2) = (-2)^4 - 2(-2)^3 - 11(-2)^2$
$+ 10(-2) - 2$
$= 16 + 16 - 44 - 20 - 2$
$= -34$

e) $f(-2) = (-2)^3 + 3(-2)^2 - 10(-2) + 6$
$= -8 + 12 + 20 + 6$
$= 30$

f) $f(-2) = 4(-2)^4 + 12(-2)^3 - 13(-2)^2$
$- 33(-2) + 18$
$= 64 - 96 - 52 + 66 + 18$
$= 0$

5. a)

$$\begin{array}{r|rrrr} \frac{5}{2} & 2 & -5 & -2 & 5 \\ & \downarrow & 5 & 0 & -5 \\ \hline & 2 & 0 & -2 & 0 \end{array}$$

$2x - 5$ is a factor because there is no remainder.

b)

$$\begin{array}{r|rrrr} \frac{5}{2} & 3 & 2 & -3 & -2 \\ & \downarrow & \frac{15}{2} & \frac{95}{4} & \frac{415}{8} \\ \hline & 3 & \frac{19}{2} & \frac{83}{4} & \frac{399}{8} \end{array}$$

$2x - 5$ is not a factor because there is a remainder.

c)

$$\begin{array}{r|rrrrr} \frac{5}{2} & 2 & -7 & -13 & 63 & -45 \\ & \downarrow & 5 & -5 & -45 & 45 \\ \hline & 2 & -2 & -18 & 18 & 0 \end{array}$$

$2x - 5$ is a factor because there is no remainder.

d)

$$\begin{array}{r|rrrrr} \frac{5}{2} & 6 & 1 & -7 & -1 & 1 \\ & \downarrow & 15 & 40 & \frac{165}{2} & \frac{815}{4} \\ \hline & 6 & 16 & 33 & \frac{163}{2} & \frac{819}{4} \end{array}$$

$2x - 5$ is not a factor because there is a remainder.

6. a) $x^3 - 3x^2 - 10x + 24$
$f(2) = 2^3 - 3(2)^2 - 10(2) + 24$
$= 8 - 12 - 20 + 24$
$= 0$

$x - 2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & \downarrow & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$= (x - 2)(x^2 - x - 12)$
$= (x - 2)(x - 4)(x + 3)$

b) $4x^3 + 12x^2 - x - 15$
$f(1) = 4(1)^3 + 12(1)^2 - 1 - 15$
$= 4 + 12 - 1 - 15$
$= 0$

$x - 1$ is a factor.

$$\begin{array}{r|rrrr} 1 & 4 & 12 & -1 & -15 \\ & \downarrow & 4 & 16 & 15 \\ \hline & 4 & 16 & 15 & 0 \end{array}$$

$= (x - 1)(4x^2 + 16x + 15)$
$= (x - 1)(4x^2 + 10x + 6x + 15)$

$= (x - 1)(2x(2x + 5) + 3(2x + 5))$

$= (x - 1)(2x + 3)(2x + 5)$

c) $x^4 + 8x^3 + 4x^2 - 48x$

$x(x^3 + 8x^2 + 4x - 48)$

For $(x^3 + 8x^2 + 4x - 48)$

$f(2) = 2^3 + 8(2)^2 + 4(2) - 48$

$= 8 + 32 + 8 - 48$

$= 0$

$x - 2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 1 & 8 & 4 & -48 \\ & \downarrow & 2 & 20 & 48 \\ \hline & 1 & 10 & 24 & 0 \end{array}$$

$= x(x - 2)(x^2 + 10x + 24)$

$= x(x - 2)(x + 4)(x + 6)$

d) $4x^4 + 7x^3 - 80x^2 - 21x + 270$

$f(-2) = 4(-2)^4 + 7(-2)^3 - 80(-2)^2$

$- 21(-2) + 270$

$= 64 - 56 - 320 + 42 + 270$

$= 0$

$x + 2$ is a factor.

$$\begin{array}{r|rrrrr} -2 & 4 & 7 & -80 & -21 & 270 \\ & \downarrow & -8 & 2 & 156 & -270 \\ \hline & 4 & -1 & -78 & 135 & 0 \end{array}$$

$= (x + 2)(4x^3 - x^2 - 78x + 135)$

For $(4x^3 - x^2 - 78x + 135)$

$f(3) = 4(3)^3 - (3)^2 - 78(3) + 135$

$= 108 - 9 - 234 + 135$

$= 0$

$x - 3$ is a factor.

$$\begin{array}{r|rrrr} 3 & 4 & -1 & -78 & 135 \\ & \downarrow & 12 & 33 & -135 \\ \hline & 4 & 11 & -45 & 0 \end{array}$$

$= (x + 2)(x - 3)(4x^2 + 11x - 45)$

$= (x + 2)(x - 3)(4x^2 + 20x - 9x - 45)$

$= (x + 2)(x - 3)(4x(x + 5) - 9(x + 5))$

$= (x + 2)(x + 5)((4x - 9)(x - 3))$

e) $x^5 - 5x^4 - 7x^3 + 29x^2 + 30x$

$x(x^4 - 5x^3 - 7x^2 + 29x + 30)$

For $(x^4 - 5x^3 - 7x^2 + 29x + 30)$

$f(-1) = (-1)^4 - 5(-1)^3 - 7(-1)^2$

$+ 29(-1) + 30$

$= 1 + 5 - 7 - 29 + 30$

$= 0$

$x + 1$ is a factor.

$$\begin{array}{r|rrrrr} -1 & 1 & -5 & -7 & 29 & 30 \\ & \downarrow & -1 & 6 & 1 & -30 \\ \hline & 1 & -6 & -1 & 30 & 0 \end{array}$$

$= x(x + 1)(x^3 - 6x^2 - x + 30)$

For $(x^3 - 6x^2 - x + 30)$

$f(3) = 3^3 - 6(3)^2 - 3 + 30$

$= 27 - 54 - 3 + 30$

$= 0$

$x - 3$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & -1 & 30 \\ & \downarrow & 3 & -9 & -30 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$= x(x + 1)(x - 3)(x^2 - 3x - 10)$

$= x(x + 2)(x + 1)(x - 3)(x - 5)$

f) $x^4 + 2x^3 - 23x^2 - 24x + 144$

$f(3) = 3^4 + 2(3)^3 - 23(3)^2 - 24(3) + 144$

$= 81 + 54 - 207 - 72 + 144$

$= 0$

$x - 3$ is a factor.

$$\begin{array}{r|rrrrr} 3 & 1 & 2 & -23 & -24 & 144 \\ & \downarrow & 3 & 15 & -24 & -144 \\ \hline & 1 & 5 & -8 & -48 & 0 \end{array}$$

$= (x - 3)(x^3 + 5x^2 - 8x - 48)$

For $x^3 + 5x^2 - 8x - 48$

$f(-4) = (-4)^3 + 5(-4)^2 - 8(-4) - 48$

$= -64 + 80 + 32 - 48$

$= 0$

$x + 4$ is a factor.

$$\begin{array}{r|rrrr} -4 & 1 & 5 & -8 & -48 \\ & \downarrow & -4 & -4 & 48 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$(x - 3)(x + 4)(x^2 + x - 12)$

$= (x - 3)(x - 3)(x + 4)(x + 4)$

7. a) $x^3 + 9x^2 + 8x - 60$

$f(2) = 2^3 + 9(2)^2 + 8(2) - 60$

$= 8 + 36 + 16 - 60$

$= 0$

$$\begin{array}{r|rrrr} 2 & 1 & 9 & 8 & -60 \\ & \downarrow & 2 & 22 & 60 \\ \hline & 1 & 11 & 30 & 0 \end{array}$$

$= (x - 2)(x^2 + 11x + 30)$

$= (x - 2)(x + 5)(x + 6)$

b) $x^3 - 7x - 6$

$f(-1) = (-1)^3 - 7(-1) - 6$

$= -1 + 7 - 6$

$= 0$

$x + 1$ is a factor.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & \downarrow & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$= (x + 1)(x^2 - x - 6)$

$= (x + 1)(x - 3)(x + 2)$

c) $x^4 - 5x^2 + 4$
$= (x^2 - 1)(x^2 - 4)$
$= (x + 1)(x - 1)(x - 2)(x + 2)$
d) $x^4 + 3x^3 - 38x^2 + 24x + 64$
$f(2) = 2^4 + 3(2)^3 - 38(2)^2 + 24(2) + 64$
$= 16 + 24 - 152 + 48 + 64$
$= 0$
$x - 2$ is a factor.

$$\begin{array}{r|rrrrr} 2 & 1 & 3 & -38 & 24 & 64 \\ & \downarrow & 2 & 10 & -56 & -64 \\ \hline & 1 & 5 & -28 & -32 & 0 \end{array}$$

$= (x - 2)(x^3 + 5x^2 - 28x - 32)$
For $x^3 + 5x^2 - 28x - 32$
$f(-1) = (-1)^3 + 5(-1)^2 - 28(-1) - 32$
$= -1 + 5 + 28 - 32$
$= 0$
$x + 1$ is a factor.

