Chapter 5
Rational Functions, Equations, and Inequalities

Getting Started, pp. 246–247

1. Examine the coefficient for each term. To factor each expression, you will need two factors of the coefficient of the third term whose sum is the coefficient of the second term. If the coefficient of the first term is not 1, then you will have to consider the factors of that coefficient too. If the expression doesn't factor easily you can use the quadratic formula.

a) $x^2 - 3x - 10$
$-5 \times 2 = -10; -5 + 2 = -3$
$(x - 5)(x + 2)$

b) $3x^2 + 12x - 15$
$3(x^2 + 4x - 5)$
$5 \times -1 = -5; 5 + -1 = 4$
$3(x + 5)(x - 1)$

c) $16x^2 - 49$
Notice that this is the difference of two squares. This means that the factored form of the expression will be $(4x - 7)(4x + 7)$.

d) $9x^2 - 12x + 4$
Notice that the coefficients of the first and third terms are squares. Since the coefficient of the second term is negative, the factors of the coefficient of the third term will also probably be negative. The factored form of the expression is $(3x - 2)(3x - 2)$.

e) $3a^2 + a - 30$
This is a trinomial of the form $ax^2 + bx + c$ where $a \neq 0$, and it has no common factor. The expression can be factored using decomposition by finding two numbers whose sum is 1 and whose product is $(3)(-30) = -90$. The numbers are -9 and 10. These numbers are used to decompose the middle term.
$3a^2 - 9a + 10a - 30$
$= 3a(a - 3) + 10(a - 3)$
$= (a - 3)(3a + 10)$

f) $6x^2 - 5xy - 21y^2$
This trinomial can again be factored by decomposition. Find two numbers whose sum is -5 and whose product is $(6)(-21) = -126$. The numbers are -14 and 9. These numbers are used to decompose the middle term.
$6x^2 - 14xy + 9xy - 21y^2$
$= 2x(3x - 7y) + 3y(3x - 7y)$
$= (2x + 3y)(3x - 7y)$

2. a) $\dfrac{12 - 8s}{4}$
$= \dfrac{4(3 - 2s)}{4}$
$= 3 - 2s$

b) $\dfrac{6m^2n^4}{18m^3n}$
$= \dfrac{6m^2n(n)^3}{6m^2n(3m)}$
$= \dfrac{n^3}{3m}$, m and $n \neq 0$

c) $\dfrac{9x^3 - 12x^2 - 3x}{3x}$
$= \dfrac{3x(3x^2 - 4x - 1)}{3x}$
$= 3x^2 - 4x - 1$
$x \neq 0$

d) $\dfrac{25x - 10}{5(5x - 2)^2}$
$= \dfrac{5(5x - 2)}{5(5x - 2)(5x - 2)}$
$= \dfrac{1}{5x - 2}$
$x \neq \dfrac{2}{5}$

e) $\dfrac{x^2 + 3x - 18}{9 - x^2}$
$= \dfrac{(x - 3)(x + 6)}{(3 - x)(3 + x)}$
$= \dfrac{-(3 - x)(x + 6)}{(3 - x)(3 + x)}$
$= -\dfrac{x + 6}{3 + x}, x \neq -3, 3$

f) $\dfrac{a^2 + 4ab - 5b^2}{2a^2 + 7ab - 15b^2}$

$= \dfrac{(a + 5b)(a - 1b)}{(2a - 3b)(a + 5b)}$

$= \dfrac{a - b}{a - 3b}, a \neq -5b, \dfrac{3b}{2}$

3. a) $\dfrac{3}{5} \times \dfrac{7}{9}$

$= \dfrac{1}{5} \times \dfrac{7}{3}$

$= \dfrac{7}{15}$

b) $\dfrac{2x}{5} \div \dfrac{x^2}{15}$

$= \dfrac{2x}{5} \times \dfrac{15}{x^2}$

$= \dfrac{2x}{1} \times \dfrac{3}{x^2}$

$= \dfrac{6x}{x^2}$

$= \dfrac{6}{x}$

$x \neq 0$

c) $\dfrac{x^2 - 4}{x - 3} \div \dfrac{x + 2}{12 - 4x}$

$= \dfrac{(x - 2)(x + 2)}{x - 3} \times \dfrac{4(3 - x)}{(x + 2)}$

$= \dfrac{(x - 2)}{x - 3} \times \dfrac{4(3 - x)}{1}$

$= \dfrac{4(3 - x)(x - 2)}{x - 3}$

$= \dfrac{-4x^2 + 20x - 6}{x - 3}$

$x \neq -2, 3$

d) $\dfrac{x^3 + 4x^2}{x^2 - 1} \times \dfrac{x^2 - 5x + 6}{x^2 - 3x}$

$= \dfrac{x^2(x + 4)}{(x - 1)(x + 1)} \times \dfrac{(x - 3)(x - 2)}{x(x - 3)}$

$= \dfrac{x^2(x + 4)}{(x - 1)(x + 1)} \times \dfrac{(x - 2)}{x}$

$= \dfrac{x^2(x + 4)(x - 2)}{x(x - 1)(x + 1)}$

$= \dfrac{x(x + 4)(x - 2)}{(x - 1)(x + 1)}$

$= \dfrac{x^3 + 2x - 8x}{x^2 - 1}, x \neq -1, 0, 1, 3$

4. a) $\dfrac{2}{3} + \dfrac{6}{7}$

$= \dfrac{7}{7} \times \dfrac{2}{3} + \dfrac{3}{3} \times \dfrac{6}{7}$

$= \dfrac{14}{21} + \dfrac{18}{21}$

$= \dfrac{32}{21}$

$= 1\dfrac{11}{21}$

b) $\dfrac{3x}{4} + \dfrac{5x}{6}$

$= \left(\dfrac{3}{3}\right)\left(\dfrac{3x}{4}\right) + \left(\dfrac{2}{2}\right)\left(\dfrac{5x}{6}\right)$

$= \dfrac{9x}{12} + \dfrac{10x}{12}$

$= \dfrac{19x}{12}$

c) $\dfrac{1}{x} + \dfrac{4}{x^2}$

$= \dfrac{x}{x} \times \dfrac{1}{x} + \dfrac{4}{x^2}$

$= \dfrac{x}{x^2} + \dfrac{4}{x^2}$

$= \dfrac{4 + x}{x^2}$

$x \neq 0$

d) $\dfrac{5}{x - 3} - \dfrac{2}{x}$

$= \dfrac{x}{x} \times \dfrac{5}{x - 3} - \dfrac{x - 3}{x - 3} \times \dfrac{2}{x}$

$= \dfrac{5x}{x^2 - 3x} - \dfrac{2x - 6}{x^2 - 3x}$

$= \dfrac{3x - 6}{x^2 - 3x}$

$x \neq 0, 3$

e) $\dfrac{2}{x - 5} + \dfrac{y}{x^2 - 25}$

$= \dfrac{2}{x - 5} + \dfrac{y}{(x - 5)(x + 5)}$

$= \left(\dfrac{x + 5}{x + 5}\right)\left(\dfrac{2}{x - 5}\right) + \dfrac{y}{(x - 5)(x + 5)}$

$= \dfrac{2x + 10}{(x + 5)(x - 5)} + \dfrac{y}{(x - 5)(x + 5)}$

$= \dfrac{2x + 10 + y}{(x + 5)(x - 5)}$

$$= \frac{2x + 10 + y}{(x^2 - 25)}$$

$x \neq 5, -5$

f) $$\frac{6}{a^2 - 9a + 20} - \frac{8}{a^2 - 2a - 15}$$

$$= \frac{6}{(a-5)(a-4)} - \frac{8}{(a-5)(a+3)}$$

$$= \left(\frac{a+3}{a+3}\right)\left(\frac{6}{(a-5)(a-4)}\right) - \left(\frac{a-4}{a-4}\right)\left(\frac{8}{(a-5)(a+3)}\right)$$

$$= \frac{6a + 18}{(a+3)(a-5)(a-4)} - \frac{8a - 32}{(a+3)(a-5)(a+3)}$$

$$= \frac{-2a + 50}{(a+3)(a-5)(a+3)}$$

$x \neq -3, 4, 5$

5. a) $$\frac{5x}{8} = \frac{15}{4}$$

$$4(5x) = 8(15)$$
$$20x = 120$$
$$\frac{20x}{20} = \frac{120}{20}$$
$$x = 6$$

b) $$\frac{x}{4} + \frac{1}{3} = \frac{5}{6}$$

$$\left(\frac{3}{3}\right)\left(\frac{x}{4}\right) + \left(\frac{4}{4}\right)\left(\frac{1}{3}\right) = \frac{5}{6}$$
$$\frac{3x}{12} + \frac{4}{12} = \frac{5}{6}$$
$$\frac{3x + 4}{12} = \frac{5}{6}$$
$$6(3x + 4) = 12(5)$$
$$18x + 24 = 60$$
$$18x + 24 - 24 = 60 - 24$$
$$18x = 36$$
$$\frac{18x}{18} = \frac{36}{18}$$
$$x = 2$$

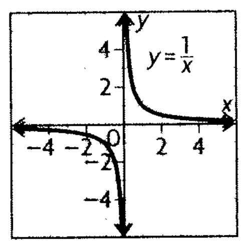
c) $$\frac{4x}{5} - \frac{3x}{10} = \frac{3}{2}$$

$$\left(\frac{2}{2}\right)\left(\frac{4x}{5}\right) - \frac{3x}{10} = \frac{3}{2}$$
$$\frac{8x}{10} - \frac{3x}{10} = \frac{3}{2}$$
$$\frac{5x}{10} = \frac{3}{2}$$
$$2(5x) = 3(10)$$
$$10x = 30$$
$$\frac{10x}{10} = \frac{30}{10}$$
$$x = 3$$

d) $$\frac{x+1}{2} + \frac{2x-1}{3} = -1$$

$$\left(\frac{3}{3}\right)\left(\frac{x+1}{2}\right) + \left(\frac{2}{2}\right)\left(\frac{2x-1}{3}\right) = -1$$
$$\frac{3x+3}{6} + \frac{4x-2}{6} = -1$$
$$\frac{7x+1}{6} = -1$$
$$6\left(\frac{7x+1}{6}\right) = 6(-1)$$
$$7x + 6 = -6$$
$$7x + 6 - 6 = -6 - 6$$
$$7x = -12$$
$$\frac{7x}{7} = \frac{-12}{7}$$
$$x = -\frac{12}{7}$$

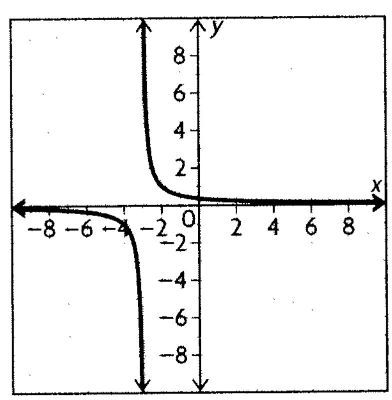
6.



The graph has vertical and horizontal asymptotes that follow the corresponding axis. The domain of the function is $\{x \in \mathbf{R} \mid x \neq 0\}$; the range of the function is $\{y \in \mathbf{R} \mid y \neq 0\}$.

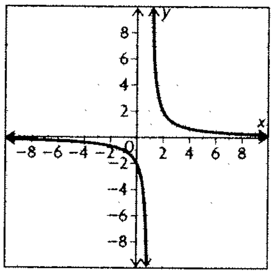
7. a) $f(x) = \dfrac{1}{x+3}$

The function is translated 3 units to the left.



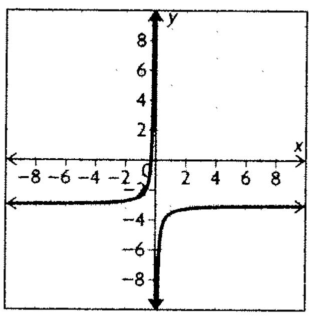
b) $f(x) = \frac{2}{x - 1}$

The graph has a vertical stretch by a factor of 2 and a horizontal translation 1 unit to the right.



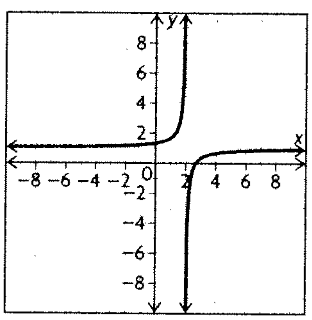
c) $f(x) = -\frac{1}{2x} - 3$

The graph has a reflection in the x-axis, vertical compression by a factor of $\frac{1}{2}$, and a vertical translation 3 units down.



d) $f(x) = \frac{2}{-3(x - 2)} + 1$

The graph has a reflection in the x-axis, a vertical compression by a factor of $\frac{2}{3}$, horizontal translation 2 units right, and a vertical translation 1 unit up.



8. Factor the expressions in the numerator and the denominator. Simplify each expression as necessary. Multiply the first expression by the reciprocal of the second.

$$\frac{9y^2 - 4}{4y - 12} \div \frac{9y^2 + 12y + 4}{18 - 6y}$$

$$= \frac{(3y + 2)(3y - 2)}{4(y - 3)} \div \frac{(3y + 2)(3y + 2)}{6(3 - y)}$$

$$= \frac{(3y + 2)(3y - 2)}{4(y - 3)} \times \frac{6(3 - y)}{(3y + 2)(3y + 2)}$$

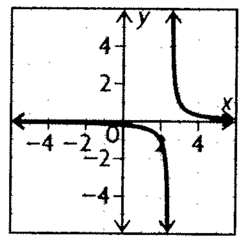
$$= -\frac{(3y - 2)(y - 3)}{2(y - 3)(3y + 2)}$$

$$= -\frac{3(3y - 2)}{2(3y + 2)}$$

5.1 Graphs of Reciprocal Functions, pp. 254–257

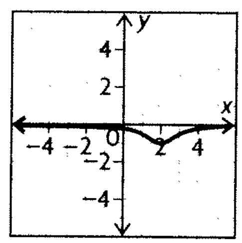
1. Graph each function and compare to the given graphs.

a)



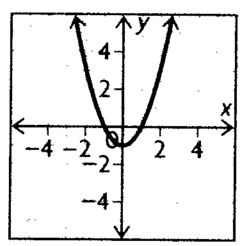
This is the graph of $y = \frac{1}{2x - 5}$. **C**

b)



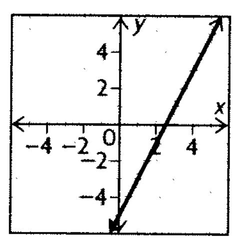
This is the graph of $y = \frac{1}{-(x - 2)^2 - 1}$. **A**

c)

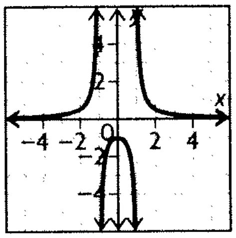


This is the graph of $y = x^2 - 1$. **D**

d)

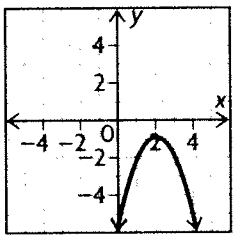


This is the graph of $y = 2x - 5$. **F** e)



This is the graph of $y = \dfrac{1}{x^2 - 1}$. **B**

f)



This is the graph of $y = -(x - 2)^2 - 1$. **E**

A and E are reciprocals. B and D are reciprocals.

C and F are reciprocals.

2. The zeros of a function occur when $f(x) = 0$.

a) $f(x) = x - 6$

$0 = x - 6$

$6 = x$

The vertical asymptote of $g(x) = \dfrac{1}{(x - 6)}$ occurs at $x = 6$.

b) $f(x) = 3x + 4$

$0 = 3x + 4$

$0 - 4 = 3x$

$-\dfrac{4}{3} = x$

The vertical asymptote of $g(x) = \dfrac{1}{(3x + 4)}$ occurs at $x = -\frac{4}{3}$.

c) $f(x) = x^2 - 2x - 15$

$0 = x^2 - 2x - 15$

$0 = (x - 5)(x + 3)$

$0 = (x - 5)$ and $0 = (x + 3)$

$0 + 5 = x - 5 + 5$ and $0 - 3 = x + 3 - 3$

$5 = x$ and $-3 = x$. The vertical asymptotes of $g(x) = \dfrac{1}{x^2 - 2x - 15}$ occur at $x = 5$ and $x = -3$.

d) $f(x) = 4x^2 - 25$

$0 = 4x^2 - 25$

$0 = (2x + 5)(2x - 5)$

$0 = (2x + 5)$ and $0 = (2x - 5)$

$0 - 5 = 2x + 5 - 5$ and

$0 + 5 = 2x - 5 + 5$

$\dfrac{-5}{2} = \dfrac{2x}{2}$ and $\dfrac{5}{2} = \dfrac{2x}{2}$

$\dfrac{-5}{2} = x$ and $\dfrac{5}{2} = x$

The vertical asymptotes of $g(x) = \dfrac{1}{4x^2 - 25}$ occur at $x = -\frac{5}{2}$ and $x = \frac{5}{2}$.

e) $f(x) = x^2 + 4$

$0 = x^2 + 4$

$-4 = x^2$

There are no real solutions to $-4 = x^2$. Therefore, there are no asymptotes for the function $\dfrac{1}{x^2 + 4}$.

f) $f(x) = 2x^2 + 5x + 3$

$0 = 2x^2 + 5x + 3$

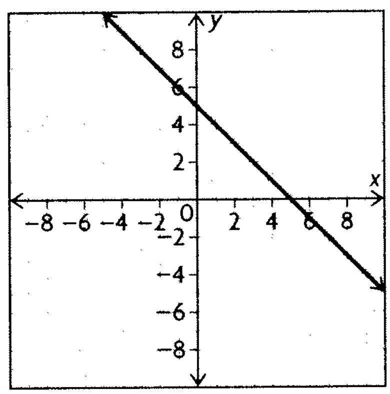
$0 = (2x + 3)(x + 1)$

$0 = (2x + 3)$ and $0 = (x + 1)$

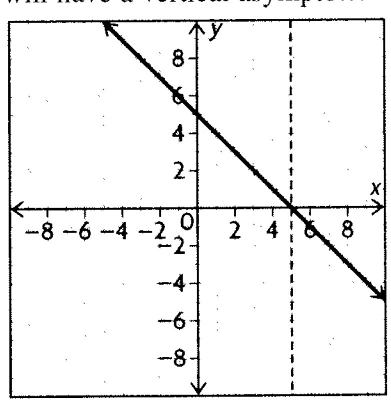
$x = -1.5$ and $x = -1$

The asymptotes for $g(x) = \dfrac{1}{(2x^2 + 5x + 3)}$ are at $x = -1.5$ and -1.