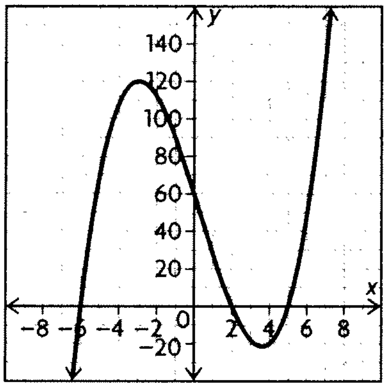
$$\begin{array}{r|rrrr} -1 & 1 & 5 & -28 & -32 \\ & \downarrow & -1 & -4 & 32 \\ \hline & 1 & 4 & -32 & 0 \end{array}$$

$= (x - 2)(x + 1)(x^2 + 4x - 32)$
$= (x - 2)(x + 1)(x + 8)(x - 4)$
e) $x^3 - x^2 + x - 1$
$x^2(x - 1) + 1(x - 1)$
$(x - 1)(x^2 + 1)$
f) $x^5 - x^4 + 2x^3 - 2x^2 + x - 1$
$f(1) = 1^5 - 1^4 + 2(1)^3 - 2(1)^2 + 1 - 1$
$= 1 - 1 + 2 - 2 + 1 - 1$
$= 0$
$x - 1$ is a factor.

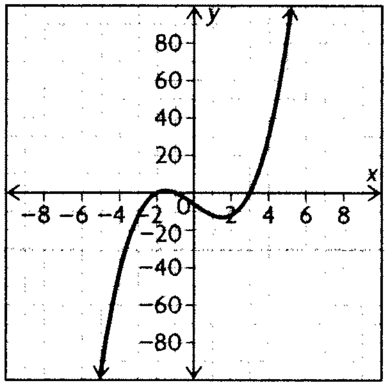
$$\begin{array}{r|rrrrrr} 1 & 1 & -1 & 2 & -2 & 1 & -1 \\ & \downarrow & 1 & 0 & 2 & 0 & 1 \\ \hline & 1 & 0 & 2 & 0 & 1 & 0 \end{array}$$

$= (x - 1)(x^4 + 2x^2 + 1)$
$= (x - 1)(x^2 + 1)(x^2 + 1)$

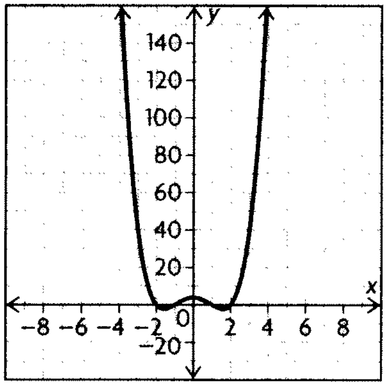
8. a)



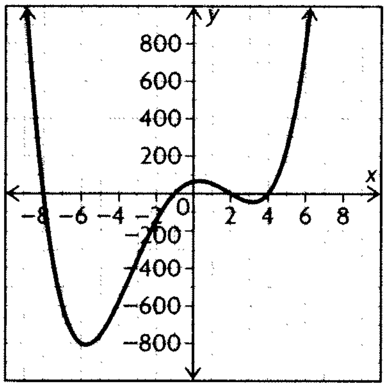
b)



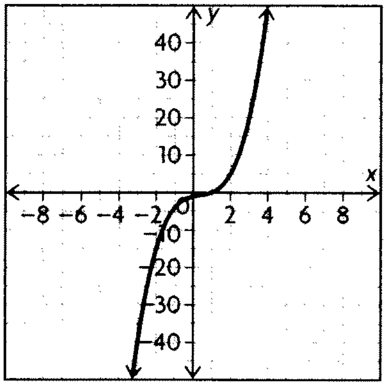
c)



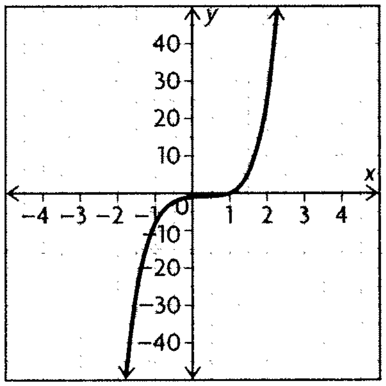
d)



e)



f)



9. Using synthetic division and filling in everything we know or can figure by working backwards, we get:

$$\begin{array}{c|cccc} \frac{1}{2} & 12 & k & -1 & -6 \\ & \downarrow & 6 & 13 & 6 \\ \hline & 12 & 26 & 12 & 0 \end{array}$$

So, $k + 6 = 26$

$k = 20$

10. Using synthetic division for both divisors and remainders, we get:

$$\begin{array}{c|cccc} 1 & a & -1 & 2 & b \\ & \downarrow & a & -1+a & 1+a \\ \hline & a & -1+a & 1+a & 10 \end{array}$$

So, $b + 1 + a = 10$

$a + b = 9$

and

$$\begin{array}{c|cccc} 2 & a & -1 & 2 & b \\ & \downarrow & 2a & -2+4a & 8a \\ \hline & a & -1+2a & 4a & 51 \end{array}$$

So, $8a + b = 51$

Solve the system of equations to determine a and b.

$$\begin{array}{r} a + b = 9 \\ \underline{-(8a + b = 51)} \\ -7a = -42 \\ a = 6 \\ 6 + b = 9 \\ b = 3 \end{array}$$

11. For $x^n - a^n$, if n is even, they're both factors. If n is odd, only $(x - a)$ is a factor. For $x^n + a^n$, if n is even, neither is a factor. If n is odd, only $(x + a)$ is a factor.

12. Using synthetic division for both divisors and remainders, we get:

$$\begin{array}{c|cccc} 2 & a & -1 & b & -24 \\ & \downarrow & 2a & -2+4a & 24 \\ \hline & a & -1+2a & 12 & 0 \end{array}$$

So, $b + (-2 + 4a) = 12$

$4a + b = 14$

and

$$\begin{array}{c|cccc} -4 & a & -1 & b & -24 \\ & \downarrow & -4a & 4+16a & 24 \\ \hline & a & -1-4a & -6 & 0 \end{array}$$

So, $4 + 16a + b = -6$

$16a + b = -10$

Solve the system of equations to determine a and b.

$$\begin{array}{r} 4a + b = 14 \\ \underline{-(16a + b = -10)} \\ -12a = 24 \\ a = -2 \\ 4(-2) + b = 14 \\ b = 22 \end{array}$$

$f(x) = -2x^3 - x^2 + 22x - 24$

Two factors are $x - 2$ and $x + 4$, so $f(x)$ is divisible by $(x - 2)(x + 4) = x^2 + 2x - 8$.

$$\begin{array}{r} -2x + 3 \\ x^2 + 2x - 8\overline{)-2x^3 - x^2 + 22x - 24} \\ \underline{-2x^3 - 4x^2 + 16x} \\ 3x^2 + 6x - 24 \\ \underline{3x^2 + 6x - 24} \end{array}$$

The other factor is $-2x + 3$.

13. Using synthetic division for both divisors and remainders, we get:

$$\begin{array}{c|cccc} -2 & 1 & 4 & k & -4 \\ & \downarrow & -2 & -4 & -2k+8 \\ \hline & 1 & 2 & k-4 & 2r \end{array}$$

So, $-2k + 8 - 4 = 2r$

$-2k - 2r = -4$

and

$$\begin{array}{c|cccc} 2 & 1 & 4 & k & -4 \\ & \downarrow & 2 & 12 & 2k+24 \\ \hline & 1 & 6 & k+12 & r \end{array}$$

So, $2k + 24 - 4 = r$

$2k - r = -20$

Solve the system of equations to determine a and b.

$$\begin{array}{r} -2k - 2r = -4 \\ \underline{2k - r = -20} \\ -3r = -24 \\ r = 8 \\ 2(k) - 8 = -20 \\ k = -6 \end{array}$$

14. $x^4 - a^4$

$= (x^2)^2 - (a^2)^2$

$= (x^2 + a^2)(x^2 - a^2)$

$= (x^2 + a^2)(x + a)(x - a)$

$x - a$ is a factor of $x^4 - a^4$.

15. Answers may vary. For example:
If $f(x) = k(x - a)$, then
$f(a) = k(a - a) = k(0) = 0$.

16. $x^2 - x - 2 = (x + 1)(x - 2)$
Check that both of these factors work for
$x^3 - 6x^2 + 3x + 10$ using the factor theorem.
$f(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$
$= -1 - 6 - 3 + 10$
$= 0$
$f(2) = (2)^3 - 6(2)^2 + 3(2) + 10$
$= 8 - 24 + 6 + 10$
$= 0$

17. Let $f(x) = (x + a)^5 + (x + c)^5 + (a - c)^5$.
$x + a$ is a factor of $f(x)$ if and only if $f(-a) = 0$.
$f(-a) = (-a + a)^5 + (-a + c)^5 + (a - c)^5$
$= 0^5 + [-1(a - c)]^5 + (a - c)^5$
$= -(a - c)^5 + (a - c)^5$
$= 0$
So $x + a$ is a factor of
$(x + a)^5 + (x + c)^5 + (a - c)^5$.

3.7 Factoring a Sum or Difference of Cubes, p. 182

1. Let $f(x) = x^3 + b^3$
$f(-b) = 0$, so $(x + b)$ is a factor.
$(x^3 + b^3) \div (x + b) = (x^2 - bx + b^2)$
$x^3 + b^3 = (x + b)(x^2 - bx + b^2)$

2. a) $x^3 - 64$
$= (x)^3 - (4)^3$
$= (x - 4)(x^2 + 4x + 16)$

b) $x^3 - 125$
$= (x)^3 - (5)^3$
$= (x - 5)(x^2 + 5x + 25)$

c) $x^3 + 8$
$= (x)^3 + (2)^3$
$= (x + 2)(x^2 - 2x + 4)$

d) $8x^3 - 27$
$= (2x)^3 - (3)^3$
$= (2x - 3)(4x^2 + 6x + 9)$

e) $64x^3 - 125$
$= (4x)^3 - (5)^3$
$= (4x - 5)(16x^2 + 20x + 25)$

f) $x^3 + 1$
$= (x)^3 + (1)^3$
$= (x + 1)(x^2 - x + 1)$

g) $27x^3 + 8$
$= (3x)^3 + (2)^3$
$= (3x + 2)(9x^2 - 6x + 4)$

h) $1000x^3 + 729$
$= (10x)^3 + (9)^3$
$= (10x + 9)(100x^2 - 90x + 81)$

i) $216x^3 - 8$
$= (6x)^3 - (2)^3$
$= (6x - 2)(36x^2 + 12x + 4)$
$= 8(3x - 1)(9x^2 + 3x + 1)$

3. a) $64x^3 + 27y^3$
$= (4x)^3 + (3y)^3$
$= (4x + 3y)(16x^2 - 12xy + 9y^2)$

b) $-3x^4 + 24x$
$= -3x(x^3 - 8)$
$= -3x((x)^3 - (2)^3)$
$= (-3x)(x - 2)(x^2 + 2x + 4)$

c) $(x + 5)^3 - (2x + 1)^3$
$= ((x + 5) - (2x + 1))$
$\times ((x + 5)^2 + (x + 5)(2x + 1) + (2x + 1)^2)$
$= (4 - x)(x^2 + 10x + 25 + 2x^2 + 11x$
$+ 5 + 4x^2 + 4x + 1)$
$= (4 - x)(7x^2 + 25x + 31)$

d) $x^6 + 64$
$= (x^2)^3 + (4)^3$
$= (x^2 + 4)(x^4 - 4x^2 + 16)$

4. a) $x^3 - 343$
$= (x)^3 - (7)^3$
$= (x - 7)(x^2 + 7x + 49)$

b) $216x^3 - 1$
$= (6x)^3 - (1)^3$
$= (6x - 1)(36x^2 + 6x + 1)$

c) $x^3 + 1000$
$= (x)^3 + (10)^3$
$= (x + 10)(x^2 - 10x + 100)$

d) $125x^3 - 512$
$= (5x)^3 - (8)^3$
$= (5x - 8)(25x^2 + 40x + 64)$

e) $64x^3 - 1331$
$= (4x)^3 - (11)^3$
$= (4x - 11)(16x^2 + 44x + 121)$

f) $343x^3 + 27$
$= (7x)^3 + (3)^3$
$= (7x + 3)(49x^2 - 21x + 9)$

g) $512x^3 + 1$
$= (8x)^3 + (8)^3$
$= (8x + 1)(64x^2 - 8x + 1)$

h) $1331x^3 + 1728$
$= (11x)^3 + (12)^3$
$= (11x + 12)(121x^2 - 132x + 144)$

i) $512 - 1331x^3$
$= (8)^3 - (11x)^3$
$= (8 - 11x)(64 + 88x + 121x^2)$
$= 8(3x - 1)(9x^2 + 3x + 1)$

5. a) $\frac{1}{27}x^3 - \frac{8}{125}$
$= \left(\frac{1}{3}x\right)^3 - \left(\frac{2}{5}\right)^3$
$= \left(\frac{1}{3}x - \frac{2}{5}\right)\left(\frac{1}{9}x^2 + \frac{2}{15}x + \frac{4}{25}\right)$

b) $-432x^5 - 128x^2$
$= (-16x^2)(27x^3 + 8)$
$= (-8x^2)((3x)^3 + (2)^3)$
$= -16x^2(3x + 2)(9x^2 - 6x + 4)$

c) $(x - 3)^3 + (3x - 2)^3$
$= (x - 3 + 3x - 2)$
$\times ((x - 3)^2 - (x - 3)(3x - 2) + (3x - 2)^2)$
$= (4x - 5)((x^2 - 6x + 9) - (3x^2 - 11x + 6)$
$+ (9x^2 - 12x + 4))$
$= (4x - 5)(7x^2 - 7x + 7)$
$= 7(4x - 5)(x^2 - x + 1)$

d) $\frac{1}{512}x^9 - 512$
$= \left(\frac{1}{8}x^3\right)^3 - (8)^3$
$= \left(\frac{1}{8}x^3 - 8\right)\left(\frac{1}{64}x^6 + x^3 + 64\right)$
$= \left(\left(\frac{1}{2}x\right)^3 - (2)^3\right)\left(\frac{1}{64}x^6 + x^3 + 64\right)$
$= \left(\frac{1}{2}x - 2\right)\left(\frac{1}{4}x^2 + x + 4\right)\left(\frac{1}{64}x^6 + x^3 + 64\right)$

6. Agree; by the formulas for factoring the sum and difference of cubes, the numerator of the fraction is equivalent to $(a^3 + b^3) + (a^3 - b^3)$. Since $(a^3 + b^3) + (a^3 - b^3) = 2a^3$, the entire fraction is equal to 1.