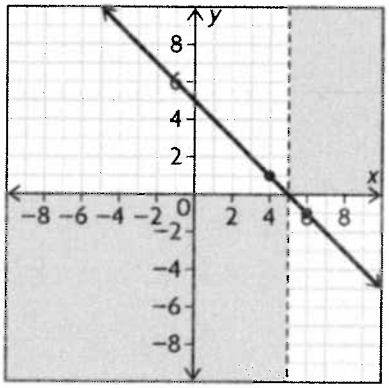
3. a) Graph the function $f(x) = 5 - x$. The y-intercept is $(0, 5)$ and the x-intercept is $(5, 0)$. Let $g(x)$ be the reciprocal function.



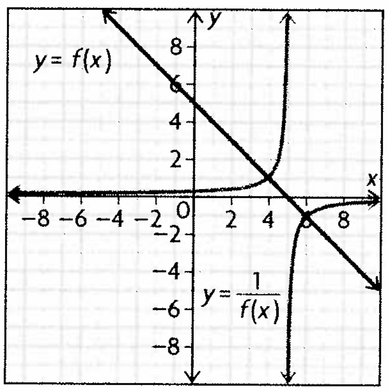
The zero of $f(x)$ is $x = 5$ and so that is where $g(x)$ will have a vertical asymptote.



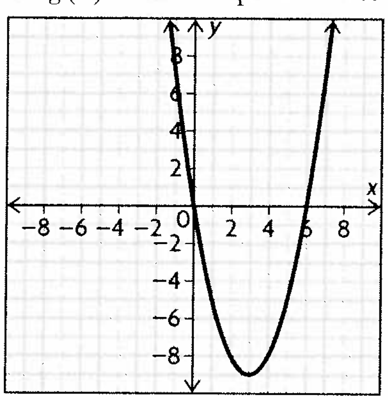
$f(x)$ is positive on $(-\infty, 5)$, negative on $(5, \infty)$, and always decreasing. Therefore, $g(x)$ positive on $(-\infty, 5)$, negative on $(5, \infty)$, and always increasing. $f(x) = 1$ at 4 and -1 at 6. The points of intersection for $f(x)$ and $g(x)$ will be at $(1, 4)$ and $(-1, 6)$.



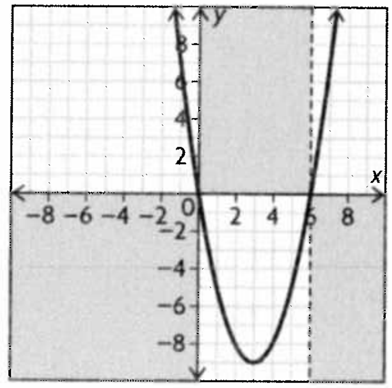
Use this information to draw the graph of $g(x)$.



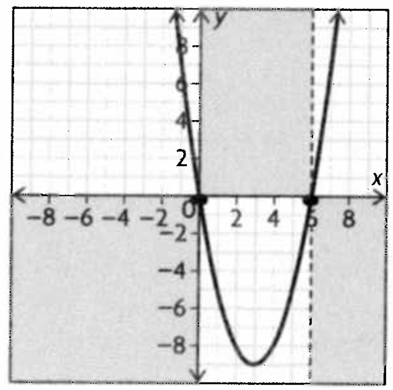
b) Graph $f(x) = x^2 - 6x$. The x-intercepts are at $x = 0$ and $x = 6$. The minimum occurs at $(3, -9)$. Let $g(x)$ be the reciprocal function.



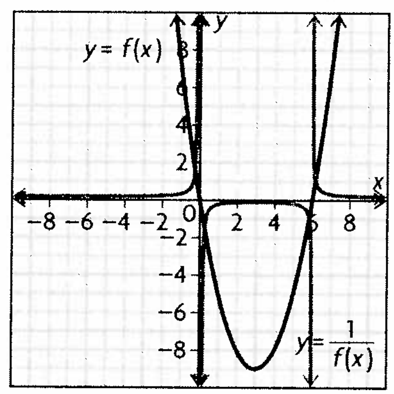
$f(x)$ is positive on $(-\infty, 0)$ and $(6, \infty)$ and negative on $(0, 6)$. It is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$. Therefore, $g(x)$ is positive on $(-\infty, 0)$ and $(6, \infty)$ and negative on $(0, 6)$. $g(x)$ is increasing on $(-\infty, 3)$ and decreasing on $(3, \infty)$. There are vertical asymptotes at $x = 0$ and 6.



$f(x) = 1$ at $x = 6.1$ and -0.2. $f(x) = -1$ at $x = 5.8$ and 0.2. These will be points of intersection for $f(x)$ and $g(x)$.



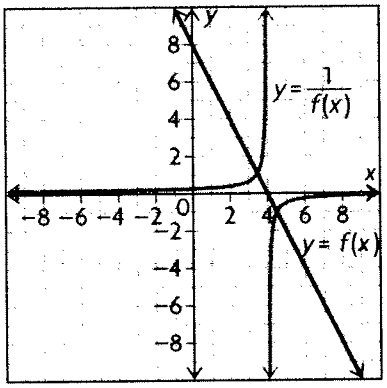
Use this information to graph $g(x)$.



4. a) Complete the table with the reciprocals of the values of $f(x)$.

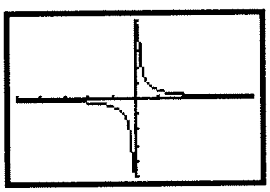
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	16	14	12	10	8	6	4	2	0	-2	-4	-6
$\frac{1}{f(x)}$	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	undefined	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{6}$

b)

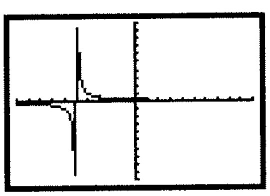


c) The first differences in the values of $f(x)$ are a constant, -2, so the function is linear. Since the y-intercept is 8, the equation for the function is $y = -2x + 8$. The equation for the reciprocal function is $y = \frac{1}{-2x + 8}$.

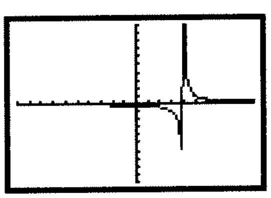
5. a) $y = \frac{1}{2x}$; vertical asymptote at $x = 0$



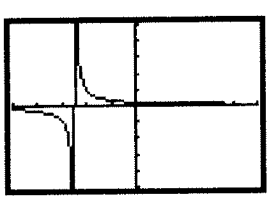
b) $y = \frac{1}{x + 5}$; vertical asymptote at $x = -5$



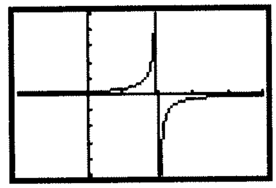
c) $y = \frac{1}{x - 4}$; vertical asymptote at $x = 4$



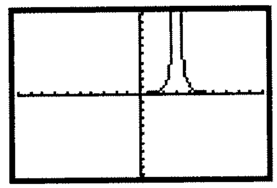
d) $y = \frac{1}{2x + 5}$; vertical asymptote at $x = -\frac{5}{2}$



e) $y = \frac{1}{-3x + 6}$; vertical asymptote at $x = 2$

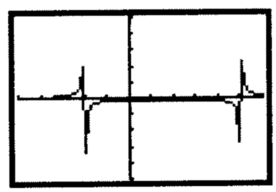


f) $y = \frac{1}{(x - 3)^2}$; vertical asymptote at $x = 3$



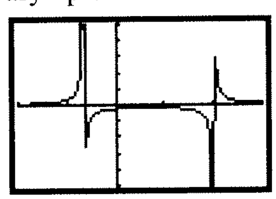
g) $y = \frac{1}{x^2 - 3x - 10}$

$x^2 - 3x - 10 = (x - 5)(x + 2)$, so the vertical asymptotes are at $x = -2$ and $x = 5$.

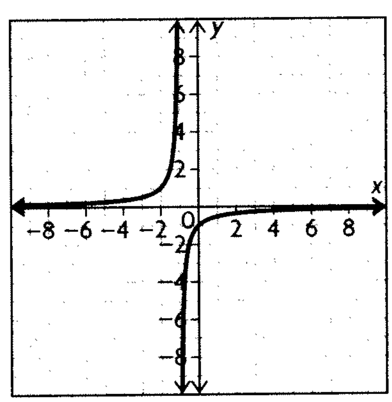


h) $y = \frac{1}{3x^2 - 4x - 4}$

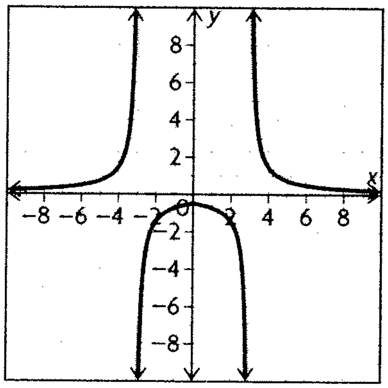
$3x^2 - 4x - 4 = (x - 2)(3x + 2)$, so the vertical asymptotes are at $x = -\frac{2}{3}$ and $x = 2$.



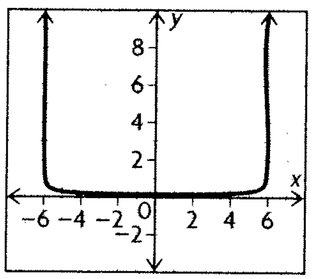
6. a) $f(x) = -x - 1$, so the reciprocal function is $y = -\frac{1}{x + 1}$.



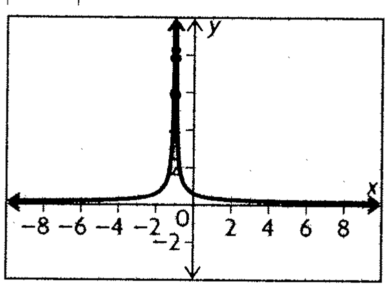
b) $f(x) = \frac{2}{3}|x| - 2$, so the reciprocal function is $y = \dfrac{1}{\frac{2}{3}|x| - 2}$.



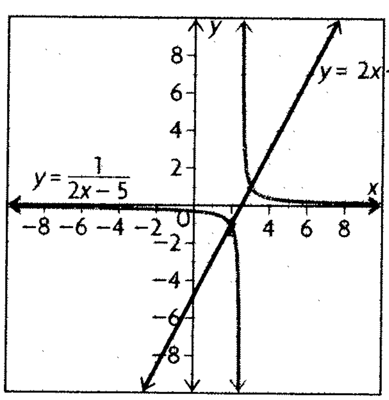
c) $f(x) = \sqrt{36 - x^2}$, so the reciprocal function is $y = \dfrac{1}{\sqrt{36 - x^2}}$.



d) $f(x) = 2|x + 1|$, so the reciprocal function is $y = \dfrac{1}{2|x + 1|}$.

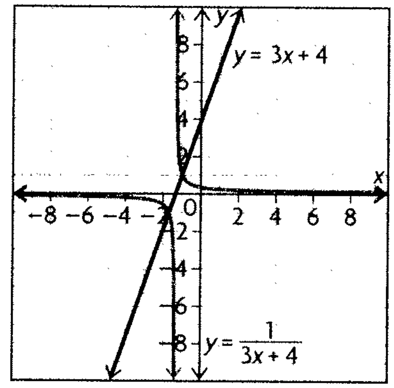


7. a)



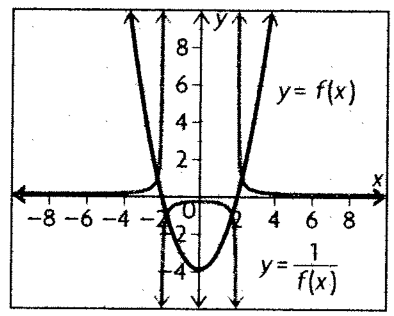
For the reciprocal function: $D = \{x \in \mathbf{R} \mid x \neq \frac{5}{2}\}$, $R = \{y \in \mathbf{R} \mid y \neq 0\}$

b)

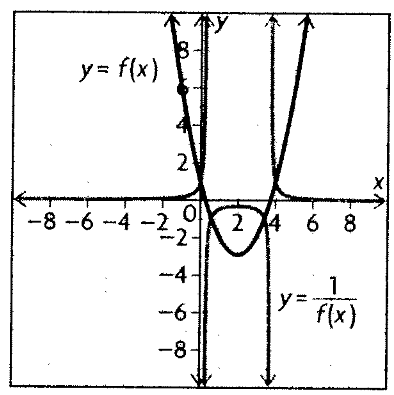


For the reciprocal function: $D = \{x \in \mathbf{R} \mid x \neq -\frac{4}{3}\}$, $R = \{y \in \mathbf{R} \mid y \neq 0\}$

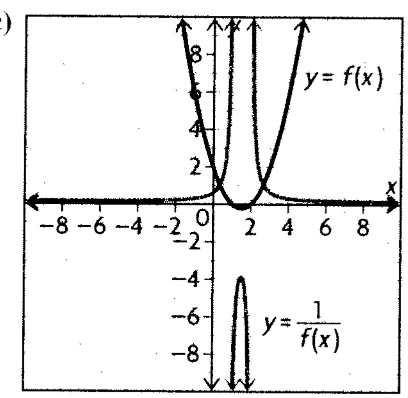
8. a)



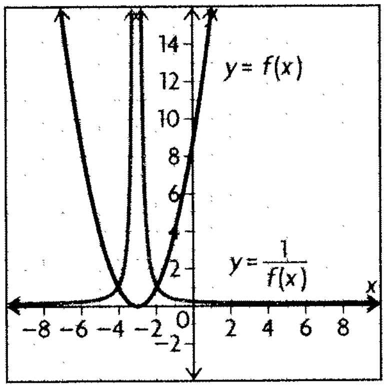
b)



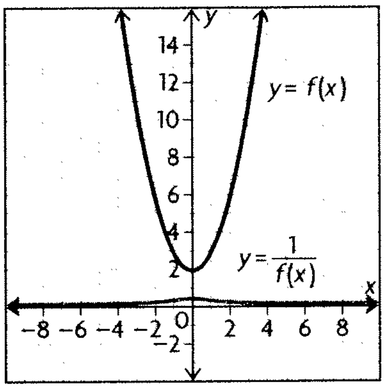
c)



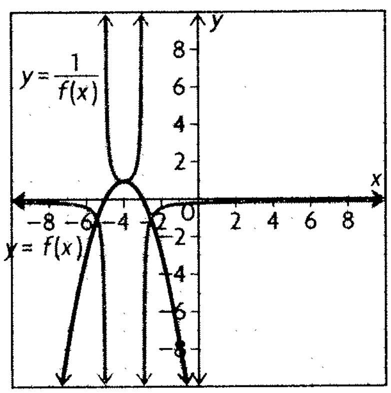
d)



e)



f)

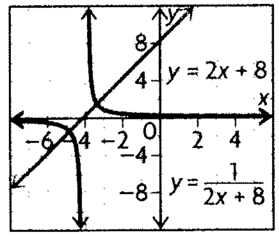


9. a) The domain and range of $f(x)$ are all real numbers because x is a linear polynomial. Also, because $f(x)$ is a linear polynomial with a positive slope, it is always increasing. The y-intercept is $2(0) + 8 = 8$. The x-intercept is

$$0 = 2x + 8$$
$$-8 = 2x$$
$$-4 = x$$

The y-intercept is $(0, 8)$ and x-intercept is $(-4, 0)$. Use the x-intercept to determine the intervals upon which the function is negative and positive. Because the function is always increasing, the function is negative on $(-\infty, -4)$ and positive on $(-4, -\infty)$.

$g(x) = \frac{1}{f(x)}$ so the reciprocal of $f(x)$ is $\frac{1}{2x + 8}$.

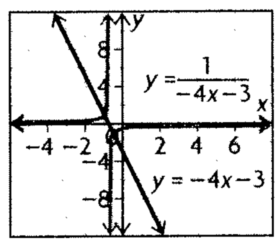


b) The domain and range of $f(x)$ are all real numbers because x is a linear polynomial. Also, because $f(x)$ is a linear polynomial with a negative slope, it is always decreasing. The y-intercept is $-4(0) - 3 = -3$. The x-intercept is

$$0 = -4x - 3$$
$$3 = -4x$$
$$-\frac{3}{4} = x$$

The y-intercept is $(0, -3)$ and x-intercept is $\left(-\frac{3}{4}, 0\right)$. Use the x-intercept to determine the intervals upon which the function is negative and positive. Because the function is always decreasing, the function is positive on $\left(-\infty, -\frac{3}{4}\right)$ and negative on $\left(-\frac{3}{4}, \infty\right)$.

$g(x) = \frac{1}{f(x)}$ so the reciprocal of $f(x)$ is $\frac{1}{-4x - 3}$.



c) The y-intercept is $f(0) = 0^2 - 0 - 12 = 12$. The x-intercepts are:

$$f(x) = x^2 - x - 12$$
$$0 = x^2 - x - 12$$
$$0 = (x - 4)(x + 3)$$
$$0 = (x - 4) \text{ and } 0 = (x + 3)$$

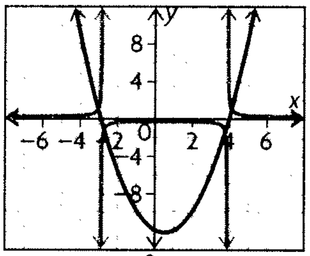
The x-intercepts are 4 and -3. The vertex is at 0.5.

$$f(0.5) = (0.5)^2 - (0.5) - 12$$
$$= 0.25 - 0.5 - 12$$
$$= -12.25$$

The leading coefficient of the function is positive and so the function opens up; the vertex $(0.5, -12.25)$ is a local minimum. The domain is all real numbers and the range is all real numbers greater than -12.25. The function is decreasing on $(-\infty, 0.5)$ and increasing on $(0.5, \infty)$. The function is positive on $(-\infty, -3)$ and on $(4, \infty)$. The function is negative on $(-3, 4)$.

$g(x) = \frac{1}{f(x)}$ so the reciprocal of $f(x)$ is $\frac{1}{x^2 - x - 12}$.

$y = \dfrac{1}{x^2 - x - 12}$



$y = x^2 - x - 12$

d) The y-intercept is $-2(0)^2 + 10(0) - 12$. The x-intercepts are:

$$\begin{aligned}
f(x) &= -2x^2 + 10x - 12\\
0 &= -2x^2 + 10x - 12\\
0 &= -2(x^2 - 5x + 6)\\
0 &= -2(x - 3)(x - 2)\\
\frac{0}{-2} &= \frac{-2(x - 3)(x - 2)}{-2}\\
0 &= (x - 3)(x - 2)\\
0 &= (x - 3) \text{ and } 0 = (x - 2)\\
x &= 3 \text{ and } x = 2
\end{aligned}$$

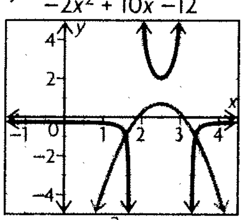
The vertex occurs at $x = 2.5$.

$$\begin{aligned}
f(2.5) &= -2(2.5)^2 + 10(2.5) - 12\\
&= -12.5 + 25 - 12\\
&= 0.5
\end{aligned}$$

The vertex is at (2.5, 0.5). Because the leading coefficient is negative, the vertex is a local maximum. This means that $f(x)$ is increasing on $(-\infty, 2.5)$ and decreasing on $(2.5, \infty)$. The domain of $f(x)$ is all real numbers and the range is all numbers less than 2.5. The function is negative on $(-\infty, 2)$ and $(3, \infty)$ and positive on $(2, 3)$.

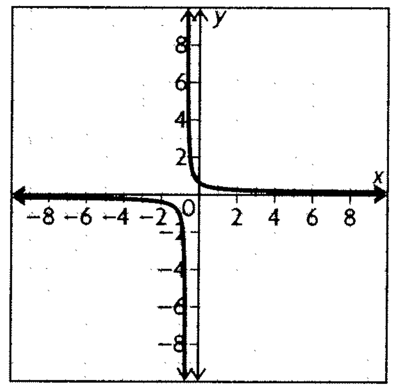
$g(x)\dfrac{1}{f(x)}$ so the reciprocal of $f(x)$ is $\dfrac{1}{-2x^2 + 10x - 12}$.

$y = \dfrac{1}{-2x^2 + 10x - 12}$

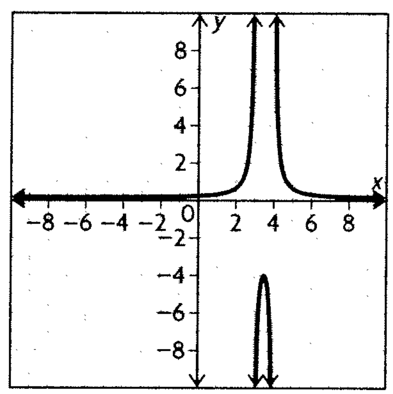


$y = -2x^2 + 10x - 12$

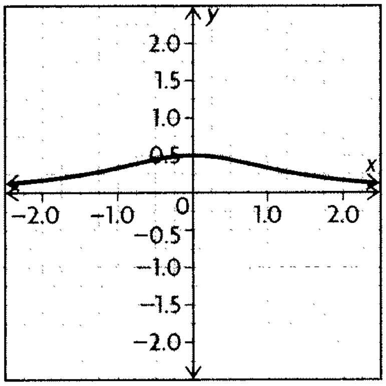
10. Answers may vary. For example: A reciprocal function creates a vertical asymptote when the denominator is equal to 0 for a specific value of x. Consider $\dfrac{1}{ax + b}$. For this expression, there is always some value of x that is $\dfrac{-b}{a}$ that will result in a vertical asymptote for the function. This is a graph of $y = \dfrac{1}{3x + 2}$ and the vertical asymptote is at $x = -\frac{2}{3}$.



Consider the function $\dfrac{1}{(x - 3)(x - 4)}$. The graph of the quadratic function in the denominator crosses the x-axis at 3 and 4 and therefore will have vertical asymptotes at 3 and 4 in the graph of the reciprocal function.



However, a quadratic function, such as $x^2 + c$, which has no real zeros, will not have a vertical asymptote in the graph of its reciprocal function. For example, this is the graph of $y = \dfrac{1}{x^2 + 2}$.



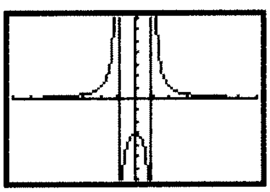
11. There are vertical asymptotes at $x = -1$ and $x = 1$, so the denominator is $(x + 1)(x - 1)$ or $x^2 - 1$.

$y = \dfrac{k}{x^2 - 1}$

Since $(0, -3)$ is on the graph,

$-3 = \dfrac{k}{0 - 1}$, or $k = 3$.

The equation is $y = \dfrac{3}{x^2 - 1}$.



12. a) Substitute 20 into the formula to find the number of bacteria that will be left after 20 seconds.

$b(t) = 10\,000\dfrac{1}{t}$

$b(20) = 10\,000\dfrac{1}{20}$

$b(20) = 500$

There will be 500 bacteria after 20 seconds.

b) Find the time t for which there will be 5000 bacteria left.

$5000 = 10\,000\dfrac{1}{t}$

$\dfrac{5000}{10\,000} = \dfrac{1}{10\,000} \times 10\,000 \times \dfrac{1}{t}$

$\dfrac{1}{2} = \dfrac{1}{t}$

$2 = t$

c) Determine the time at which there will be only 1 bacteria left.

$1 = 10\,000\dfrac{1}{t}$

$\dfrac{1}{10\,000} = \dfrac{1}{10\,000} \times 10\,000 \times \dfrac{1}{t}$

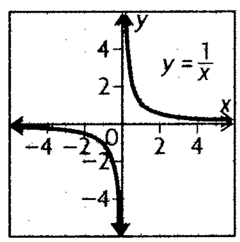
$\dfrac{1}{10\,000} = \dfrac{1}{t}$

$10\,000 = t$

d) If you were to use a value of t that was less than one, the equation would tell you that the number of bacteria was increasing as opposed to decreasing. Also, after time $t = 10\,000$ the formula indicates that there is a smaller and smaller fraction of 1 bacteria left.

e) Because the time less than 1 second and greater than 10 000 is inaccurate, the domain should be $\{x \in \mathbf{R} \mid 1 < x < 10\,000\}$. Because you cannot have a negative number of bacteria and because there are 10 000 bacteria at the beginning of the trial, the range should be $\{y \in \mathbf{R} \mid 1 < y < 10\,000\}$.

13. a) Think of the general shape of the reciprocal of a linear function—it will have a horizontal and a vertical asymptote.



The vertical asymptote occurs when the original function is equal to 0.

$0 = x + n - n$

$0 - n = x + n - n$

$-n = x$

Because there is a vertical asymptote at $x = -n$, the domain does not include $-n$.

Domain $= \{x \in \mathbf{R} \mid x \neq -n\}$.

Examine the end behaviours of the function to find the vertical asymptote. As $x \to \infty$, $g(x)$ will get closer and closer to 0 because no matter how small n is $x + n$ will always get closer to ∞. The same holds true for $g(x)$ as $x \to -\infty$. Range $= \{y \in \mathbf{R} \mid y \neq 0\}$.

b) The vertical asymptote occurs at $x = -n$. Changes in n in the $f(x)$ family cause changes in the y-intercept—an increase in n causes the intercept to move up the y-axis and a decrease causes it to move down the y-axis. Changes in n in the $g(x)$ family cause changes in the vertical asymptote of the function—an increase in n causes the asymptote to move down the x-axis and a decrease in n causes it to move up the x-axis.

c) The functions will intersect when $f(x) = g(x)$.

$$x + n = \frac{1}{x + n}$$
$$(x + n)(x + n) = \frac{1}{x + n}(x + n)$$
$$(x + n)^2 = 1$$
$$\sqrt[2]{(x + n)^2} = \sqrt{1}$$
$$x + n = 1 \text{ or } x + n = -1$$
$$x = 1 - n \text{ or } x = -1 - n$$

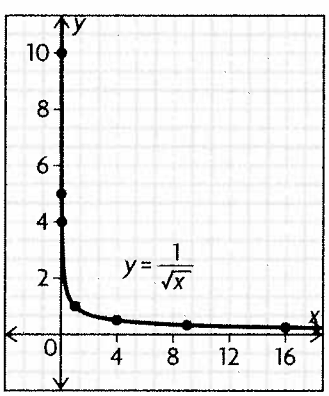
The functions intersect at $x = 1 - n$ and $x = -1-n$.

14. Answers may vary. For example: 1) Determine the zero(s) of the function $f(x)$—these will be the asymptote(s) for the reciprocal function $g(x)$.
2) Determine where the function $f(x)$ is positive and where it is negative—the reciprocal function $g(x)$ will have the same characteristics. 3) Determine where the function $f(x)$ is increasing and where it is decreasing—the reciprocal function $g(x)$ will have opposite characteristics.

15. Use a series of tables to help you graph the functions.

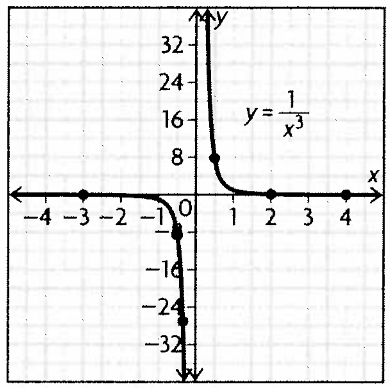
a)

$y = \frac{1}{\sqrt{x}}$	
x	**y**
$\frac{1}{100}$	10
$\frac{1}{25}$	5
$\frac{1}{16}$	4
1	1
4	$\frac{1}{2}$
9	$\frac{1}{3}$
16	$\frac{1}{4}$



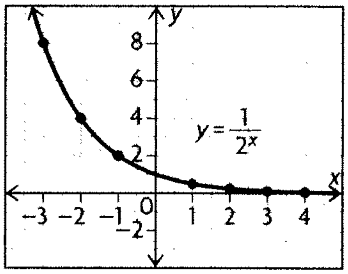
b)

$y = \frac{1}{x^3}$	
x	**y**
−3	$-\frac{1}{27}$
$-\frac{1}{2}$	−8
$-\frac{1}{3}$	−27
$\frac{1}{2}$	8
1	1
2	$\frac{1}{8}$
4	$\frac{1}{64}$



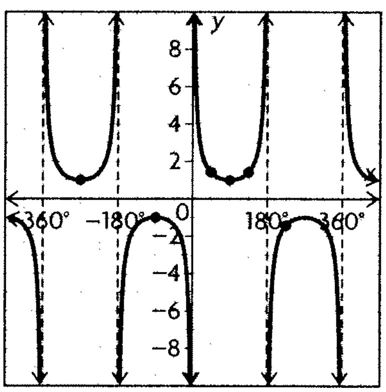
c)

$y = \frac{1}{2^x}$	
x	**y**
−3	8
−2	4
−1	−2
1	$\frac{1}{2}$
2	$\frac{1}{14}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$



d)

$y=\frac{1}{\sin x}$	
x	y
−360°	undefined
−270°	1
−180°	undefined
−90°	−1
0°	undefined
45°	$\sqrt{2}$
90°	1
135°	$\sqrt{2}$
180°	undefined
225°	$-\sqrt{2}$



16. The graph looks like the graph of $f(x) = \frac{1}{x}$, but translated 4 units to the left and 1 unit down. Therefore, the equation of the function shown in the graph is $y = \frac{1}{x+4} - 1$.

5.2 Exploring Quotients of Polynomial Functions, p. 262

1. a) A; $y = \frac{-1}{x-3}$; The function has a zero at 3 and the reciprocal function has a vertical asymptote at $x = 3$. The function is positive for $x < 3$ and negative for $x > 3$.

b) C; $y = \frac{x^2-9}{x-3}$; The function in the numerator factors to $(x+3)(x-3)$. $(x-3)$ factors out of both the numerator and the denominator. The equation simplifies to $y = (x+3)$, but has a hole at $x = 3$.

c) F; $y = \frac{1}{(x+3)^2}$; The function in the denominator has a zero at $x = -3$, so there is a vertical asymptote at $x = -3$. The function is always positive.

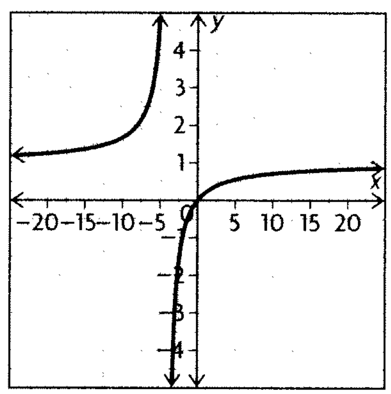
d) D; $y = \frac{x}{(x-1)(x+3)}$; The function in the denominator has zeros at $y = 1$ and $y = -3$. The rational function has vertical asymptotes at $x = 1$ and $x = -3$.

e) B; $y = \frac{1}{x^2+5}$; The function has no zeros and no vertical asymptotes or holes.

f) E; $y = \frac{x^2}{x-3}$; The function in the denominator has a zero at $x = 3$ and the rational function has a vertical asymptote at $x = 3$. The degree of the numerator is exactly 1 more than the degree of the denominator, so the graph has an oblique asymptote.

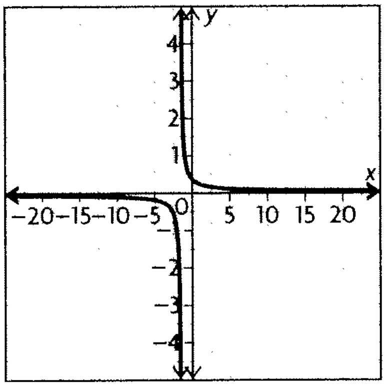
2. Use the graph of each equation to find the equations of any vertical asymptotes, the location of any holes, and the existence of any horizontal or oblique asymptotes.

a)



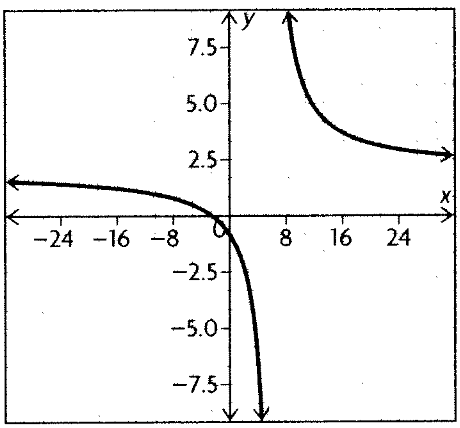
vertical asymptote at $x = -4$; horizontal asymptote at $y = 1$

b)



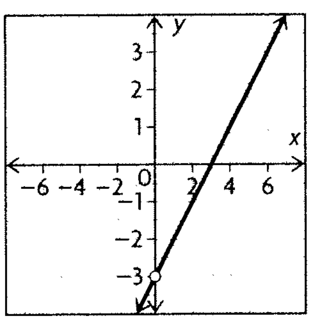
vertical asymptote at $x = -\frac{3}{2}$; horizontal asymptote at $y = 0$

c)



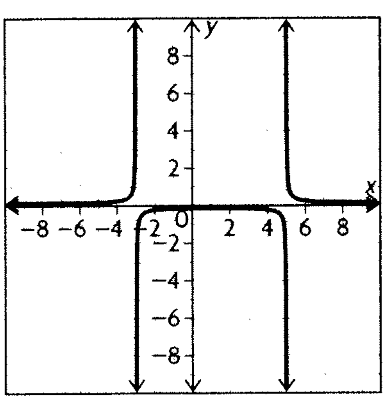
vertical asymptote at $x = 6$; horizontal asymptote at $y = 2$

d)



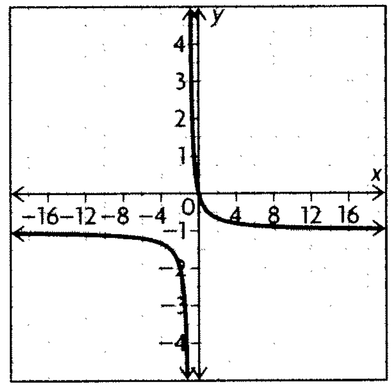
hole at $x = -3$

e)



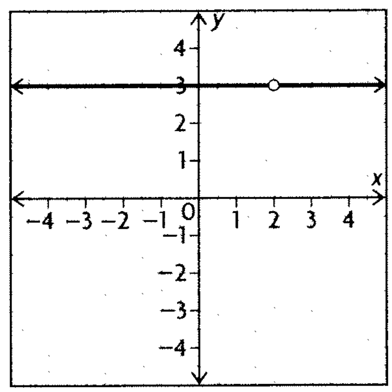
vertical asymptotes at $x = -3$ and 5; horizontal asymptote at $y = 0$

f)