7. $1^3 + 12^3$
$= (1 + 12)(1^2 - (1)(12) + 12^2)$
$= (1 + 12)(1 - 12 + 144)$
$= (13)(133)$
$= 1729$
$9^3 + 10^3$
$= (9 + 10)(9^2 - (9)(10) + 10^2)$
$= (9 + 10)(81 - 90 + 100)$
$= (19)(91)$
$= 1729$

8. $x^9 + y^9$
$= x^{18} + 2x^9y^9 + y^{18}$
$= (x^{18} + y^{18}) + 2x^9y^9$
$= (x^6 + y^6)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$
$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x^{12} - x^6y^6 + y^{12})$
$+ 2x^9y^9$

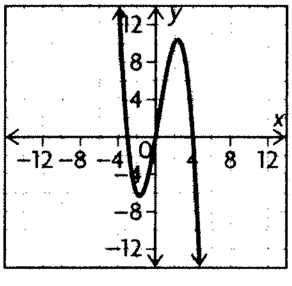
9. Answers may vary. For example, this statement is true because $a^3 - b^3$ is the same as $a^3 + (-b)^3$.

10. a) A taxicab number (TN) is the smallest number that can be expressed as a sum of two positive cubes in n distinct ways.

b) Yes;
$\text{TN}(1) = 2 = 1^3 + 1^3$
$\text{TN}(2) = 1729 = 1^3 + 12^3$
$= 9^3 + 10^3$
$\text{TN}(3) = 87\,539\,319$
$= 167^3 + 436^3$
$= 228^3 + 423^3$
$= 255^3 + 414^3$
$\text{TN}(4) = 6\,963\,472\,309\,248$
$= 2421^3 + 19\,083^3$
$= 5436^3 + 18\,948^3$
$= 10\,200^3 + 18\,072^3$
$= 13\,322^3 + 16\,630^3$
$\text{TN}(5) = 48\,988\,659\,276\,962\,496$
$= 38\,787^3 + 365\,757^3$
$= 107\,839^3 + 362\,753^3$
$= 205\,292^3 + 342\,952^3$
$= 221\,424^3 + 336\,588^3$
$= 231\,518^3 + 331\,954^3$
$\text{TN}(6) = 24\,153\,319\,581\,254\,312\,065\,344$
$= 582\,162^3 + 28\,906\,206^3$
$= 3\,064\,173^3 + 28\,894\,806^3$
$= 8\,519\,281^3 + 28\,657\,487^3$
$= 16\,218\,068^3 + 27\,093\,208^3$
$= 17\,492\,496^3 + 26\,590\,452^3$
$= 18\,289\,922^3 + 26\,224\,366^3$

Chapter Review, pp. 184–185

1.



2. Since it is of odd degree and the leading coefficient is positive, the end behaviour is: as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$.

3. a) There are 2 turning points which means that the degree is $2 + 1$ or 3. Based on the end behaviour of the function, which is as $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$, the leading coefficient is positive.

b) There are 3 turning points which means that the degree is $3 + 1$ or 4. Based on the end behaviour of the function which is $x \to \pm\infty$, $y \to \infty$ the leading coefficient is positive.

4. a) Answers may vary. For example,
$f(x) = (x + 3)(x - 6)(x - 4)$,
$f(x) = 10(x + 3)(x - 6)(x - 4)$, and
$f(x) = -4(x + 3)(x - 6)(x - 4)$.

b) Answers may vary. For example,
$f(x) = (x - 5)(x + 1)(x + 2)$,
$f(x) = -6(x - 5)(x + 1)(x + 2)$, and
$f(x) = 9(x - 5)(x + 1)(x + 2)$.

c) Answers may vary. For example,
$f(x) = (x + 7)(x - 2)(x - 3)$,
$f(x) = \frac{1}{4}(x + 7)(x - 2)(x - 3)$, and
$f(x) = 3(x + 7)(x - 2)(x - 3)$.

d) Answers may vary. For example,
$f(x) = (x - 9)(x + 5)(x + 4)$,
$f(x) = 7(x - 9)(x + 5)(x + 4)$, and
$f(x) = -\frac{1}{3}(x - 9)(x + 5)(x + 4)$.

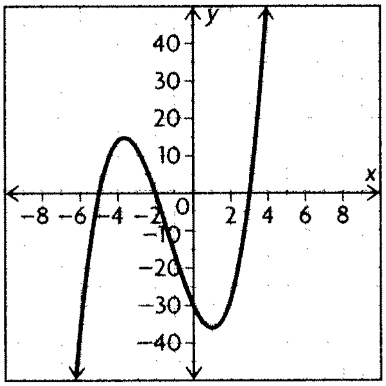
5. a) Answers may vary. For example,
$f(x) = (x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = 2(x + 6)(x - 2)(x - 5)(x - 8)$, and
$f(x) = -8(x + 6)(x - 2)(x - 5)(x - 8)$.

b) Answers may vary. For example,
$f(x) = (x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = \frac{3}{4}(x - 4)(x + 8)(x - 1)(x - 2)$, and
$f(x) = -12(x - 4)(x + 8)(x - 1)(x - 2)$.

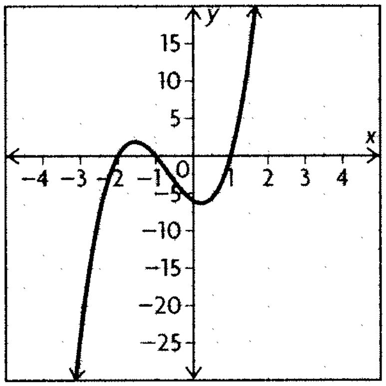
c) Answers may vary. For example,
$f(x) = x(x + 1)(x - 9)(x - 10)$,
$f(x) = 5x(x + 1)(x - 9)(x - 10)$, and
$f(x) = -3x(x + 1)(x - 9)(x - 10)$.

d) Answers may vary. For example,
$f(x) = (x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = \frac{2}{5}(x + 3)(x - 3)(x + 6)(x - 6)$, and
$f(x) = -10(x + 3)(x - 3)(x + 6)(x - 6)$.

6. The zeros are 3, -2, and -5. The function is a cubic with a positive leading coefficient (1).



7. $y = a(x - 1)(x + 1)(x + 2)$
$-6 = a(0 - 1)(0 + 1)(0 + 2)$
$-6 = -2a$
$3 = a$
$y = 3(x - 1)(x + 1)(x + 2)$



8. a) $y = x^2$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 1 unit to the right, and vertically translated 23 units up.

b) $y = x^2$ has been horizontally stretched by a factor of $\frac{13}{12}$, horizontally translated 9 units to the left, and vertically translated 14 units down.

c) $y = x^2$ has been horizontally translated 4 units to the right.

d) $y = x^2$ has been horizontally translated $\frac{3}{7}$ units to the left.

e) $y = x^2$ has been vertically stretched by a factor of 40, reflected in the y-axis, horizontally compressed by a factor of $\frac{1}{7}$, horizontally translated 10 units to the right, and vertically translated 9 units up.

9. a) $y = 25\left(\frac{6}{5}(x - 3)\right)^3$

Answers may vary. For example:

$x = -2$

$y = 25\left(\frac{6}{5}((-2) - 3)\right)^3 = 25(-6)^3 = -5400$

$x = 3$

$y = 25\left(\frac{6}{5}(3 - 3)\right)^3 = 25(0)^3 = 0$

$x = 8$

$y = 25\left(\frac{6}{5}(8 - 3)\right)^3 = 25(6)^3 = 5400$

$(-2, -5400)$, $(3, 0)$, and $(8, 5400)$

b) $y = -\left(\frac{1}{7}x\right)^3 - 19$

Answers may vary. For example:

$x = -7$

$y = -\left(\frac{1}{7}(-7)\right)^3 - 19 = 1 - 19 = -18$

$x = 0$

$y = -\left(\frac{1}{7}(0)\right)^3 - 19 = 0 - 19 = -19$

$x = 7$

$y = -\left(\frac{1}{7}(7)\right)^3 - 19 = -1 - 19 = -20$

$(-7, -18)$, $(0, -19)$, and $(7, -20)$

c) $y = \frac{6}{11}(-(x + 5))^3 + 16$

Answers may vary. For example:

$x = -6$

$y = \frac{6}{11}(-(-6 + 5))^3 + 16 = \frac{6}{11} + 16 = \frac{182}{11}$

$x = -5$

$y = \frac{6}{11}(-(-5 + 5))^3 + 16 = 0 + 16 = 16$

$x = -4$

$y = \frac{6}{11}(-(-4 + 5))^3 + 16 = -\frac{6}{11} + 16 = \frac{170}{11}$

$\left(-6, \frac{182}{11}\right)$, $(-5, 16)$, and $\left(-4, \frac{170}{11}\right)$

d) $y = 100\left(\frac{1}{2}x\right)^3 + 14$

Answers may vary. For example:

$x = -2$

$y = 100\left(\frac{1}{2}(-2)\right)^3 + 14 = -100 + 14 = -86$

$x = 0$

$y = 100\left(\frac{1}{2}(0)\right)^3 + 14 = 0 + 14 = 14$

$x = 2$

$y = 100\left(\frac{1}{2}(2)\right)^3 + 14 = 100 + 14 = 114$

$(-2, -86)$, $(0, 14)$, and $(2, 114)$

e) $y = -(x)^3 - 45$

Answers may vary. For example:

$x = -1$

$y = -(-1)^3 - 45 = 1 - 45 = -44$

$x = 0$

$y = -(0)^3 - 45 = 0 - 45 = -45$

$x = 1$

$y = -(1)^3 - 45 = -1 - 45 = -46$

$(-1, -44)$, $(0, -45)$, and $(1, -46)$

f) $y = \left(-\frac{10}{7}(x - 12)\right)^3 + 6$

Answers may vary. For example:

$x = 5$

$y = \left(-\frac{10}{7}(5 - 12)\right)^3 + 6 = 1000 + 6$

$x = 12$

$y = \left(-\frac{10}{7}(12 - 12)\right)^3 + 6 = 0 + 6 = 6$

$x = 19$

$y = \left(-\frac{10}{7}(19 - 12)\right)^3 + 6 = -1000 + 6 = -994$

$(5, 1006)$, $(12, 6)$, and $(19, -994)$

10. a)

$$\begin{array}{rl}
 & \;2x^2 - 5x + 28 \\
 & x + 5\overline{)2x^3 + 5x^2 + 3x - 4} \\
2x^2(x + 5) \rightarrow & \underline{2x^3 + 10x^2} \;\downarrow \;\; \downarrow \\
 & \qquad -5x^2 + \;\; 3x \\
-5x(x + 5) \rightarrow & \qquad \underline{-5x^2 - 25x} \\
 & \qquad\qquad 28x - \;\;\; 4 \\
28(x + 5) \rightarrow & \qquad\qquad \underline{28x + 140} \\
 & \qquad\qquad\qquad -144
\end{array}$$

$2x^2 - 5x + 28$ remainder -144

b)

$$\begin{array}{rl}
 & \; x^2 + 4x + 5 \\
 & x^2 - 8\big)\overline{x^4 + 4x^3 - 3x^2 - 6x - 7} \\
x^2(x^2 - 8) \rightarrow & \underline{x^4 \qquad - 8x^2} \quad \downarrow \quad \downarrow \\
 & 4x^3 + 5x^2 - 6x \\
4x(x^2 - 8) \rightarrow & \underline{4x^3 \qquad - 32x} \\
 & 5x^2 + 26x - 7 \\
5(x^2 - 8) \rightarrow & \underline{5x^2 \qquad - 40} \\
 & 26x + 33
\end{array}$$

$x^2 + 4x + 5$ remainder $26x + 33$

c)

$$\begin{array}{rl}
 & \; 2x - 6 \\
 & x^3 + 3x^2 + 3x - 3\big)\overline{2x^4 + 0x^3 - 2x^2 + 3x - 16} \\
2x(x^3 + 3x^2 + 3x - 3) \rightarrow & \underline{2x^4 + 6x^3 + 6x^2 - 6x} \quad \downarrow \\
 & -6x^3 - 8x^2 + 9x - 16 \\
-6(x^3 + 3x^2 + 3x - 3) \rightarrow & \underline{-6x^3 - 18x^2 - 18x + 18} \\
 & 10x^2 + 27x - 34
\end{array}$$

$2x - 6$ remainder $10x^2 + 27x - 34$

d)

$$\begin{array}{rl}
 & \; x - 4 \\
 & x^4 + 4x^3 + 4x^2 - x - 3\big)\overline{x^5 + 0x^4 - 8x^3 + 0x^2 - 7x - 6} \\
x(x^4 + 4x^3 + 4x^2 - x - 3) \rightarrow & \underline{x^5 + 4x^4 + 4x^3 - x^2 - 3x} \quad \downarrow \\
 & -4x^4 - 12x^3 + x^2 - 4x - 6 \\
-4(x^4 + 4x^3 + 4x^2 - x - 3) \rightarrow & \underline{-4x^4 - 16x^3 - 16x^2 + 4x + 12} \\
 & 4x^3 + 17x^2 - 8x - 18
\end{array}$$

$x - 4$ remainder $4x^3 + 17x^2 - 8x - 18$

11. a)

$$\begin{array}{r|rrrr}
-2 & 2 & 5 & -1 & -5 \\
 & \downarrow & -4 & -2 & 6 \\
\hline
 & 2 & 1 & -3 & 1
\end{array}$$

$(x + 2)(2x^2 + x - 3)$ remainder 1

b)

$$\begin{array}{r|rrrr}
-2 & 3 & 13 & 17 & 3 \\
 & \downarrow & -6 & -14 & -6 \\
\hline
 & 3 & 7 & 3 & -3
\end{array}$$

$(x + 2)(3x^2 + 7x + 3)$ remainder -3

c)

$$\begin{array}{r|rrrrr}
-2 & 2 & 5 & -16 & -45 & -18 \\
 & \downarrow & -4 & -2 & 36 & 18 \\
\hline
 & 2 & 1 & -18 & -9 & 0
\end{array}$$

$(x + 2)(2x^3 + x^2 - 18x - 9)$ remainder 0

d)

$$\begin{array}{r|rrrr}
-2 & 2 & 4 & -5 & -4 \\
 & \downarrow & -4 & 0 & 10 \\
\hline
 & 2 & 0 & -5 & 6
\end{array}$$

$(x + 2)(2x^2 - 5)$ remainder 6

12. a) Divisor: $x - 9$; Quotient: $2x^2 + 11x - 8$; Remainder: 3

$= (x - 9)(2x^2 + 11x - 8) + 3$
$= 2x^3 + 11x^2 - 8x - 18x^2 - 99x + 72 + 3$
$= 2x^3 - 7x^2 - 107x + 75$

b) Divisor: $4x + 3$; Quotient: $x^3 - 2x + 7$; Remainder: -4

$= (4x + 3)(x^3 - 2x + 7) - 4$
$= 4x^4 - 8x^2 + 28x + 3x^3 - 6x + 21 - 4$
$= 4x^4 + 3x^3 - 8x^2 + 22x + 17$

c) Divisor: $3x - 4$; Quotient: $x^3 + 6x^2 - 6x - 7$; Remainder: 5

$= (3x - 4)(x^3 + 6x^2 - 6x - 7) + 5$
$= 3x^4 + 18x^3 - 18x^2 - 21x - 4x^3 - 24x^2 + 24x + 28 + 5$
$= 3x^4 + 14x^3 - 42x^2 + 3x + 33$

d) Divisor: $3x^2 + x - 5$;
Quotient: $x^4 - 4x^3 + 9x - 3$; Remainder: $2x - 1$

$= (3x^2 + x - 5)(x^4 - 4x^3 + 9x - 3) + (2x - 1)$
$= (3x^6 - 12x^5 + 27x^3 - 9x^2 + x^5 - 4x^4 + 9x^2 - 3x - 5x^4 + 20x^3 - 45x + 15) + (2x - 1)$
$= 3x^6 - 11x^5 - 9x^4 + 47x^3 - 46x + 14$

13. $f(-2) = (-2)^3 + 2(-2)^2 - 6(-2) + 1$
$= -8 + 8 + 12 + 1$
$= 13$

14. a) $x^3 - 5x^2 - 22x - 16$

$f(-1) = (-1)^3 - 5(-1)^2 - 22(-1) - 16$
$= -1 - 5 + 22 - 16$
$= 0$

$x + 1$ is a factor.

$$\begin{array}{r|rrrr} -1 & 1 & -5 & -22 & -16 \\ & \downarrow & -1 & 6 & 16 \\ \hline & 1 & -6 & -16 & 0 \end{array}$$