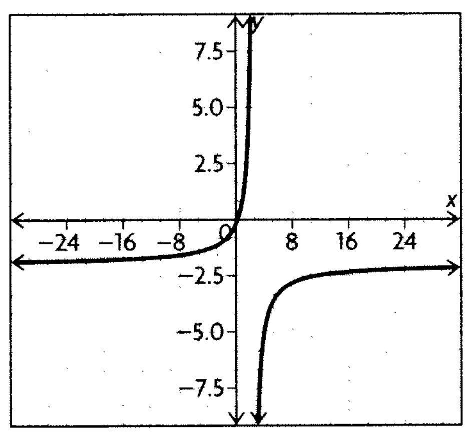
vertical asymptote at $x = -1$; horizontal asymptote at $y = -1$

g)



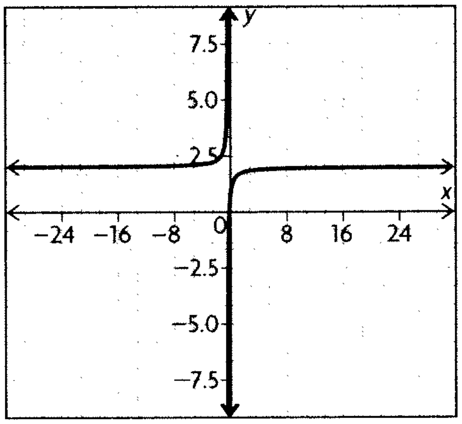
hole at $x = 2$

h)



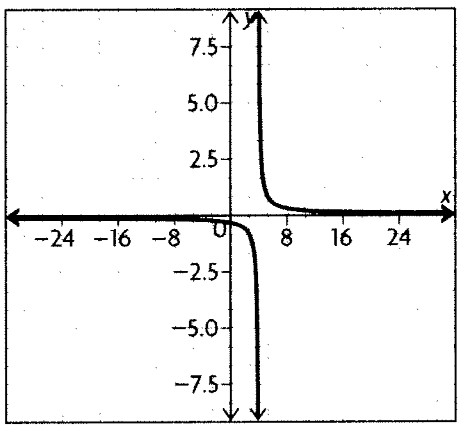
vertical asymptote at $x = \frac{5}{2}$; horizontal asymptote at $y = -2$

i)



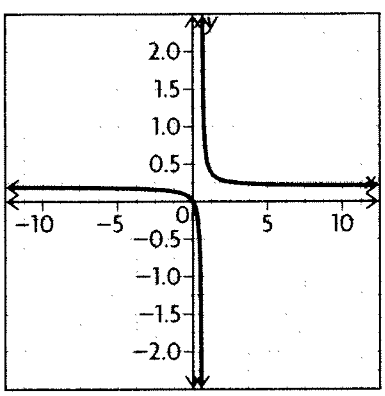
vertical asymptote at $x = -\frac{1}{4}$; horizontal asymptote at $y = 1$

j)



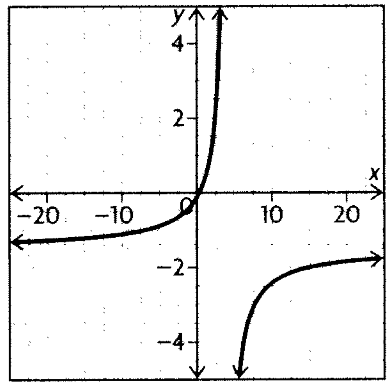
vertical asymptote at $x = 4$; hole at $x = -4$; horizontal asymptote at $y = 0$

k)



vertical asymptote at $x = \frac{3}{5}$; horizontal asymptote at $y = \frac{1}{5}$

l)



vertical asymptote at $x = 4$; horizontal asymptote at $y = -\frac{3}{2}$

3. a) The graph of the rational function has a hole at $x = 1$. This means that $(x - 1)$ must be a factor in both the numerator and the denominator of the function. Answers may vary. For example:

$$y = \frac{x - 1}{x^2 + x - 2}$$

b) The graph of the rational function has a vertical asymptote anywhere. This means that the polynomial that makes up the denominator must have a zero. The graph of the function also has a horizontal asymptote along the x-axis. This means that the numerator of the function must be a constant and the denominator must be a polynomial. Answers may vary. For example:

$$y = \frac{1}{x^2 - 4}$$

c) The graph of the rational function has a hole at $x = -2$. This means that $(x + 2)$ is a factor in both the numerator and the denominator. The graph also has a vertical asymptote at $x = 1$. This means that the ratio between the leading terms of the polynomials in the numerator and denominator must be 1. Answers may vary. For example:

$$y = \frac{x^2 - 4}{x^2 + 3x + 2}$$

d) The graph has a vertical asymptote at $x = -1$. This means that $(x + 1)$ must be a factor in the denominator of the rational function. The graph also has a horizontal asymptote at $y = 2$. This means that the ratio between the leading coefficients of the numerator and the denominator must be 2. Answers may vary. For example:

$$y = \frac{2x}{x + 1}$$

e) The graph of the function has an oblique asymptote. This means that the degree of the polynomial in the numerator must be one greater than that of the denominator. The rational function also has no vertical asymptote. This means that the polynomial in the numerator must have no real zeros. Answers may vary. For example:

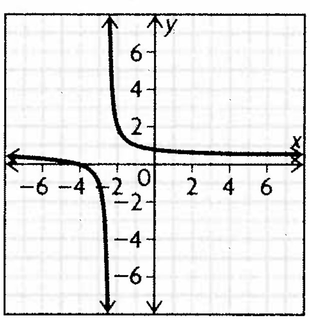
$$y = \frac{x^3}{x^2 + 5}$$

5.3 Graphs of Rational Functions of the Form $f(x) = \dfrac{ax + b}{cx + d}$, pp. 272–274

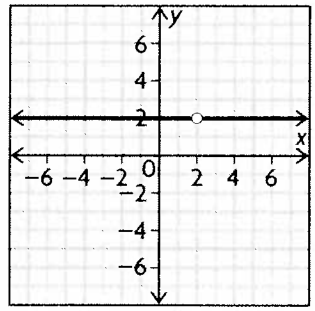
1. a) The rational function $h(x) = \dfrac{x + 4}{2x + 5}$ would have a vertical asymptote at the point where the function in the denominator has a zero.

$$2x + 5 = 0$$
$$2x + 5 - 5 = 0 - 5$$
$$2x = -5$$
$$\frac{2x}{2} = \frac{-5}{2}$$
$$x = -2.5$$

The function has a vertical asymptote at $x = -2.5$. This is graph **A**.



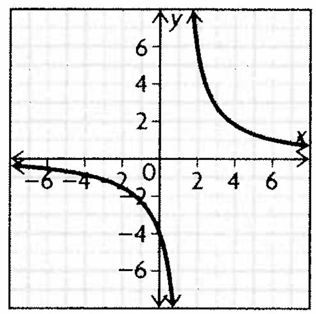
b) The rational function $h(x) = \dfrac{2x - 4}{x - 2}$ has $x - 2$ as a factor of both the numerator and the denominator. This means that the function has a hole at $x = 2$. This is graph **C**.



c) The rational function $h(x) = \dfrac{3}{x - 1}$ would have a vertical asymptote at the point where the function in the denominator has a zero.

$$x - 1 = 0$$
$$x - 1 + 1 = 0 + 1$$
$$x = 1$$

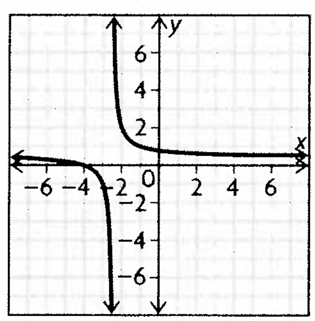
The function has a vertical asymptote at $x = 1$. This is graph **D**.



d) The rational function $h(x) = \dfrac{2x - 3}{x + 2}$ would have a vertical asymptote at the point where the function in the denominator has a zero.

$$x + 2 = 0$$
$$x + 2 - 2 = 0 - 2$$
$$x = -2$$

The function has a vertical asymptote at $x = -2$. This is graph **B**.



2. a) The rational function $f(x) = \frac{3}{x-2}$ will have a vertical asymptote at the point where the function in the denominator has a zero.

$$x - 2 = 0$$
$$x - 2 + 2 = 0 + 2$$
$$x = 2$$

The equation has a vertical asymptote at $x = 2$.

b) Use a table to examine the values of $f(x)$ as $x \to 2$. Use small increments between values of x.

x	f(x)
2.25	12
2.10	30
1.90	−30
1.75	−12
1.5	−6
1	−3

As x approaches 2 from the right, the values of $f(x)$ get larger. As x approaches 2 from the left, the values become larger in magnitude but are negative.

c) As $x \to \pm\infty$, the value of $f(x)$ approaches $\frac{3}{\infty}$ or 0.

d) Use a table to examine the values of $f(x)$ as x approaches $\pm\infty$. Use large increments between values of x.

x	f(x)
5000	0.000 600 24
2500	0.001 200 961
250	0.012 096 774
−250	−0.011 904 762
−2500	−0.001 199 041
−5000	−0.000 599 76

As x grows larger, $f(x)$ gets closer and closer to 0.

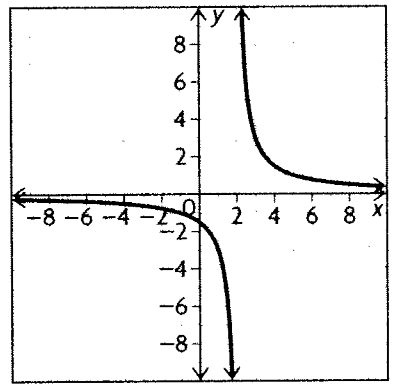
e) The domain is all real numbers except for 3 and the range is all real numbers except for 0.

f) Examine the equation $y = \frac{3}{x-2}$. The numerator is always positive. A positive number divided by a positive number is always positive. When $x - 2$ is positive, $y = \frac{3}{x-2}$ will be positive. A positive number divided by a negative number is always negative. When $x - 2$ is negative, $y = \frac{3}{x-2}$ will be negative.

$$x - 2 > 0$$
$$x - 2 + 2 > 0 + 2$$
$$x > 2$$

When $x > 2$, $y = \frac{3}{x-2}$ is positive. When $x < 2$, $y = \frac{3}{x-2}$ is negative.

g)



3. a) The rational function $f(x) = \frac{4x-3}{x+1}$ will have a vertical asymptote at the point where the function in the denominator has a zero.

$$x + 1 = 0$$
$$x + 1 - 1 = 0 - 1$$
$$x = -1$$

The equation has a vertical asymptote at $x = -1$.

b) Use a table to examine the values of $f(x)$ as x approaches 2. Use small increments between values of x.

x	f(x)
−0.1	−3.777 777 778
−0.5	−10
−0.75	−24
−1.25	32
−1.5	18

As $x \to -1$ from the left, $y \to \infty$.
As $x \to -1$ from the right, $y \to -\infty$.

c) The equation of the horizontal asymptote can be found by dividing the leading coefficients of the equation in the numerator and the denominator.

$$y = \frac{4x}{x} = 4$$

The equation of the horizontal asymptote is $y = 4$.

d) Use a table to examine the values of $f(x)$ as x approaches ∞ and $-\infty$. Use large increments between values of x.

x	f(x)
5000	3.998 600 28
2500	3.997 201 12
250	3.972 111 554
−250	4.028 112 45
−2500	4.002 801 12
−5000	4.001 400 28

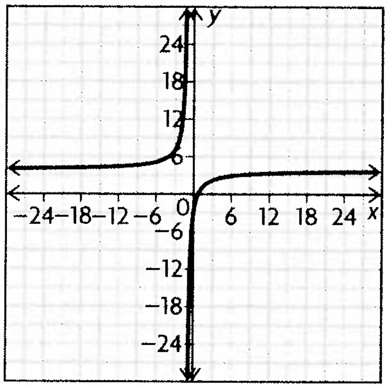
As $x \to \pm\infty$, $f(x)$ gets closer and closer to 4.

e) $D = \{x \in \mathbf{R} \mid x \neq -1\}$
$R = \{y \in \mathbf{R} \mid y \neq 4\}$

f) Use a table to help you determine the positive and negative intervals.

	$x < -1$	$-1 < x < \frac{3}{4}$	$x > \frac{3}{4}$
$4x - 3$	−	−	+
$x + 1$	−	+	+
$\frac{4x-3}{x+1}$	+	−	+

g)



4. The vertical asymptote of a rational function can be found by finding the zero(s) of the denominator.

a) $y = \dfrac{2}{x + 3}$

$x + 3 = 0$
$x + 3 - 3 = 0 - 3$
$x = -3$

The equation of the vertical asymptote of $y = \dfrac{2}{x + 3}$ is $x = -3$.
When $x = -3.1$, $y = -20$. So as $x \to -3$, $y \to -\infty$ on the left.
When $x = -2.9$, $y = 20$. So as $x \to -3$, $y \to \infty$ on the right.

b) $y = \dfrac{x - 1}{x - 5}$

$x - 5 = 0$
$x = 5$

The equation of the vertical asymptote of $y = \dfrac{x - 1}{x - 5}$ is $x = 5$.
When $x = 4.9$, $y = -39$. So as $x \to 5$, $y \to -\infty$ on the left.
When $x = 5.1$, $y = 41$. So as $x \to 5$, $y \to \infty$ on the right.

c) $y = \dfrac{2x + 1}{2x - 1}$

$2x - 1 + 1 = 0 + 1$
$2x = 1$
$x = \dfrac{1}{2}$

The equation of the vertical asymptote of $y = \dfrac{2x + 1}{2x - 1}$ is $x = \frac{1}{2}$.
When $x = 0.49$, $y = -99$. So as $x \to \frac{1}{2}$, $y \to -\infty$ on the left.
When $x = 0.51$, $y = 101$. So as $x \to \frac{1}{2}$, $y \to \infty$ on the right.

d) $y = \dfrac{3x + 9}{4x + 1}$

$4x + 1 - 1 = 0 - 1$
$4x = -1$
$x = -\dfrac{1}{4}$

The equation of the vertical asymptote of $y = \dfrac{3x + 9}{4x + 1}$ is $x = -\frac{1}{4}$.
When $x = -0.26$, $y = -205.5$. So as $x \to -\frac{1}{4}$, $y \to -\infty$ on the left.
When $x = -0.24$, $y = 207$. So as $x \to -\frac{1}{4}$, $y \to \infty$ on the right.

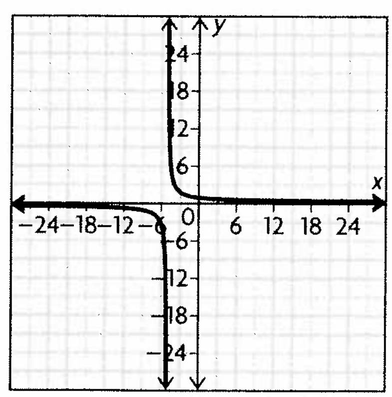
5. a) The function is $f(x) = \dfrac{3}{x + 5}$.
$f(x) = \dfrac{3}{x + 5}$ will have a vertical asymptote at $x = -5$. The horizontal asymptote will be $y = 0$. Therefore, the domain will be $\{x \in \mathbf{R} \mid x \neq -5\}$. The range will be $\{y \in \mathbf{R} \mid y \neq 0\}$. Because the horizontal asymptote is $y = 0$, there is no x-intercept. Substitute 0 for x to find the y-intercept.

$\dfrac{3}{0 + 5} = \dfrac{3}{5}$

The y-intercept is $\frac{3}{5}$. Use a table to determine the positive and negative intervals.

	$x < -5$	$x > -5$
3	+	+
$x + 5$	−	+
$\frac{3}{x+5}$	−	+

$f(x)$ is negative on $(-\infty, -5)$ and positive on $(-5, \infty)$.
The graph of the function is:



Examine the graph to determine where the function is increasing or decreasing. The function is decreasing on $(-\infty, -5)$ and on $(-5, \infty)$. The function is never increasing.

b) The function is $\frac{10}{2x - 5}$.

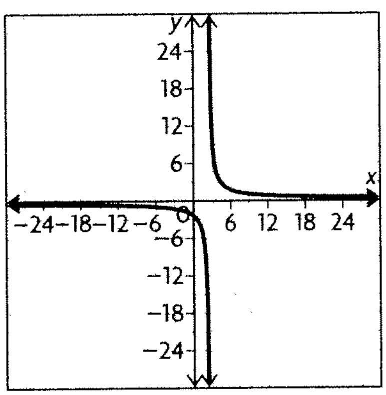
$f(x) = \frac{10}{2x - 5}$ will have a vertical asymptote at $x = \frac{5}{2}$. The horizontal asymptote will be $y = 0$. Therefore, the domain will be $\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\}$ and the range will be $\{y \in \mathbf{R} \mid y \neq 0\}$. Because the horizontal asymptote is $y = 0$, there is no x-intercept. Substitute 0 for x to find the y-intercept.

$$\frac{10}{2(0) - 5} = -2$$

The y intercept is -2. Use a table to determine the positive and negative intervals.

	$x < \frac{5}{2}$	$x > \frac{5}{2}$
10	+	+
$2x - 5$	−	+
$\frac{10}{2x - 5}$	−	+

$f(x)$ is negative on $(-\infty, \frac{5}{2})$ and positive on $(\frac{5}{2}, \infty)$. The graph of the function is:



Examine the graph to determine where the function is increasing or decreasing. The function is decreasing on $(-\infty, \frac{5}{2})$ and on $(\frac{5}{2}, \infty)$. The function is never increasing.

c) The function is $f(x) = \frac{x + 5}{4x - 1}$.

$f(x) = \frac{x + 5}{4x - 1}$ will have a vertical asymptote at $x = \frac{1}{4}$. The horizontal asymptote will be $y = \frac{1}{4}$. Therefore, the domain will be $\{x \in \mathbf{R} \mid x \neq \frac{1}{4}\}$ and the range will be $\{y \in \mathbf{R} \mid y \neq \frac{1}{4}\}$.