$= (x + 1)(x^2 - 6x - 16)$
$= (x + 1)(x - 8)(x + 2)$

b) $2x^3 + x^2 - 27x - 36$

$f(4) = 2(4)^3 + (4)^2 - 27(4) - 36$
$= 128 + 16 - 108 - 36$
$= 0$

$x - 4$ is a factor.

$$\begin{array}{r|rrrr} 4 & 2 & 1 & -27 & -36 \\ & \downarrow & 8 & 36 & 36 \\ \hline & 2 & 9 & 9 & 0 \end{array}$$

$= (x - 4)(2x^2 + 9x + 9)$
$= (x - 4)(2x^2 + 6x + 3x + 9)$
$= (x - 4)(2x(x + 3) + 3(x + 3))$
$= (x - 4)(2x + 3)(x + 3)$

c) $3x^4 - 19x^3 + 38x^2 - 24x$

$x(3x^3 - 19x^2 + 38x - 24)$

For $3x^3 - 19x^2 + 38x - 24$:

$f(2) = 3(2)^3 - 19(2)^2 + 38(2) - 24$
$= 24 + 76 + 76 - 24$
$= 0$

$x - 2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 3 & -19 & 38 & -24 \\ & \downarrow & 6 & -26 & 24 \\ \hline & 3 & -13 & 12 & 0 \end{array}$$

$= x(x - 2)(3x^2 - 13x + 12)$
$= x(x - 2)(3x^2 - 9x - 4x + 12)$
$= x(x - 2)(3x(x - 3) - 4(x - 3))$
$= x(x - 2)(x - 3)(3x - 4)$

d) $x^4 + 11x^3 + 36x^2 + 16x - 64$

$f(1) = 1^4 + 11(1)^3 + 36(1)^2 + 16(1) - 64$
$= 1 + 11 + 36 + 16 - 64$
$= 0$

$x - 1$ is a factor.

$$\begin{array}{r|rrrrr} 1 & 1 & 11 & 36 & 16 & -64 \\ & \downarrow & 1 & 12 & 48 & 64 \\ \hline & 1 & 12 & 48 & 64 & 0 \end{array}$$

$= (x - 1)(x^3 + 12x^2 + 48x + 64)$

For $x^3 + 12x^2 + 48x + 64$:

$f(-4) = (-4)^3 + 12(-4)^2 + 48(-4) + 64$
$= -64 + 192 - 192 + 64$
$= 0$

$$\begin{array}{r|rrrr} -4 & 1 & 12 & 48 & 64 \\ & \downarrow & -4 & -32 & -64 \\ \hline & 1 & 8 & 16 & 0 \end{array}$$

$= (x - 1)(x + 4)(x^2 + 8x + 16)$
$= (x - 1)(x + 4)(x + 4)(x + 4)$

15. a) $8x^3 - 10x^2 - 17x + 10$

$f(2) = 8(2)^3 - 10(2)^2 - 17(2) + 10$
$= 64 - 40 - 34 + 10$
$= 0$

$x - 2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 8 & -10 & -17 & 10 \\ & \downarrow & 16 & 12 & -10 \\ \hline & 8 & 6 & -5 & 0 \end{array}$$

$= (x - 2)(8x^2 + 6x - 5)$
$= (x - 2)(8x^2 + 10x - 4x - 5)$
$= (x - 2)(2x(4x + 5) - 1(4x + 5))$
$= (x - 2)(4x + 5)(2x - 1)$

b) $2x^3 + 7x^2 - 7x - 30$

$f(2) = 2(2)^3 + 7(2)^2 - 7(2) - 30$
$= 16 + 28 - 14 - 30$
$= 0$

$x - 2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 2 & 7 & -7 & -30 \\ & \downarrow & 4 & 22 & 30 \\ \hline & 2 & 11 & 15 & 0 \end{array}$$

$= (x - 2)(2x^2 + 11x + 15)$
$= (x - 2)(2x^2 + 6x + 5x + 15)$
$= (x - 2)(2x(x + 3) + 5(x + 3))$
$= (2x + 5)(x - 2)(x + 3)$

c) $x^4 - 7x^3 + 9x^2 + 27x - 54$

$f(3) = 3^4 - 7(3)^3 + 9(3)^2 + 27(3) - 54$
$= 81 - 189 + 81 + 81 - 54$
$= 0$

$x - 3$ is a factor.

$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 9 & 27 & -54 \\ & \downarrow & 3 & -12 & -9 & 54 \\ \hline & 1 & -4 & -3 & 18 & 0 \end{array}$$

$= (x - 3)(x^3 - 4x^2 - 3x + 18)$

For $x^3 - 4x^2 - 3x + 18$:

$f(-2) = (-2)^3 - 4(-2)^2 - 3(-2) + 18$
$= -8 - 16 + 6 + 18$
$= 0$

$x + 2$ is a factor.

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ & \downarrow & -2 & 12 & -18 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$= (x - 3)(x + 2)(x^2 - 6x + 9)$
$= (x - 3)(x - 3)(x - 3)(x + 2)$

d) $4x^4 + 4x^3 - 35x^2 - 36x - 9$

$f(3) = 4(3)^4 + 4(3)^3 - 35(3)^2 - 36(3) - 9$
$= 324 + 108 - 315 - 108 - 9$
$= 0$

$x - 3$ is a factor.

$$\begin{array}{r|rrrrr} 3 & 4 & 4 & -35 & -36 & -9 \\ & \downarrow & 12 & 48 & 39 & 9 \\ \hline & 4 & 16 & 13 & 3 & 0 \end{array}$$

$= (x - 3)(4x^3 + 16x^2 + 13x + 3)$

For $4x^3 + 16x^2 + 13x + 3$:

$f(-3) = 4(-3)^3 + 16(-3)^2 + 13(-3) + 3$
$= -108 + 144 - 39 + 3$
$= 0$

$x + 3$ is a factor.

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 13 & 3 \\ & \downarrow & -12 & -12 & -3 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$= (x - 3)(x + 3)(4x^2 + 4x + 1)$
$= (x - 3)(x + 3)(4x^2 + 2x + 2x + 1)$
$= (x - 3)(x + 3)(2x(2x + 1) + 1(2x + 1))$
$= (2x + 1)(2x + 1)(x - 3)(x + 3)$

16. a) $64x^3 - 27$
$= (4x)^3 - (3)^3$
$= (4x - 3)(16x^2 + 12x + 9)$

b) $512x^3 - 125$
$= (8x)^3 - (5)^3$
$= (8x - 5)(64x^2 + 40x + 25)$

c) $343x^3 - 1728$
$= (7x)^3 - (12)^3$
$= (7x - 12)(49x^2 + 84x + 144)$

d) $1331x^3 - 1$
$= (11x)^3 - (1)^3$
$= (11x - 1)(121x^2 + 11x + 1)$

17. a) $1000x^3 + 343$
$= (10x)^3 + (7)^3$
$= (10x + 7)(100x^2 - 70x + 49)$

b) $1728x^3 + 125$
$= (12x)^3 + (5)^3$
$= (12x + 5)(144x^2 - 60x + 25)$

c) $27x^3 + 1331$
$= (3x)^3 + (11)^3$
$= (3x + 11)(9x^2 - 33x + 121)$

d) $216x^3 + 2197$
$= (6x)^3 + (13)^3$
$= (6x + 13)(36x^2 - 78x + 169)$

18. a) $(x^6 - y^6) = (x^3 - y^3)(x^3 + y^3)$
$= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$

b) $(x^6 - y^6) = (x^2 - y^2)(x^4 + x^2y^2 + y^4)$
$= (x - y)(x + y)(x^4 + x^2y^2 + y^4)$

c) Both methods produce factors of $(x - y)$ and $(x + y)$; however, the other factors are different. Since the two factorizations must be equal to each other, this means that $(x^4 + x^2y^2 + y^4)$ must be equal to $(x^2 + xy + y^2)(x^2 - xy + y^2)$.

Chapter Self-Test, p. 186

1. a) $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number. The degree of the function is n; the leading coefficient is a_n.

$y = x^4 - 2x^3 + x^2 - 2x + 8$.

The degree of this function is 4, and the leading coefficient is 1.

b) The maximum number of turning points is $n - 1$.

c) The function may have at most n zeros (the same as the degree).

d) If the least number of zeros is one, it is an odd degree function.

e) The function is an even degree function with a negative leading coefficient.

2. $y = a(x + 4)(x + 2)(x - 2)$
$-16 = a(0 + 4)(0 + 2)(0 - 2)$
$-16 = a(4)(2)(-2)$
$-16 = -16a$
$1 = a$
$y = (x + 4)(x + 2)(x - 2)$

3. a) $2x^3 - x^2 - 145x - 72$

$f(9) = 2(9)^3 - (9)^2 - 145(9) - 72$
$= 1458 - 81 - 1305 - 72$
$= 0$

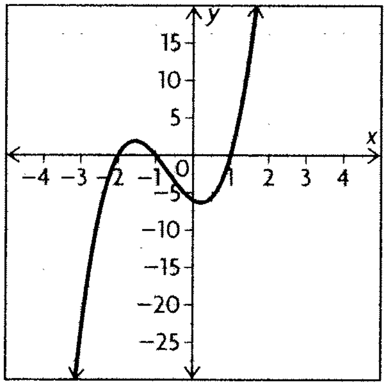
$x - 9$ is a factor.

$$\begin{array}{r|rrrr} 9 & 2 & -1 & -145 & -72 \\ & \downarrow & 18 & 153 & 72 \\ \hline & 2 & 17 & 8 & 0 \end{array}$$

$= (x - 9)(2x^2 + 17x + 8)$
$= (x - 9)(2x^2 + 16x + 1x + 8)$
$= (x - 9)(2x(x + 8) + 1(x + 8))$
$= (x - 9)(x + 8)(2x - 1)$

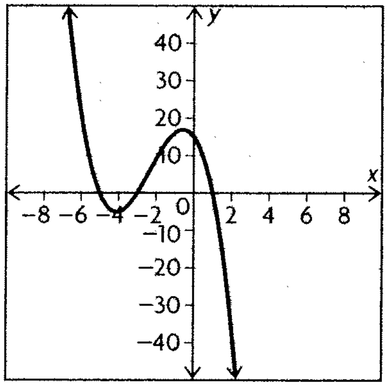
b) $(x - 7)^3 + (2x + 3)^3$
$= (x - 7 + 2x + 3)((x - 7)^2 - (x - 7)$
$(2x + 3) + (2x + 3)^2)$
$= (3x - 4)((x^2 - 14x + 49) -$
$(2x^2 - 11x - 21) + (4x^2 + 12x + 9))$
$= (3x - 4)(3x^2 + 9x + 79)$

4.



By shifting the original function up, the new function has more zeros.

5.



x is below the x-axis in the intervals of $-5 < x < -3$; $x > 1$.

6.
$$\begin{array}{c|cccc} \frac{1}{2} & 6 & 1 & -12 & 5 \\ & \downarrow & 3 & 2 & -5 \\ \hline & 6 & 4 & -10 & 0 \end{array}$$

$2x - 1$ is a factor because there is no remainder.

7. a) $y = 5(2(x - 2))^3 + 4$

b) $\left(\frac{x}{k} + d, ay + c\right)$

$= \left(\frac{1}{2} + 2, (5)(1) + 4\right) = (2.5, 9)$

8.
$$\begin{array}{rl} & x + 5 \\ x^3 - 2x^2 + x - 5 & \overline{)x^4 + 3x^3 - 9x^2 + 0x + 6} \\ x(x^3 - 2x^2 + x - 5) \rightarrow & \underline{x^4 - 2x^3 - x^2 - 5x \quad \downarrow} \\ & 5x^3 - 10x^2 + 5x + 6 \\ 5(x^3 - 2x^2 + x - 5) \rightarrow & \underline{5x^3 - 10x^2 + 5x - 25} \\ & 31 \end{array}$$

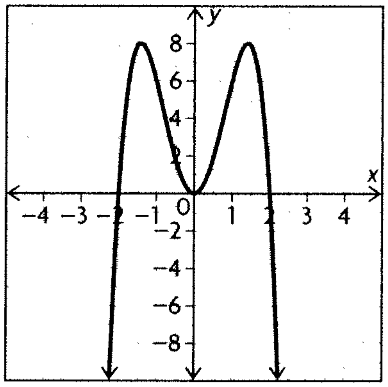
Julie divided by $x + 5$.

9. To determine a, use synthetic division and work backwards.

$$\begin{array}{c|ccccc} 2 & a & 0 & 8 & 0 & 0 \\ & \downarrow & -4 & -8 & 0 & 0 \\ \hline & -2 & -4 & 0 & 0 & 0 \end{array}$$

$a = -2$
$f(x) = -2x^4 + 8x^2$
$= -2x^2(x^2 - 4)$
$= -2x^2(x + 2)(x - 2)$

The zeros are located at 0, -2, and 2.



Chapters 1–3 Cumulative Review, pp. 188–191

1. The domain is all real numbers except 5, since 5 would make the denominator 0.
So, the domain is $\{x \in \mathbf{R} \mid x \neq 5\}$.
The correct answer is **b)**.

2. $x = -y^2$ is not a function of x because the points $(-1, 1)$ and $(-1, -1)$ are both in the relation.
The correct answer is **a)**.

3. The parent graph is $f(x) = |x|$. This function is horizontally compressed by a factor of $\frac{1}{2}$ and then translated 2 units to the right.
The new function after the transformations is $f(x) = |2(x - 2)|$ or $|2x - 4|$.
The correct answer is **c)**.

4. $f(-x) = (-x - 2)(-x + 2)$
$$= -1(x + 2)(-1)(x - 2)$$
$$= (x + 2)(x - 2)$$
$f(-x) = f(x)$, so the function is even.
The correct answer is **b)**.
5. The function was horizontally stretched by a factor of 3 and then translated 2 units to the right.
The correct answer is **b)**.
6. The parent function is $g(x) = \frac{1}{x}$
So, the correct answer is **d)**.
7. The horizontal stretch transforms the function to $2^{\frac{1}{5}x}$, and the translation 3 units down transforms the function to $2^{\frac{1}{5}x} - 3$.
The correct answer is **d)**.
8. $y = 2x^2 + 5$
Determine the inverse.
$$x = 2y^2 + 5$$
$$x - 5 = 2y^2$$
$$\frac{x - 5}{2} = y^2$$
$$\pm\sqrt{\frac{x - 5}{2}} = y$$
So, the domain is $\{x \in \mathbf{R} | x \geq 5\}$
The correct answer is **a)**.
9. $f(x) = 2x^2 - 4$
$$y = 2x^2 - 4$$
$$x = 2y^2 - 4$$
$$x + 4 = 2y^2$$
$$\frac{x + 4}{2} = y^2$$
$$\pm\sqrt{\frac{x + 4}{2}} = y$$
The correct answer is **c)**.
10. Look at the endpoints to determine which function is continuous.
For the first function, there is an open circle at (1, 2) and a closed circle at (1, 2).
For the second function, there is an closed circle at (2, 0) and a open circle at (2, 0).
For the third function, there is an open circle at (−1, −1) and a closed circle at (−1, −1).
For the fourth function, there is an open circle at (2, 5) and a closed circle at (2, 4).
So, the fourth function is not continuous.
The correct answer is **d)**.
11. $f(-1) = (-1)^3 - 2(-1)^2 + 7$
$$= -1 - 2 + 7$$
$$= 4$$
So, the ordered pair is (−1, 4).
$f(3) = (3)^3 - 2(3)^2 + 7$
$$= 27 - 18 + 7$$
$$= 16$$
The ordered pair is (3, 16).
So, the rate of change is
$(16 - 4) \div (3 - (-1)) = 12 \div 4$ or 3.
The correct answer is **a)**.
12. Kristin's rate of change: $(5 - 0.1) \div 3 \doteq 1.63$
Husain's rate of change: $(15 - 0.1) \div 10 = 1.49$
Kristin's grew faster.
The correct answer is **a)**.
13. rate of change:
$(27.015\,002 - 27) \div 0.001 = 15.002$
This is the average rate of change over 0.001 s, so the best estimate for the instantaneous rate of change is 15 m/s.
The correct answer is **c)**.
14. $f(-1) = 2^{-1} - 2(-1) + 1$
$$= 0.5 + 2 + 1$$
$$= 3.5$$
$f(-1.0001) = 2^{-1.0001} - 2(-1.0001) + 1$
$$\doteq 3.500\,165$$
The rate of change is
$(3.500\,165 - 3.5) \div -0.0001 = -1.65$.
The correct answer is **d)**.
15. Draw the tangent line and find the slope of the line. The slope appears to be about −2.
The correct answer is **c)**.
16. The slope of the first part should be greater than the slope of the second part. The slope of the last part should be the greatest. None of the slopes should be 0.
The correct answer is **c)**.
17. $\frac{-b}{2a} = \frac{-13}{2(-1.3)}$
$$= \frac{-13}{-2.6}$$
$$= 5$$
The coefficient of x^2 is negative, so the graph opens downward. The point is a maximum.
The correct answer is **a)**.