Substitute 0 for y to find the x-intercept.

$$0 = \frac{x + 5}{4x - 1}$$

$$(4x - 1) \times 0 = (4x - 1) \times \frac{x + 5}{4x - 1}$$

$$0 = x + 5$$

$$-5 = x$$

The x-intercept is $x = -5$.

Substitute 0 for x to find the y-intercept.

$$\frac{0 + 5}{4(0) - 1} = -5$$

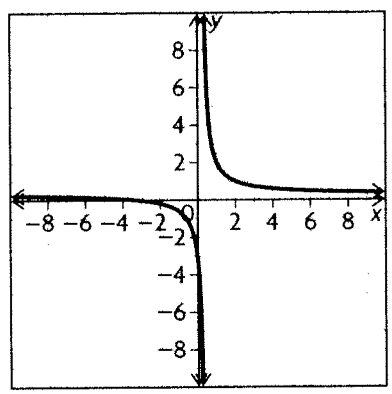
The y-intercept is -1.

Use a table to determine the positive and negative intervals.

	$x < -5$	$-5 < x < \frac{1}{4}$	$x > \frac{1}{4}$
$x + 5$	−	+	+
$4x - 1$	−	−	+
$\frac{x + 5}{4x - 1}$	+	−	+

$f(x)$ is positive on $(-\infty, -5)$ and $(\frac{1}{4}, \infty)$, and negative on $(-5, \frac{1}{4})$.

The graph of the function is:

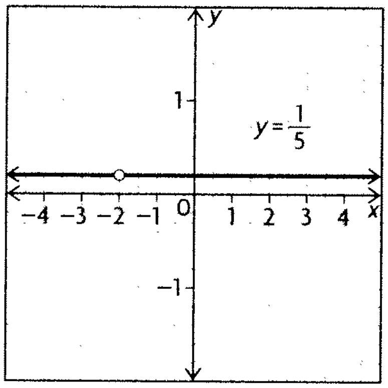


Examine the graph to determine where the function is increasing or decreasing. The function is decreasing on $(-\infty, \frac{1}{4})$ and on $(\frac{1}{4}, \infty)$. The function is never increasing.

d) The function is $f(x) = \frac{x + 2}{5(x + 2)}$. The function has the factor $(x + 2)$ in both the numerator and the denominator.

Examine the function. For any value of x, $f(x)$ will always be $\frac{1}{5}$. Because the function in the denominator will have zero at $x = -2$, $f(x)$ will have a hole at $x = -2$. The domain is $\{x \in \mathbf{R} \mid x \neq -2\}$. The range is $\{y = \frac{1}{5}\}$. The y-intercept is $y = \frac{1}{5}$. There is no x-intercept. The function will always be positive.

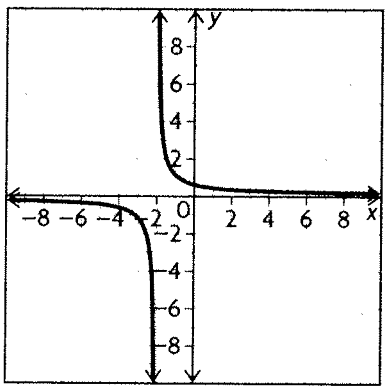
The graph of the function is:



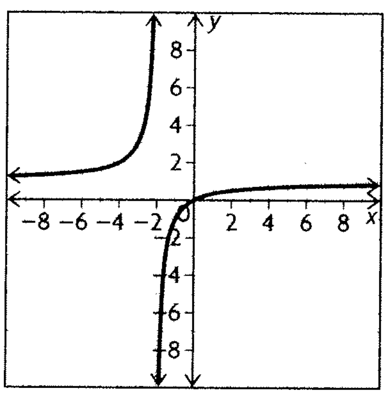
The function is neither increasing nor decreasing; it is constant.

6. a) Answers may vary. For example:

$$f(x) = \frac{1}{x + 2}$$



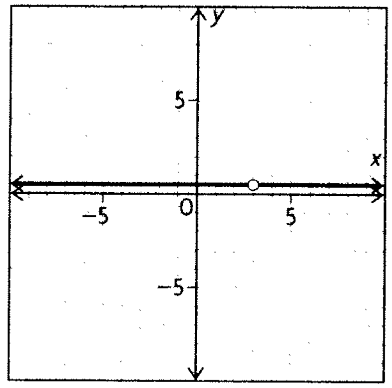
b) Answers may vary. For example: $f(x) = \frac{x}{x + 2}$



c) Answers may vary. For example:

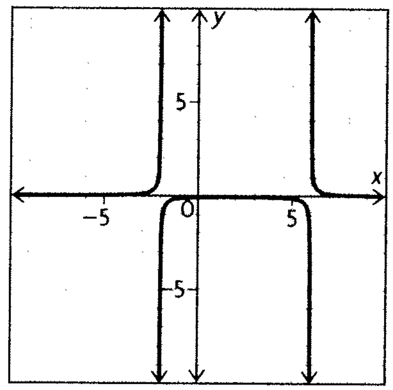
$$f(x) = \frac{x - 3}{2x - 6}$$

This has a hole at $x = 3$ and a y-intercept of 0.5.

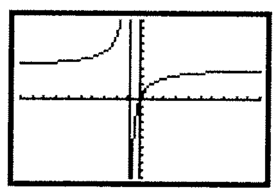


d) Answers may vary. For example:

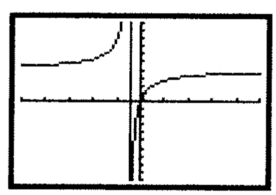
$$f(x) = \frac{1}{x^2 - 4x - 12}$$



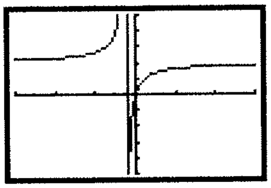
7. a) Graph the function $f(x) = \frac{8x}{nx + 1}$ for each value of n, $n = 1, 2, 4$, and 8.



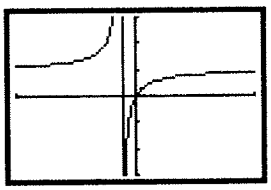
The graph is shown in a window from −10 to 10 by 1's on the horizontal axis and from −20 to 20 by 2's on the vertical axis.



The graph is shown in a window from −5 to 5 by 1's on the horizontal axis and from −10 to 10 by 1's on the vertical axis.



The graph is shown in a window from -3 to 3 by 1's on the horizontal axis and from -5 to 5 by 1's on the vertical axis.



The graph is shown in a window from -1 to 1 by 1's on the horizontal axis and from -3 to 3 by 1's on the vertical axis.
Use this information to discuss the differences between the graphs. The equation has a general vertical asymptote at $x = -\frac{1}{n}$. The function has a general horizontal asymptote $y = \frac{8}{n}$. The vertical asymptotes are $-\frac{1}{8}$, $-\frac{1}{4}$, $-\frac{1}{2}$, and -1. The horizontal asymptotes are 8, 4, 2, and 1. The function contracts as n increases. The function is always increasing. The function is positive on $\left(-\infty, -\frac{17}{n}\right)$ and $\left(\frac{3}{10}, \infty\right)$ The function is negative on $\left(-\frac{17}{n}, \frac{3}{10}\right)$.
b) The horizontal and vertical asymptotes both approach 0 as the value of n increases; the x- and y-intercepts do not change nor do the positive and negative characteristics or the increasing and decreasing characteristics.
c) The vertical asymptote becomes $x = \frac{17}{n}$ and the horizontal becomes $x = -\frac{10}{n}$. The function is always increasing. The function is positive on $\left(-\infty, \frac{3}{10}\right)$ and $\left(\frac{17}{n}, \infty\right)$. The function is negative on $\left(\frac{3}{10}, \frac{17}{n}\right)$. The rest of the characteristics do not change.
8. $f(x)$ will have a vertical asymptote at $x = 1$; $g(x)$ will have a vertical asymptote at $x = -\frac{3}{2}$. $f(x)$ will have a horizontal asymptote at $x = 3$; $g(x)$ will have a vertical asymptote at $x = \frac{1}{2}$.

9. Substitute the values of t to find the value of the investment over a given period of time. The function is $I(t) = \frac{15t + 25}{t}$.
a) $I(2) = \frac{15(2) + 25}{2} = \frac{55}{2} = 27.5$
The investment will be worth \$27 500 after 2 years.
b) $I(1) = \frac{15(1) + 25}{1} = 40$
The investment will be worth \$40 000 after 1 year.
c) $I(0.5) = \frac{15(0.5) + 25}{0.5} = 65$
The investment will be worth \$65 000 after 0.5 years.
d) No, the value of the investment at $t = 0$ should be the original value invested.
e) The function is probably not accurate at very small values of t because as $t \rightarrow 0$ from the right, $x \rightarrow \infty$.
f) The horizontal asymptote will indicate where the value of the investment will settle over time. Divide the leading terms to find the equation of the horizontal asymptote.
$y = \frac{15t}{t} = 15$
The value of the investment will settle at around \$15 000.
10. Use a table to help you examine the concentration of the chlorine in the pool over the 24-hour period.

t	$c(t)$
1	0.666 666 667
2	1
3	1.2
4	1.333 333 333
8	1.6
12	1.714 285 714
16	1.777 777 778
20	1.818 181 818
24	1.846 153 846

The concentration increases over the 24 hour period and approaches approximately 1.89 mg/L.
11. Answers may vary. For example: The rational functions will all have vertical asymptotes at $x = -\frac{d}{c}$. They will all have horizontal asymptotes at $y = \frac{a}{c}$. They will intersect the y-axis at $y = \frac{b}{d}$.

The rational functions will have an x-intercept at $x = -\frac{b}{a}$. You can use $a = 1$, $b = 2$, $c = 3$, and $d = 4$ to illustrate this. The function is

$$f(x) = \frac{1x + 2}{3x + 4}$$

Vertical Asymptote:

$$0 = 3x + 4$$
$$0 - 4 = 3x + 4 - 4$$
$$-4 = 3x$$
$$\frac{-4}{3} = \frac{3x}{3}$$
$$\frac{-4}{3} = x = \frac{-d}{c}$$

Horizontal Asymptote:

$$y = \frac{1x}{3x} = \frac{1}{3} = \frac{a}{c}$$

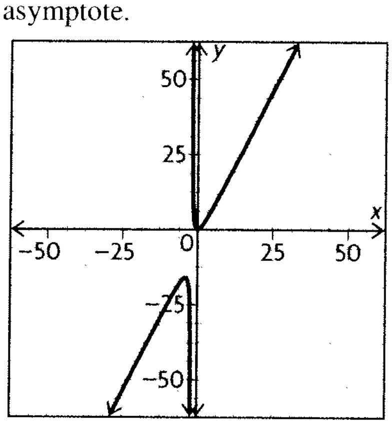
y-intercept:

$$y = \frac{1(0) + 2}{3(0) + 4} = \frac{2}{4} = \frac{b}{d}$$

x-intercept:

$$0 = \frac{1x + 2}{3x + 4}$$
$$(3x + 4) \times 0 = (3x + 4) \times \frac{1x + 2}{3x + 4}$$
$$0 = 1x + 2$$
$$0 - 2 = 1x + 2 - 2$$
$$-2 = 1x$$
$$\frac{-2}{1} = \frac{1x}{1}$$
$$\frac{-2}{1} = x = \frac{-b}{a}$$

12. Answers may vary. For example: The function $f(x) = \frac{2x^2}{2 + x}$ will have an oblique asymptote. Examine the graph below to see the oblique asymptote.

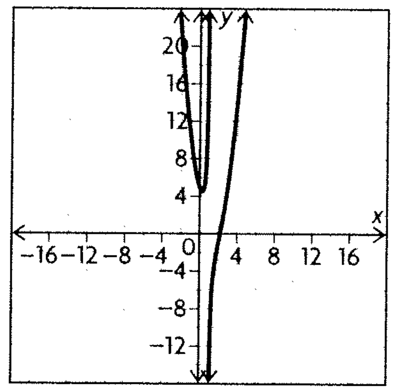


13. Use synthetic division to help you rewrite $f(x) = \frac{2x^3 - 7x^2 + 8x - 5}{x - 1}$.

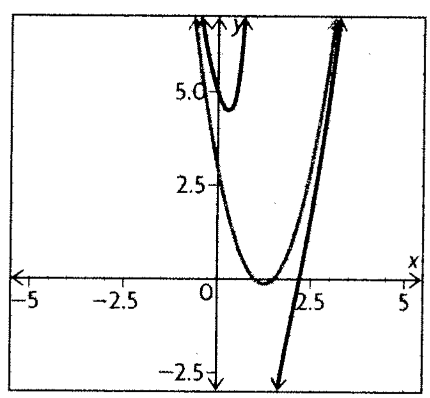
$$\begin{array}{r|rrrr} 1 & 2 & -7 & 8 & -5 \\ & \downarrow & 2 & -5 & 3 \\ \hline & 2 & -5 & 3 & -2 \end{array}$$

$$f(x) = \frac{2x^3 - 7x^2 + 8x - 5}{x - 1}$$
$$= 2x^2 - 5x + 3 - \frac{2}{x - 1}$$

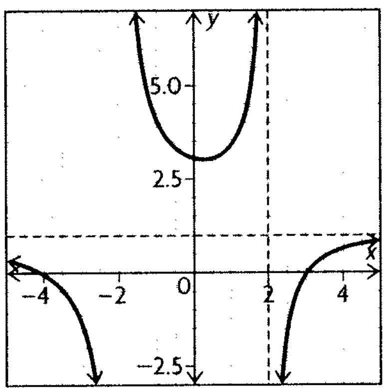
As $x \to \pm\infty$, $f(x) \to \infty$.



The equation of the vertical asymptote would be $x = 1$. When you find the equation of a horizontal asymptote of a rational function $f(x)$, you find a constant value that the equation approaches as $x \to \infty$. In this case, $f(x)$ isn't approaching a constant value, but rather an equation or a parabola. For $f(x)$, the equation of the oblique asymptote would be $y = 2x^2 - 5x + 3$, which is the quotient without the remainder because the remainder is what keeps the rational function from reaching the parabola. Graph both equations to verify this.



14. a) $f(x)$ would have a horizontal asymptote because the degree of the numerator is less than that of the denominator.
b) $g(x)$ and $h(x)$ would have an oblique asymptote because the degree of the numerator is greater than that of the denominator.
c) $g(x)$ has no vertical asymptote because the numerator has no real zero.
d) Use the horizontal and vertical asymptotes to help you draw the graph. The denominator has zeros at $x = 2$ and -2—these will be the vertical asymptotes. Divide the leading coefficients to find the horizontal asymptote of $y = 1$.



Mid-Chapter Review, p. 277

1. The reciprocal of a function, $f(x)$, is equal to $\frac{1}{f(x)}$. Vertical asymptotes can be found using the zeros of $f(x)$.

a) $f(x) = x - 3$

$$\frac{1}{f(x)} = \frac{1}{x-3}$$
$$0 = x - 3$$
$$0 + 3 = x - 3 + 3$$
$$3 = x$$

The equation of the vertical asymptote is $x = 3$.

b) $f(q) = -4q + 6$

$$\frac{1}{f(q)} = \frac{1}{-4q+6}$$
$$0 = -4q + 6$$
$$0 - 6 = -4q + 6 - 6$$
$$-6 = -4q$$
$$\frac{-6}{-4} = \frac{-4q}{-4}$$
$$\frac{3}{2} = q$$

The equation of the vertical asymptote is $q = \frac{3}{2}$.

c) $f(z) = z^2 + 4z - 5$

$$\frac{1}{f(z)} = \frac{1}{z^2 + 4z - 5}$$
$$0 = z^2 + 4z - 5$$
$$0 = (z + 5)(z - 1)$$
$$0 = z + 5$$
$$0 - 5 = z + 5 - 5$$
$$-5 = z$$
$$0 + 1 = z - 1 + 1$$
$$1 = z$$

The equations of the vertical asymptotes are $z = -5$ and 1.

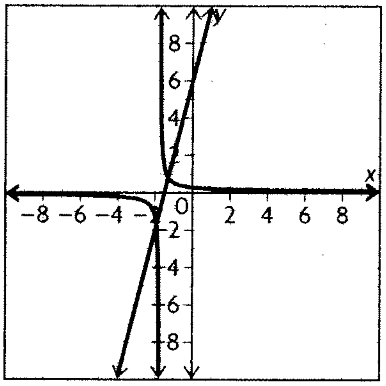
d) $f(d) = 6d^2 + 7d - 3$

$$\frac{1}{f(d)} = \frac{1}{6d^2 + 7d - 3}$$
$$0 = 6d^2 + 7d - 3$$
$$0 = (3d - 1)(2d + 3)$$
$$0 = 3d - 1$$
$$0 + 1 = 3d - 1 + 1$$
$$1 = 3d$$
$$\frac{1}{3} = \frac{3d}{3}$$
$$\frac{1}{3} = d$$
$$0 = 2d + 3$$
$$0 - 3 = 2d + 3 - 3$$
$$-3 = 2d$$
$$\frac{-3}{2} = \frac{2d}{2}$$
$$-\frac{3}{2} = d$$

2. a) The function is $f(x) = 4x + 6$. The function is a straight line. The domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R}\}$. Find the x-intercept to determine the positive and negative intervals.

$$0 = 4x + 6$$
$$0 - 6 = 4x + 6 - 6$$
$$-6 = 4x$$
$$\frac{-6}{4} = \frac{4x}{4}$$
$$-\frac{3}{2} = x$$

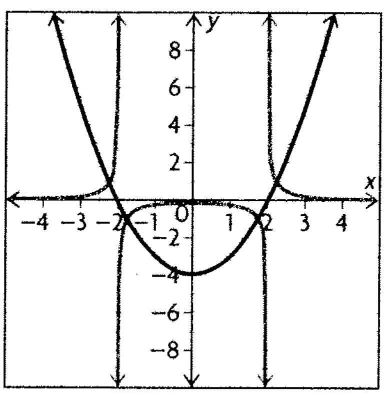
Choose a number on either side of $-\frac{3}{2}$ and substitute them into the function to find the positive. The function is negative on $\left(-\infty, -\frac{3}{2}\right)$ and positive on $\left(-\frac{3}{2}, \infty\right)$. Because the function is linear, and because the slope of the function is positive, the function is always positive. Use this information to graph the function and its reciprocal.