18. $\frac{f(3 + h) - f(3)}{h}$

$= \frac{2(3 + h)^2 - 3(3 + h) + 9 - (2(3)^2 - 3(3) + 9)}{h}$

$= \frac{2((3)^2 + 2(3)h + h^2) - 3(3) - 3h + 9 - 2(3)^2 + 3(3) - 9}{h}$

$= \frac{2(9 + 6h + h^2) - 9 - 3h + 9 - 18 + 9 - 9}{h}$

$= \frac{12h + 2h^2 - 3h}{h}$

$= 9 + 2h$

The correct answer is **d)**.

19. The maximum will be at $y = 35(1.7)^8$ or 2 441.5.
The correct answer is **b)**.

20. The function $f(x) = 2^x - 3$ is not a polynomial function.
The correct answer is **c)**.

21. A cubic function cannot be represented by the graph because the graph does not show the same end behaviour as a cubic, for which the end behaviours are opposite.
The correct answer is **b)**.

22. In a cubic function, as $x \rightarrow \pm\infty$, the signs of y are opposite.
The correct answer is **b)**.

23. The function shows a cubic function that has been translated 3 units to the right, so the best match is choice **b)**.

24. $f(x) = a(x + 2)x(x - 1)(x - 3)$

$16 = a(2 + 2)(2)(2 - 1)(2 - 3)$

$16 = a(4)(2)(1)(-1)$

$16 = -8a$

$-2 = a$

So, the equation is $-2(x + 2)x(x - 1)(x - 3)$.

$-2(x + 2)x(x - 1)(x - 3)$

$= -2(x^2 + 2x)(x^2 - 4x + 3)$

$= -2(x^4 - 4x^3 + 3x^2 + 2x^3 - 8x^2 + 6x)$

$= -2x^4 + 4x^3 + 10x^2 - 12x$

The correct answer is **a)**.

25. Since the function is stretched horizontally by a factor of 2, $y = \left(\frac{1}{2}x\right)^3$. Since the function is translated 3 units to the right, the new function is $y = \left(\frac{1}{2}(x - 3)\right)^3$. The correct answer is **c)**.

26.

$$\begin{array}{rl}
 & \underline{\qquad\qquad\qquad\quad x + 1} \\
x^2 - 3x + 4 & \overline{)x^3 - 2x^2 + 7x + 12} \\
x(x^2 - 3x + 4) \rightarrow & \underline{x^3 - 3x^2 + 4x} \quad \downarrow \\
 & \qquad x^2 + 3x + 12 \\
1(x^2 - 3x + 4) \rightarrow & \qquad \underline{x^2 - 3x + \ \ 4} \\
 & \qquad\qquad 6x + \ \ 8
\end{array}$$

The remainder is $6x + 8$.
The correct answer is **c)**.

27. By the remainder theorem, when $x^4 - 5x^2 + 12x + 16$ is divided by $x + 3$ or $x - (-3)$, the remainder is the value of the polynomial evaluated at -3.

$(-3)^4 - 5(-3)^2 + 12(-3) + 16$

$= 81 - 5(9) - 36 + 16$

$= 81 - 45 - 36 + 16$

$= 16$

The remainder is 16.
The correct answer is **d)**.

28. Because $x - 3$ is a factor of the polynomial, there is no remainder when the polynomial is divided by $x - 3$.
Show the synthetic division and fill in what is known. Work backward from the 0 remainder.

$$\begin{array}{r|rrrr}
3 & 2 & k & -3 & 18 \\
 & \downarrow & 6 & -3 & -18 \\
\hline
 & 2 & -1 & -6 & 0
\end{array}$$

$k + 6 = -1$

$k = -7$

The correct answer is **b)**.

29. $27x^3 - 216 = 27(x^3 - 8)$

$= 27(x - 2)(x^2 + 2x + 4)$

The correct answer is **c)**.

30. $(x + 3)^3 + 8 = (x + 3)^3 + 2^3$

$= (x + 3 + 2)((x + 3)^2 - 2(x + 3) + 4)$

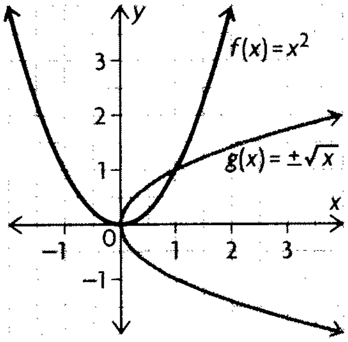
$= (x + 5)(x^2 + 6x + 9 - 2x - 6 + 4)$

$= (x + 5)(x^2 + 4x + 7)$

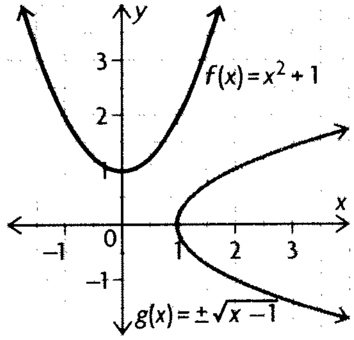
The correct answer is **c)**.

31. The graph pictured in choice c represents the height of the water. The height will change slowly at first, then speed up, then become constant.
The correct answer is **c)**.

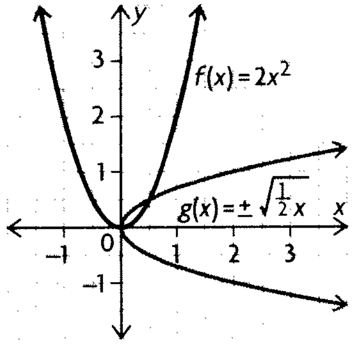
32. a)



b) Answers may vary. For example, vertical translation up produces horizontal translation of the inverse to the right.



Vertical stretch produces horizontal stretch of inverse.



c) Answers may vary. For example, if the vertex of the inverse is (a, b), restrict the value of y to either $y \geq b$ or $y \leq b$.

33. Answers may vary. For example, average rates of change vary between -2 and 4, depending on the interval; instantaneous rates of change are 9 at (0, 1), 0 at (1, 5), -3 at (2, 3), 0 at (3, 1), 9 at (4, 5); instantaneous rate of change is 0 at maximum (1, 5) and at minimum (3, 1).

34. a) $-24 = k(1 + 1)^2(1 - 2)(1 - 4)$

$$-24 = k(2)^2(-1)(-3)$$
$$-24 = 12k$$
$$-2 = k$$

So, the equation is

$f(x) = -2(x + 1)^2(x - 2)(x - 4)$.

b) $(3, p)$ is a point on the graph of $f(x)$

$$f(x) = -2(x + 1)^2(x - 2)(x - 4)$$
$$p = -2(4)^2(1)(-1)$$
$$p = 32$$

(3, 32) is a point on the graph of $f(x)$.

c) Quartic with the same end behaviours; $k < 0$, so as $x \rightarrow \pm\infty$, $f(x) \rightarrow -\infty$; from the given factors, the zeros are -1, 2, and 4

d) $f(0) = -2(0 + 1)^2(0 - 2)(0 - 4)$

$$= -2(1)^2(-2)(-4)$$
$$= -16$$

e)