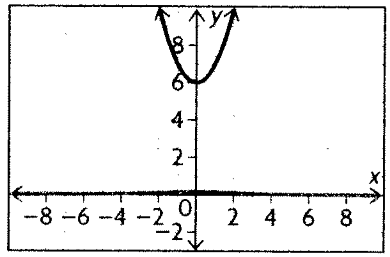
b) The function is $f(x) = x^2 - 4$. The function is a parabola. The coefficient of the first term is positive and so the parabola points up. This means that the domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} \mid y > -4\}$. Find the x-intercepts to help you determine the positive and negative intervals.

$0 = x^2 - 4$
$0 = (x + 2)(x - 2)$
$0 - 2 = x + 2 - 2$
$-2 = x$
$0 = x - 2$
$0 + 2 = x - 2 + 2$
$2 = x$

The x-intercepts are 2 and -2. This means that the vertex is at 0. The function is decreasing on $(-\infty, 0)$ and increasing $(0, \infty)$. That means that the function is positive on $(-\infty, -2)$ and $(2, \infty)$. The function is negative on $(-2, 2)$. Use this information to help you graph the function and its reciprocal.



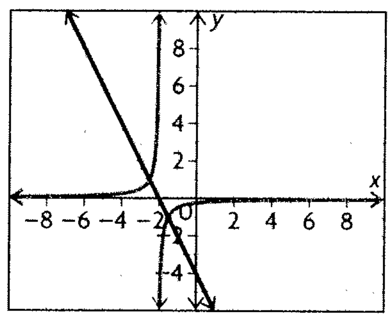
c) The function is $f(x) = x^2 + 6$. The function is a parabola. The coefficient of the first term is positive and so the parabola points up. This means that the range is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} \mid y > 6\}$. Because the range is $\{y \in \mathbf{R} \mid y > 6\}$, there are no x-intercepts and the function will never be negative. The vertex of the parabola will be $(0, 6)$. This means that the function will be decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$. Use this information to help you sketch the graph of $f(x) = x^2 + 6$ and its reciprocal.



d) The function is $f(x) = -2x - 4$. The function is a straight line. The domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R}\}$. Find the x-intercept to determine the positive and negative intervals.

$0 = -2x - 4$
$0 + 4 = -2x - 4 + 4$
$4 = -2x$
$\frac{4}{-2} = \frac{-2x}{-2}$
$-2 = x$

The x-intercept is $x = -2$. Examine the slope of the function. The slope is negative and therefore the function is always decreasing. This means that the function is positive on $(-\infty, -2)$ and negative on $(-2, \infty)$. Use this information to help you graph the function and the reciprocal.



3. Answers may vary. For example: (1) Hole: Both the numerator and the denominator contain a common factor, resulting in $\frac{0}{0}$ for a specific value of x.
(2) Vertical asymptote: A value of x causes the denominator of a rational function to be 0.
(3) Horizontal asymptote: A horizontal asymptote is created by the ratio between the numerator and the denominator of a rational function as the function approaches ∞ and $-\infty$. A continuous rational function is created when the denominator of the rational function has no zeros.

4. Vertical asymptotes can be found by finding the zero of the denominator. An equation will have a horizontal asymptote if the degree of the expression in the numerator is less than or equal to the degree of the expression in the denominator. An equation will have an oblique asymptote if the degree of the numerator is 1 greater than the degree of the denominator. A hole occurs when there is a common linear factor in the numerator and denominator.

a) The function is $y = \frac{x}{x-2}$.

$$0 = x - 2$$
$$0 + 2 = x - 2 + 2$$
$$2 = x$$

The equation of the vertical asymptote is $x = 2$.

b) The function is $y = \frac{x-1}{3x-3}$. In both the numerator and the denominator, $x - 1$ is a factor. This means that the function will have a straight line at $y = \frac{1}{3}$ and a hole at $x - 1$.

c) The function is $y = \frac{-7x}{4x+2}$.

$$0 = 4x + 2$$
$$0 - 2 = 4x + 2 - 2$$
$$-2 = 4x$$
$$-\frac{2}{4} = x$$
$$-\frac{1}{2} = x$$

The equation of the vertical asymptote is $x = -\frac{1}{2}$. The degrees of the numerator and denominator are equal. This means that there is a horizontal asymptote.

d) The function is $y = \frac{x^2+2}{x-6}$.

$$0 = x - 6$$
$$0 + 6 = x - 6 + 6$$
$$6 = x$$

The equation of the vertical asymptote is $x = 6$. The degree of the expression in the numerator is 1 larger than the expression in the denominator. This means that the function will have an oblique asymptote.

e) The equation is $y = \frac{1}{x^2 + 2x - 15}$.

$$0 = x^2 + 2x - 15$$
$$0 = (x + 5)(x - 3)$$
$$0 = x + 5$$
$$0 - 5 = x + 5 - 5$$
$$-5 = x$$
$$0 = x - 3$$
$$0 + 3 = x - 3 + 3$$
$$3 = x$$

The equations of the vertical asymptotes are $x = -5$ and $x = 3$. Since the numerator is a constant, the horizontal asymptote will be $x = 0$.

5. The functions that had horizontal asymptotes were: $y = \frac{x}{x-2}$, $y = \frac{-7x}{4x+2}$, and $y = \frac{1}{x^2+2x-15}$. Begin with $y = \frac{1}{x^2+2x-15}$. Because there is a constant in the numerator, the horizontal asymptote for this equation will be $x = 0$. For the other two equations, divide the first terms of the expressions in the numerator and denominator to find the equation of the horizontal asymptotes.

$$y = \frac{x}{x} = 1$$
$$y = \frac{-7x}{4x} = -\frac{7}{4}$$

6. a) The function is $f(x) = \frac{5}{x-6}$.

Vertical Asymptote:

$$0 = x - 6$$
$$0 + 6 = x - 6 + 6$$
$$6 = x$$

The horizontal asymptote will be $y = 0$ because the numerator is a constant.

x-intercept:

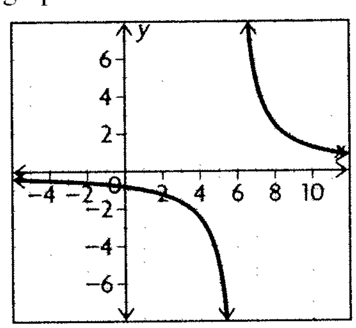
$$0 = \frac{5}{x-6}$$
$$(x - 6) \times 0 = \frac{5}{x-6}(x - 6)$$
$$0 = 5$$

The equation will not have an x-intercept.

y-intercept:

$$y = \frac{5}{0-6} = -\frac{5}{6}$$

Because the numerator is a constant, the function will be negative when the denominator is negative and positive when the numerator is positive. $x - 6$ is negative on $x < 6$. $f(x)$ is negative on $(-\infty, 6)$ and positive on $(6, \infty)$. Use this information to graph the function.



From the graph, it can be seen that the function is always decreasing.

b) The function is $f(x) = \dfrac{3x}{x + 4}$.

Vertical Asymptote:

$$0 = x + 4$$
$$0 - 4 = x + 4 - 4$$
$$-4 = x$$

Horizontal Asymptote:

$$y = \frac{3x}{x} = 3$$

x-intercept:

$$0 = \frac{3x}{x + 4}$$
$$(x + 4) \times 0 = \frac{3x}{x + 4} \times (x + 4)$$
$$0 = 3x$$
$$\frac{0}{3} = \frac{3x}{3}$$
$$0 = x$$

The x-intersect is $x = 0$.

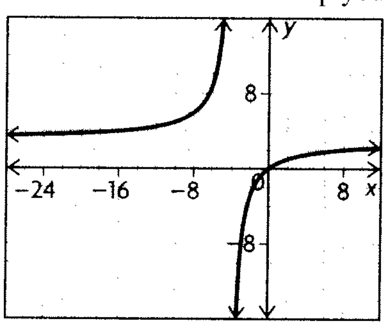
y-intercept:

$$f(0) = \frac{3(0)}{(0) + 4}$$
$$f(0) = 0$$

Use a table to determine when the equation is positive and negative.

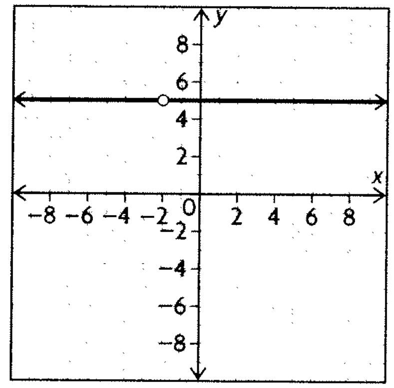
	$x < -4$	$-4 < x < 0$	$x > 0$
$3x$	$-$	$-$	$+$
$x + 4$	$-$	$+$	$+$
$\dfrac{3x}{x + 4}$	$+$	$-$	$+$

Use this information to help you graph the function.



From the graph it can be seen that the function is always increasing.

c) The function is $f(x) = \dfrac{5x + 10}{x + 2}$. Both the numerator and the denominator have a common factor, $x + 2$. $f(x) = \dfrac{5(x + 2)}{x + 2} = 5$. The function is a straight, horizontal line with a hole at $x = -2$. It will always be positive and will never increase or decrease. Use this information to graph the function.



d) The function is $f(x) = \dfrac{x - 2}{2x - 1}$.

Vertical Asymptote:

$$0 = 2x - 1$$
$$0 + 1 = 2x - 1 + 1$$
$$1 = 2x$$
$$\frac{1}{2} = \frac{2x}{2}$$
$$\frac{1}{2} = x$$

Horizontal Asymptote:

$$y = \frac{x}{2x} = \frac{1}{2}$$

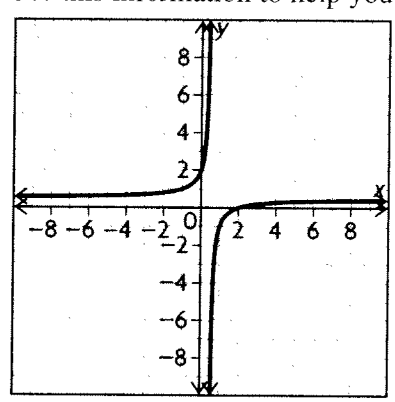
x-intercept:

$$0 = \frac{x - 2}{2x - 1}$$
$$(2x - 1) \times 0 = \frac{x - 2}{2x - 1} \times (2x - 1)$$
$$0 = x - 2$$
$$0 + 2 = x - 2 + 2$$
$$2 = x$$

y-intercept:

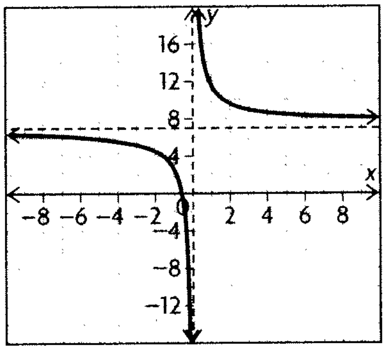
$$f(0) = \frac{5(0) + 10}{(0) + 2}$$
$$f(0) = 5$$

Use this information to help you graph the function.

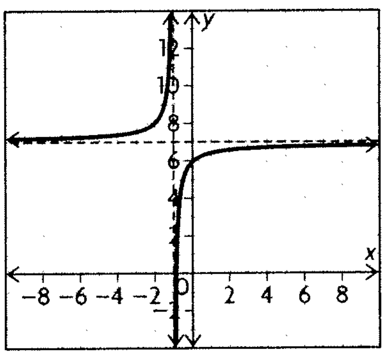


The function is always increasing.

7. Answers may vary. For example: Examine the function $y = \frac{7x + 6}{x}$.



The horizontal asymptote is $y = 7$ and the vertical asymptote is $x = 0$. Changing the function to $y = \frac{7x + 6}{x + 1}$ changes the graph.



The function now has a vertical asymptote at $x = -1$ and still has a horizontal asymptote at $y = 7$. However, the function is now constantly increasing instead of decreasing. The new function still has an x-intercept at $x = -\frac{6}{7}$, but now has a y-intercept at $y = 6$. To explain why the new function is always increasing instead of decreasing, examine the function near their respective asymptotes.

For $\frac{7x + 6}{x}$, when you have values of x that are less than 1, you have a number greater than 1 being divided by a fraction—which leads to larger and larger values of y as x gets closer to 0. But with $y = \frac{7x + 6}{x + 1}$ you don't have that. When $0 < x < 1$, you're dividing a number that is greater than 1 by another number that is greater than 1—giving you a fraction. When $-1 < x < 0$, then you have a number that is greater than 1 being divided by fraction. Additionally, when $-1 < x < -\frac{6}{7}$ the fraction is negative and approaching ∞ as x gets closer to -1. This is why the function is always decreasing instead of increasing.

8. Vertical asymptotes can be found by finding the zeros of the denominator of the rational function. In this case, $x = 6$.

$$\begin{aligned} 0 &= 2 - n(6) \\ -2 + 0 &= -2 + 2 - n(6) \\ -2 &= -n(6) \\ \frac{-2}{6} &= \frac{-n(6)}{6} \\ \frac{-1}{3} &= -n \\ n &= \frac{1}{3} \end{aligned}$$

The x-intercept of a function occurs when $y = 0$. In this case, the y-intercept will be $(5, 0)$.

$$\begin{aligned} 0 &= \frac{7(5) - m}{2 - n(5)} \\ (2 - n(5)) \times 0 &= \frac{7(5) - m}{2 - n(5)} \times 2 - n(5) \\ 0 &= 35 - m \\ 0 + m &= 35 - m + m \\ m &= 35 \end{aligned}$$

9. A graph that has a factor of $(x + 2)$ in both the numerator and denominator will have a domain of $\{x \in \mathbf{R} \mid x \neq -2\}$. If the only other factor in the denominator is a constant, there will also be no vertical asymptote. Answers may vary. For example:

$$f(x) = \frac{4x + 8}{x + 2}$$

The graph of the function will be a horizontal line at $y = 4$ with a hone at $x = -2$.

5.4 Solving Rational Equations, pp. 285–287

1. Substitute both values into the equation. If both sides are equal, then the two values are solutions. The equation is

$$\frac{2}{x} = \frac{x - 1}{3}$$

$$\frac{2}{x} = \frac{x - 1}{3}$$

$$\frac{2}{3} = \frac{3 - 1}{3}$$

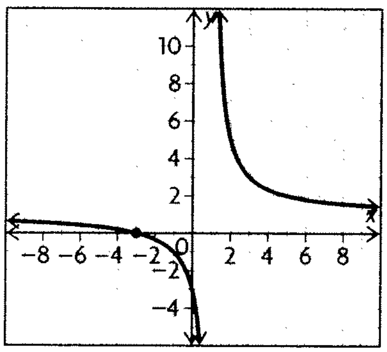
$$\frac{2}{3} = \frac{2}{3}$$

3 is a solution of the equation.

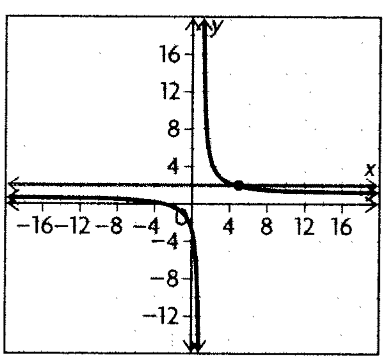
$$\frac{2}{x} = \frac{x-1}{3}$$
$$\frac{2}{-2} = \frac{-2-1}{3}$$
$$\frac{2}{-2} = \frac{-3}{3}$$
$$-1 = -1$$

-2 is a solution of the equation.

2. a)
$$\frac{x+3}{x-1} = 0$$
$$(x-1) \times \frac{x+3}{x-1} = 0 \times (x-1)$$
$$x + 3 = 0$$
$$x = -3$$

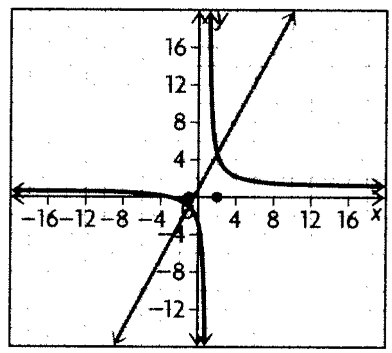


b)
$$\frac{x+3}{x-1} = 2$$
$$(x-1) \times \frac{x+3}{x-1} = 2 \times (x-1)$$
$$x + 3 = 2x - 2$$
$$x + 3 - 3 = 2x - 2 - 3$$
$$x = 2x - 5$$
$$x - 2x = 2x - 2x - 5$$
$$-x = -5$$
$$x = 5$$

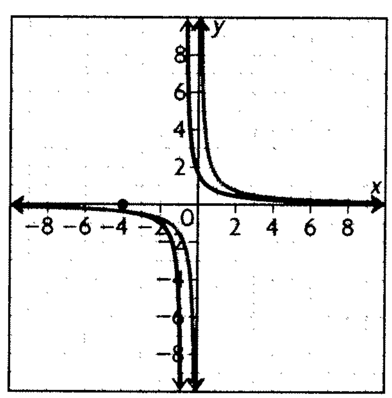


c)
$$\frac{x+3}{x-1} = 2x + 1$$
$$(x-1) \times \frac{x+3}{x-1} = (2x+1) \times (x-1)$$
$$x + 3 = (2x+1)(x-1)$$
$$x + 3 = 2x^2 + x - 2x - 1$$
$$x + 3 = 2x^2 - x - 1$$
$$0 = 2x^2 - 2x - 4$$
$$0 = 2(x^2 - x - 2)$$
$$0 = (x^2 - x - 2)$$
$$0 = (x-2)(x+1)$$
$$0 = x + 1 \text{ or } 0 = x - 2$$
$$-1 = x \qquad 2 = x$$

The solutions are $x = -1$ and 2.



d)
$$\frac{3}{3x+2} = \frac{6}{5x}$$
$$\frac{3}{3x+2} = \frac{6}{5x}$$
$$6(3x+2) = 3(5x)$$
$$18x + 12 = 15x$$
$$18x - 18x + 12 = 15x - 18x$$
$$12 = -3x$$
$$\frac{12}{-3} = \frac{-3x}{-3}$$
$$-4 = x$$



3. Move all expressions to one side, so that one side of the equation is 0.

a) $\dfrac{x-3}{x+3} - 2 = 0$

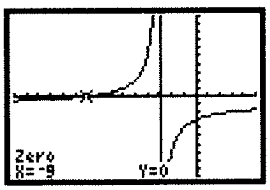
b) $\dfrac{3x-1}{x} - \dfrac{5}{2} = 0$

c) $\dfrac{x-1}{x} - \dfrac{x+1}{x+3} = 0$

d) $\dfrac{x-2}{x+3} - \dfrac{x-4}{x+5} = 0$

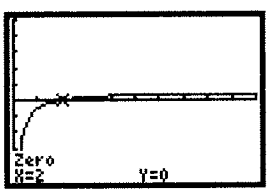
4. a) $\dfrac{x-3}{x+3} = 2$

$$(x+3) \times \frac{x-3}{x+3} = 2 \times (x+3)$$
$$x - 3 = 2x + 6$$
$$-9 = x$$



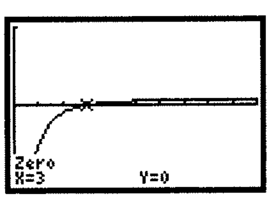
b) $\dfrac{3x-1}{x} = \dfrac{5}{2}$

$$(3x-1)2 = 5(x)$$
$$6x - 2 = 5x$$
$$x - 2 = 0$$
$$x = 2$$



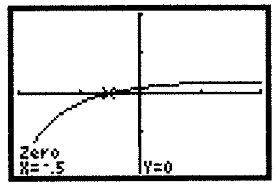
c) $\dfrac{x-1}{x} = \dfrac{x+1}{x+3}$

$$(x-1)(x+3) = x(x+1)$$
$$x^2 + 3x - x - 3 = x^2 + x$$
$$x^2 + 2x - 3 = x^2 + x$$
$$2x - 3 = x$$
$$x = 3$$



d) $\dfrac{x-2}{x+3} = \dfrac{x-4}{x+5}$

$$(x-2)(x+5) = (x+3)(x-4)$$
$$x^2 + 3x - 10 = x^2 - x - 12$$
$$4x = -2$$
$$x = -\frac{1}{2}$$



5. a) $\dfrac{2}{x} + \dfrac{5}{3} = \dfrac{7}{x}$

$$(3x)\left(\frac{2}{x} + \frac{5}{3} = \frac{7}{x}\right)$$
$$6 + 5x = 21$$
$$5x = 15$$
$$x = 3$$

b) $\dfrac{10}{x+3} + \dfrac{10}{3} = 6$

$$3(x+3)\left(\frac{10}{x+3} + \frac{10}{3} = 6\right)$$
$$30 + 10(x+3) = 18(x+3)$$
$$30 + 10x + 30 = 18x + 54$$
$$60 + 10x = 18x + 54$$
$$-8x = -6$$
$$x = \frac{3}{4}$$

c) $\dfrac{2x}{x-3} = 1 - \dfrac{6}{x-3}$

$$(x-3)\left(\frac{2x}{x-3} = 1 - \frac{6}{x-3}\right)$$
$$2x = 1(x-3) - 6$$
$$2x = (x-3) - 6$$
$$2x = x - 9$$
$$x = -9$$

d) $\dfrac{2}{x+1} + \dfrac{1}{x+1} = 3$

$$(x+1)\left(\frac{2}{x+1} + \frac{1}{x+1} = 3\right)$$
$$2 + 1 = 3x + 3$$
$$3 = 3x + 3$$
$$0 = 3x$$
$$x = 0$$

e) $$\frac{2}{2x+1} = \frac{5}{4-x}$$
$$\begin{aligned}
2(4-x) &= 5(2x+1)\\
8-2x &= 10x+5\\
8-5-2x &= 10x+5-5\\
3-2x &= 10x\\
3-2x+2x &= 10x+2x\\
3 &= 12x\\
\frac{3}{12} &= \frac{12x}{12}\\
\frac{1}{4} &= x
\end{aligned}$$

f) $$\frac{5}{x-2} = \frac{4}{x+3}$$
$$\begin{aligned}
5(x+3) &= 4(x-2)\\
5x+15 &= 4x-8\\
5x+15-15 &= 4x-8-15\\
5x+15-15 &= 4x-8-15\\
5x &= 4x-23\\
5x-4x &= 4x-4x-23\\
x &= -23
\end{aligned}$$

6. a) $$\frac{2x}{2x+1} = \frac{5}{4-x}$$
$$\begin{aligned}
2x(4-x) &= 5(2x+1)\\
8x-2x^2 &= 10x+5\\
8x-2x^2+2x^2 &= 2x^2+10x+5\\
8x &= 2x^2+10x+5\\
8x-8x &= 2x^2+10x-8x+5\\
0 &= 2x^2+2x+5
\end{aligned}$$
Examine the equation. Notice that it will not have any real zeros. Therefore, the function will have no real solutions.

b) $$\frac{3}{x} + \frac{4}{x+1} = 2$$
$$\begin{aligned}
x(x+1)\frac{3}{x} + x(x+1)\frac{4}{x+1} &= x(x+1)2\\
3+7x &= 2x^2+2x\\
3-3+7x-7x &= 2x^2+2x-7x-3\\
0 &= 2x^2-5x-3
\end{aligned}$$
Use the quadratic formula to solve the quadratic equation.
$x = 3$ and $x = -0.5$

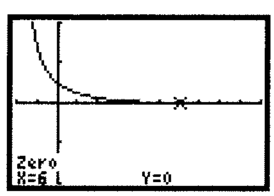
c) $$\frac{2x}{5} = \frac{x^2-5x}{5x}$$
$$\begin{aligned}
2x \times 5x &= 5(x^2-5x)\\
10x^2 &= 5x^2-25x\\
10x^2-5x^2 &= 5x^2-5x^2-25x\\
5x^2+25x &= -25x+25x\\
5x^2+25x &= 0\\
5x(x+5) &= 0
\end{aligned}$$
$5x = 0$ or $x+5 = 0$
$x+5-5 = 0-5$
$x = 0$ $\quad x = -5$

d) $$x + \frac{x}{x-2} = 0$$
$$\begin{aligned}
(x-2)x + (x-2)\frac{x}{x-2} &= (x-2)0\\
x^2-2x+x &= 0\\
x^2+x &= 0\\
x(x+1) &= 0
\end{aligned}$$
$x = 0$ or $x+1 = 0$
$$\begin{aligned}
x+1 &= 0\\
x+1-1 &= 0-1\\
x &= -1
\end{aligned}$$

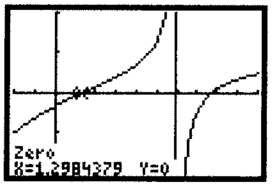
e) $$\frac{1}{x+2} + \frac{24}{x+3} = 13$$
$$\begin{aligned}
&(x+2)(x+3)\frac{1}{x+2} + (x+2)(x+3)\frac{24}{x+3}\\
&= 13(x+2)(x+3)\\
x+3+24(x+2) &= 13(x^2+5x+6)\\
25x+5 &= 13x^2+65x+78\\
25x-25x+5-5 &= 13x^2+65x-25x+78-5\\
0 &= 13x^2+65x-25x+78-5\\
0 &= 13x^2+40x+73
\end{aligned}$$
Examine the quadratic equation. There are no real zeros and that means that the original equation has no real solutions.

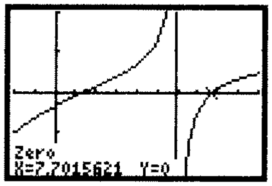
f) $$\frac{-2}{x-1} = \frac{x-8}{x+1}$$
$$\begin{aligned}
-2(x+1) &= (x-8)(x-1)\\
-2x-2 &= x^2-9x+8\\
0 &= x^2-7x+10\\
0 &= (x-5)(x-2)
\end{aligned}$$
$0 = x-5$ or $0 = x-2$
$x = 5$ $\quad x = 2$

7. Move all terms to one side of the equation so that one side of the equation is 0. Graph the expression on the other side and use the zero function of the calculator to solve.

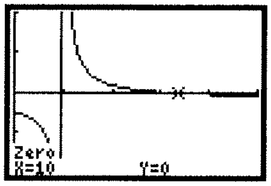
a)


$x = 6$

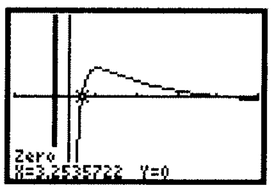
b) 

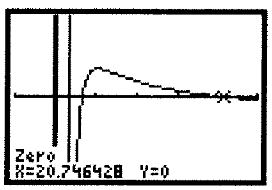


$x = 1.30, 7.70$

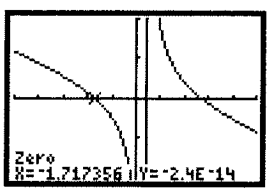
c) 

$x = 10$

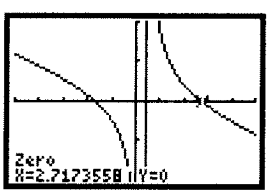
d) 

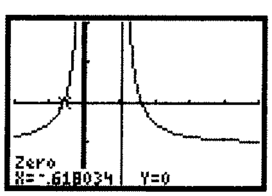


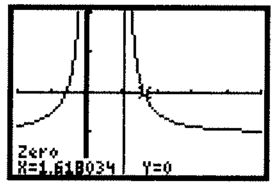
$x = 3.25, 20.75$

e) 

$x = -1.71, 2.71$



f) 



$x = -0.62, 1.62$

8. a) $\dfrac{x+1}{x-2} = \dfrac{x+3}{x-4}$

Multiply both sides of the equation by the LCD, $(x-2)(x-4)$.

$$(x-2)(x-4)\left(\frac{x+1}{x-2}\right) = (x-2)(x-4)\left(\frac{x+3}{x-4}\right)$$

$$(x-4)(x+1) = (x-2)(x+3)$$

Simplify.

$$x^2 - 3x - 4 = x^2 + x - 6$$

Simplify the equation so that 0 is on one side of the equation.

$$x^2 - x^2 - 3x - x - 4 + 6 = x^2 - x^2 + x - x - 6 + 6$$

$$-4x + 2 = 0$$

$$-2(2x - 1) = 0$$

Since the product is equal to 0 one of the factors must be equal to 0. It must be $2x - 1$ because 2 is a constant.

$$2x - 1 = 0$$

$$2x - 1 + 1 = 0 + 1$$

$$2x = 1$$

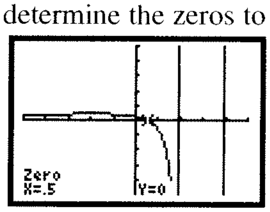
$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

b) Substitute $x = \frac{1}{2}$ to verify the solution.

$$\frac{\frac{1}{2}+1}{\frac{1}{2}-2} = -1 \text{ and } \frac{\frac{1}{2}+3}{\frac{1}{2}-4} = -1$$

c) Graph the equation $\dfrac{(x+1)}{(x-2)} - \dfrac{(x+3)}{(x-4)}$ and determine the zeros to verify the solution.



9. Multiply both sides by the LCD, $w(15 - w)$

$$w(15 - w)\left(\frac{15}{w}\right) = w(15 - w)\left(\frac{w}{15 - w}\right)$$

$$(15 - w)(15) = w^2$$

$$225 - 15w = w^2$$

$$225 - 225 - 15w + 15w = w^2 - 15w - 225$$
$$0 = w^2 - 15w - 225$$
$$w^2 - 15w - 225 = 0$$

Use the quadratic equation to help you solve the quadratic formula.

$w = 9.271$ and $w = -24.27$

Since a width has to be positive, $w = 9.271$.

10. Machine A has a rate of $\frac{1}{s}$ boxes/minute.

Machine B has a rate of $\frac{1}{s + 10}$ boxes/minute.

Their combined rate is $\frac{1}{s} + \frac{1}{s + 10} = 15$. Solve this equation for s.

$$\frac{1}{s} + \frac{1}{s + 10} = \frac{1}{15}$$
$$15s(s + 10)\left(\frac{1}{s} + \frac{1}{s + 10} = \frac{1}{15}\right)$$
$$15(s + 10) + 15s = s(s + 10)$$
$$30s + 150 = s^2 + 10s$$
$$30s - 30s + 150 - 150 = s^2 + 10s - 30s - 150$$
$$0 = s^2 - 20s - 150$$

Use the quadratic formula to help you solve the quadratic equation.

$s = 25.8$

Machine A takes 25.8 and Machine B takes 35.8 minutes.

11. The price per comic in the box that Tayla purchase is $\frac{300}{s}$, where s is the number of comics in the box. She gave 15 away, and so the number of comics in the box becomes $s - 15$. The price per comic in the box when she resold the box on the Internet then is $\frac{330}{s - 15}$. Tayla made a profit of \$1.50 on each comic, which is the sale price per comic minus the original purchase price per comic. Solve the equation $\frac{330}{s - 15} - \frac{300}{s} = 1.50$ to find the original number of comics.

$$\frac{330}{s - 15} - \frac{300}{s} = 1.50$$
$$s(s - 15)\left(\frac{330}{s - 15} - \frac{300}{s} = 1.50\right)$$
$$330s - 300(s - 15) = 1.5s(s - 15)$$
$$330s - 300s + 4500 = 1.5s^2 - 22.5s$$
$$30s + 4500 = 1.5s^2 - 22.5s$$
$$30s - 30s + 4500 - 4500 = 1.5s^2 - 30s - 22.5s - 4500$$
$$0 = 1.5s^2 - 52.5s - 4500$$

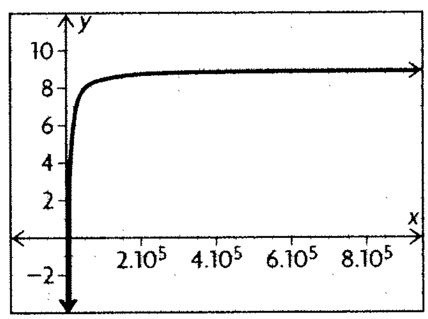
The roots are 75.00 and -40. Since you can't have a negative number of comics, the correct answer would be 75. The original price per comic would be $\frac{300}{75} = \$4$. The resale price per comic would be $\frac{300}{60} = \$5.50$.

12. a) Substitute 6 into the formula for $c(t)$ and solve for t.

$$6 = 9 - 90\,000\left(\frac{1}{10\,000 + 3t}\right)$$
$$6 - 9 = 9 - 9 - 90\,000\left(\frac{1}{10\,000 + 3t}\right)$$
$$-3(10\,000 + 3t) = -90\,000\left(\frac{1}{10\,000 + 3t}\right)(10\,000 + 3t)$$
$$-30\,000 - 9t = -90\,000$$
$$-30\,000 + 30\,000 - 9t = -90\,000 + 30\,000$$
$$-9t = -60\,000$$
$$\frac{-9t}{-9} = \frac{-60\,000}{-9}$$
$$t = 6666.67$$

After 6666.67 seconds the concentration will be 6 kg/m^3.

b) Graph the function to help you understand the function's behaviour over time.



The function appears to approach 9 kg/m^3 as time increases.

13. a) Tom can fill $\frac{1}{s}$ of an order in 1 minute. Paco and Carl's rates are similar: $\frac{1}{s - 2}$ and $\frac{1}{s + 1}$.

Working together Tom and Paco can fill an order in about 1 minute and 20 seconds, or about 1.33 minutes. Solve $\frac{1}{s - 2} + \frac{1}{s} = \frac{1}{1.33}$ to find how long it takes each person to fill an order.

$$\frac{1}{s - 2} + \frac{1}{s} = \frac{1}{1.33}$$
$$\left(\frac{s}{s}\right)\frac{1}{s - 2} + \frac{1}{s}\left(\frac{s - 2}{s - 2}\right) = \frac{1}{1.33}$$

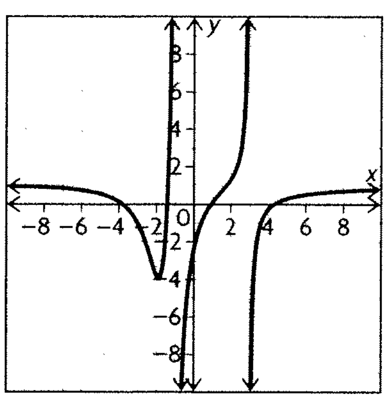
$$\frac{s}{(s)(s-2)} + \frac{s-2}{(s)(s-2)} = \frac{1}{1.33}$$
$$\frac{2s-2}{(s)(s-2)} = \frac{1}{1.33}$$
$$1.33(2s-2) = s^2 - 2s$$
$$0 = s^2 - 4.66 + 2.66$$
Use the quadratic formula to solve this equation. The roots of the function are $s = 3.994$ or 0.66. Because Paco can fill the order in 2 minutes less than Tom, and because you can't have a negative amount of time, Tom's time must be 3.994, or about 4 minutes. So, Paco can fill the order in about 2 minutes and Carl can fill the order in about 5.

b) Add their rates together to determine how long it would take them to fill the order working together.
$$\frac{1}{5} + \frac{1}{4} + \frac{1}{2} = 0.95$$
$$\frac{1}{0.95} = 1.05$$
Working together they can fill the order in about 1.05 minutes.

14. Answers may vary. For example: You can use either algebra or graphing technology to solve a rational equation. With algebra, solving the equation takes more time, but you get an exact answer. With graphing technology, you can solve the equation quickly, but you do not always get an exact answer.

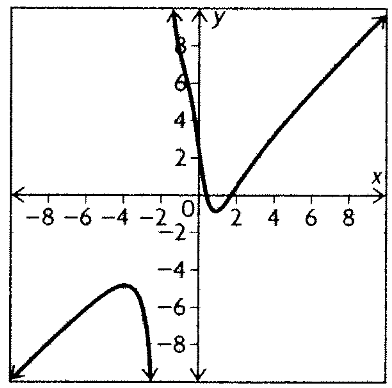
15. $\frac{x^2 - 6x + 5}{x^2 - 2x - 3} = \frac{2 - 3x}{x^2 + 3x + 3}$

Turn this into an equation that you can graph to find the solutions.
$$y = \frac{x^2 - 6x + 5}{x^2 - 2x - 3} - \frac{2 - 3x}{x^2 + 3x + 3}$$
Graph the equation.



The solutions are approximately $x = -3.80, -1.42, 0.90$, and 4.33.

16. a) The graphs will have the same position when their equations are equal. Set the two equations equal to each other and then solve for t. Graph the equation to help you.
$$\frac{7t}{t^2 + 1} = t + \frac{5}{t + 2}$$
$$y = t + \frac{5}{t + 2} - \frac{7t}{t^2 + 1}$$
Graph the equation.

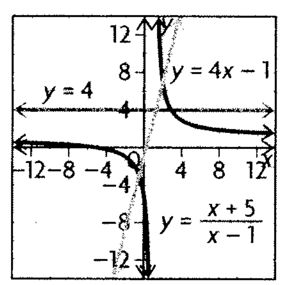


Examine the graph. The zeros are $x = 0.438$ and 1.712.

b) Object A is closer to the origin than object B when $\frac{7t}{t^2 + 1} < t + \frac{5}{t + 2}$ or when $0 < t + \frac{5}{t + 2} - \frac{7t}{t^2 + 1}$. Examine the graph and find when the function is positive to solve the inequality. The graph shows that the inequality is true on $(0, 0.438)$ and $(1.712, \infty)$.

5.5 Solving Rational Inequalities, pp. 295–297

1. Use the graph given to help you solve the inequalities.



a) $\frac{x + 5}{x - 1} < 4$

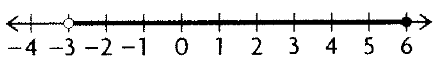
Examine the graph. To determine when $\frac{x + 5}{x - 1} < 4$, determine when the green curve is below the blue line. This is true on the intervals $(\infty, 1)$ and $(3, \infty)$.

b) $4x - 1 > \dfrac{x+5}{x-1}$

Examine the graph. To determine when $4x - 1 > \frac{x+5}{x-1}$, determine when the red line is above the green curve. This is true on the intervals $(-0.5, 1)$ and $(2, \infty)$.

2. a) Solve the inequality for x.

$$\frac{6x}{x+3} \le 4$$
$$\frac{6x}{x+3} - 4 \le 0$$
$$\frac{6x}{x+3} - 4\frac{x+3}{x+3} \le 0$$
$$\frac{6x - 4x - 12}{x+3} \le 0$$
$$\frac{2x - 12}{x+3} \le 0$$
$$\frac{2(x-6)}{x+3} \le 0$$

b) 

c) The solution is $(-3, 6]$.

3. a)
$$x + 2 > \frac{15}{x}$$
$$x + 2 - \frac{15}{x} > 0$$
$$\frac{x^2}{x} + \frac{2x}{x} - \frac{15}{x} > 0$$
$$\frac{x^2 + 2x - 15}{x} > 0$$
$$\frac{(x+5)(x-3)}{x} > 0$$

b)

	$x < -5$	$-5 < x < 0$	$0 < x < 3$	$x > 3$
$x + 5$	−	+	+	+
$x - 3$	−	−	−	+
x	−	−	+	+
$\frac{(x+5)(x-3)}{x}$	−	+	−	+

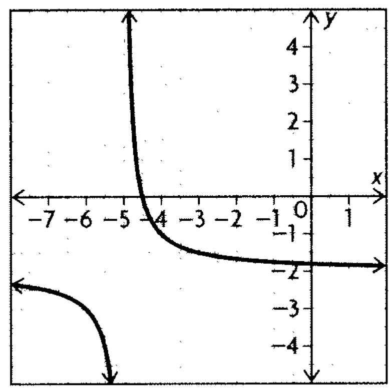
The equation is negative on $x < -5$ and $0 < x < 3$ and positive on $-5 < x < 0$ and $x > 3$.

c) The solution to the equation is $\{x \in \mathbf{R} \mid x > -5\}$. This can also be written as $(-5, \infty)$.

4. a)
$$\frac{1}{x+5} > 2$$
$$\frac{1}{x+5} - 2 > 0$$
$$\frac{1}{x+5} - 2\left(\frac{x+5}{x+5}\right) > 0$$
$$\frac{1}{x+5} + \frac{-2x - 10}{x+5} > 0$$
$$\frac{-2x - 9}{x+5} > 0$$

	$x < -5$	$-5 < x < -4.5$	$x > -4.5$
$-2x - 9$	+	+	−
$x + 5$	−	+	+
$\frac{-2x-9}{x+5}$	−	+	−

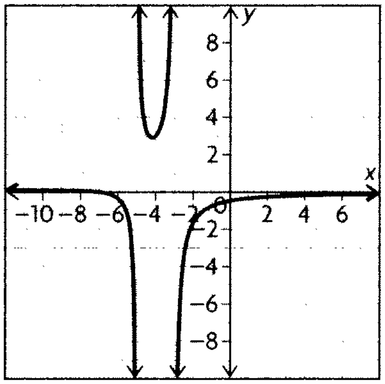
The inequality is true on $-5 < x < -4.5$.



b)
$$\frac{1}{2x+10} < \frac{1}{x+3}$$
$$\frac{1}{x+5} - \frac{1}{x+3} < 0$$
$$\left(\frac{x+3}{x+3}\right)\frac{1}{x+5} - \frac{1}{x+3}\left(\frac{x+5}{x+5}\right) < 0$$
$$\frac{x+3}{(x+5)(x+3)} + \frac{-2x - 10}{(x+3)(x+5)} < 0$$
$$\frac{-x - 7}{(x+5)(x+3)} < 0$$

	$x < -7$	$-7 < x < -5$	$-5 < x < -3$	$x > -3$
$-x - 7$	+	−	−	−
$x + 5$	−	−	+	+
$x + 3$	−	−	−	+
$\frac{-x-7}{(x+5)(x+3)}$	+	−	+	−

The inequality is true on $-7 < x < -5$ and $x > -3$.



c)

$$\frac{3}{x-2} < \frac{4}{x}$$

$$\frac{3}{x-2} - \frac{4}{x} < 0$$

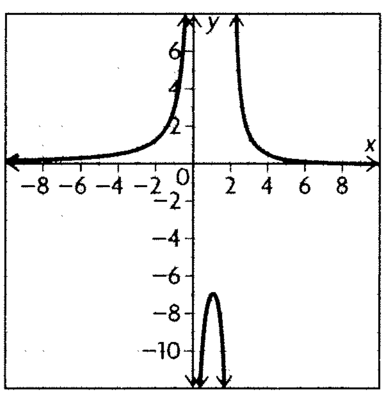
$$\left(\frac{x}{x}\right)\frac{3}{x-2} - \frac{4}{x}\left(\frac{x-2}{x-2}\right) < 0$$

$$\frac{3x}{x(x-2)} + \frac{-4x+8}{x(x-2)} < 0$$

$$\frac{-x+8}{x(x-2)} < 0$$

	$x < 0$	$0 < x < 2$	$2 < x < 8$	$x > 8$
$-x + 8$	+	+	+	−
x	−	+	+	+
$x - 2$	−	−	+	+
$\frac{-x+8}{x(x-2)}$	+	−	+	−

The inequality is true on $0 < x < 2$ and $x > 8$.



d)

$$\frac{7}{x-3} \geq \frac{2}{x+4}$$

$$\frac{7}{x-3} - \frac{2}{x+4} \geq 0$$

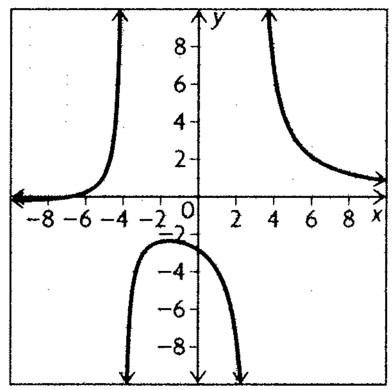
$$\left(\frac{x+4}{x+4}\right)\frac{7}{x-3} - \frac{2}{x+4}\left(\frac{x-3}{x-3}\right) \geq 0$$

$$\frac{7x+28}{(x+4)(x-3)} + \frac{-2x+6}{(x+4)(x-3)} \geq 0$$

	$x < -6.8$	$-6.8 < x < -4$	$-4 < x < 3$	$x > 3$
$5x + 34$	−	+	+	+
$x + 4$	−	−	+	+
$x - 3$	−	−	−	+
$\frac{5x+34}{(x+4)(x-3)}$	−	+	−	+

$$\frac{5x+34}{(x+4)(x-3)} \geq 0$$

The inequality is true on $-6.8 \leq x < -4$ and $x > 3$.



e)

$$\frac{-6}{x+1} > \frac{1}{x}$$

$$\frac{-6}{x+1} - \frac{1}{x} > 0$$

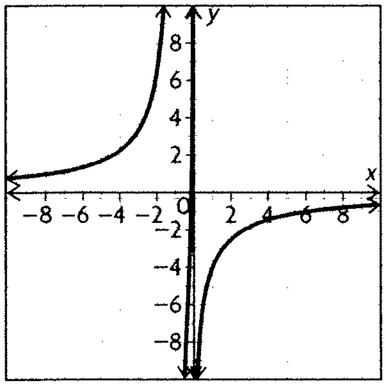
$$\left(\frac{x}{x}\right)\frac{-6}{x+1} - \frac{1}{x}\left(\frac{x+1}{x+1}\right) > 0$$

$$\frac{-6x}{x(x+1)} + \frac{-x-1}{x(x+1)} > 0$$

$$\frac{-7x-1}{x(x+1)} > 0$$

	$x < -1$	$-1 < x < -\frac{1}{7}$	$-\frac{1}{7} < x < 0$	$x > 0$
$-7x - 1$	+	+	−	−
x	−	−	−	+
$x + 1$	−	+	+	+
$\frac{-7x-1}{x(x+1)}$	+	−	+	−

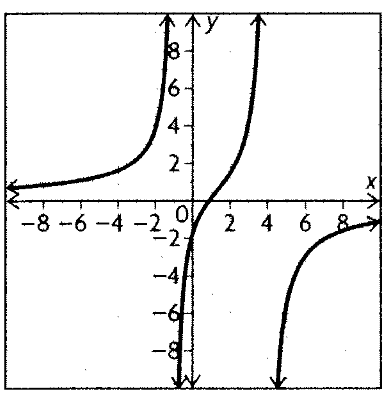
The inequality is true on $x < -1$ and $-\frac{1}{7} < x < 0$.



f)
$$\frac{-5}{x-4} < \frac{3}{x+1}$$
$$\frac{-5}{x-4} - \frac{3}{x+1} < 0$$
$$\left(\frac{x+1}{x+1}\right)\frac{-5}{x-4} - \frac{3}{x+1}\left(\frac{x-4}{x-4}\right) < 0$$
$$\frac{-5x-5}{(x+1)(x-4)} + \frac{-3x+12}{(x+1)(x-4)} < 0$$
$$\frac{-8x+7}{(x+1)(x-4)} < 0$$

	$x<-1$	$-1<x<\frac{7}{8}$	$\frac{7}{8}<x<4$	$x>4$
$-8x+7$	+	+	−	−
$x-4$	−	−	−	+
$x+1$	−	+	+	+
$\frac{-8x+7}{(x+1)(x-4)}$	+	−	+	−

The inequality is true on $-1 < x < \frac{7}{8}$ and $x < 4$.



5. a) $\frac{t^2-t-12}{t-1} < 0$

$$\frac{t^2-t-12}{t-1} = \frac{(t-4)(t+3)}{t-1}$$

	$t<-3$	$-3<t<1$	$1<t<4$	$t>4$
$t-4$	−	−	−	+
$t+3$	−	+	+	+
$t-1$	−	−	+	+
$\frac{(t-4)(t+3)}{t-1}$	−	+	−	+

The inequality is true on $t < -3$ or $1 < t < 4$.

b) $\frac{t^2+t-6}{t-4} \geq 0$

$$\frac{(t+3)(t-2)}{t-4} \geq 0$$

	$t<-3$	$-3<t<2$	$2<t<4$	$t>4$
$t+3$	−	+	+	+
$t-2$	−	−	+	+
$t-4$	−	−	−	+
$\frac{(t+3)(t-2)}{t-4}$	−	+	−	+

The inequality is true on $-3 \leq t \leq 2$ and $t > 4$.

c) $\frac{6t^2-5t+1}{2t+1} > 0$

$$\frac{(3t-1)(2t-1)}{2t-1} > 0$$

The function is true on $-\frac{1}{2} < x < \frac{1}{3}$ and $x > \frac{1}{2}$

	$t<-\frac{1}{2}$	$-\frac{1}{2}<t<\frac{1}{3}$	$\frac{1}{3}<t<\frac{1}{2}$	$t>\frac{1}{2}$
$3t-1$	−	−	+	+
$2t-1$	−	−	−	+
$2t+1$	−	+	+	+
$\frac{(3t-1)(2t-1)}{2t+1}$	−	+	−	+

d) $t - 1 < \frac{30}{5t}$

$$\left(\frac{5t}{5t}\right)t - \left(\frac{5t}{5t}\right)1 - \frac{30}{5t} < 0$$
$$\frac{5t^2}{5t} - \frac{5t}{5t} - \frac{30}{5t} < 0$$
$$\frac{5t^2-5t-30}{5t} < 0$$
$$\frac{5(t^2-t-6)}{5t} < 0$$
$$\frac{(t-3)(t+2)}{t} < 0$$

	$t<-2$	$-2<t<0$	$0<t<3$	$t>3$
$t-3$	−	−	−	+
$t+2$	−	+	+	+
t	−	−	+	+
$\frac{(t-3)(t+2)}{t}$	−	+	−	+

The inequality is true for $t < -2$ and $0 < t < 3$.

e)
$$\frac{2t-10}{t} > t+5$$
$$\frac{2t-10}{t} - t - 5 > 0$$
$$\frac{2t-10}{t} - t\left(\frac{t}{t}\right) - 5\left(\frac{t}{t}\right) > 0$$
$$\frac{2t-10-t^2-5t}{t} > 0$$
$$\frac{-t^2-7t-10}{t} > 0$$
$$\frac{t^2+7t+10}{t} < 0$$
$$\frac{(t+5)(t+2)}{t} < 0$$

	$t<-5$	$-5<t<-2$	$-2<t<0$	$t>0$
$t+5$	−	+	+	+
$t+2$	−	−	+	+
t	−	−	−	+
$\frac{(t+5)(t+2)}{t}$	−	+	−	+

The inequality is true on $t < -5$ and $-2 < t < 0$.

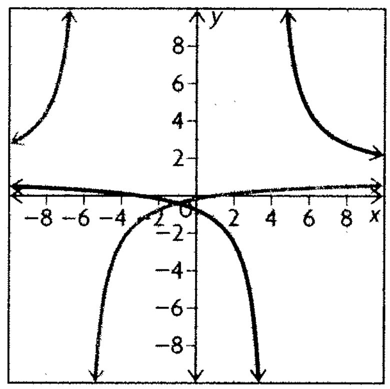
f)
$$\frac{-t}{4t-1} \geq \frac{2}{t-9}$$
$$\frac{-t}{4t-1} - \frac{2}{t-9} \geq 0$$
$$\left(\frac{t-9}{t-9}\right)\frac{-t}{4t-1} - \frac{2}{t-9}\left(\frac{4t-1}{4t-1}\right) \geq 0$$
$$\frac{-t^2+9t}{(4t-1)(t-9)} + \frac{-8t+2}{(4t-1)(t-9)} \geq 0$$
$$\frac{-t^2+t+2}{(4t-1)(t-9)} \geq 0$$
$$\frac{(-t+2)(t+1)}{(4t-1)t-9)} \geq 0$$

	$t<-1$	$-1<t<0.25$	$0.25<t<2$	$2<t<9$	$t>9$
$t+1$	−	+	+	+	+
$-t+2$	+	+	+	−	−
$4t-1$	−	−	+	+	+
$t-9$	−	−	−	−	+
$\frac{(-t+2)(t+1)}{(4t-1)(t-9)}$	−	+	−	+	−

The inequality is true on $-1 \leq t < 0.25$ and $2 \leq t < 9$.

6. Graph each expression to determine when the inequality is true.

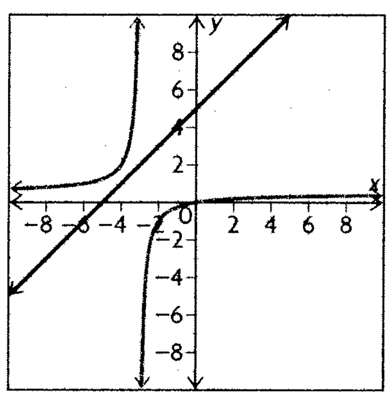
a) $\frac{x+3}{x-4} \geq \frac{x-1}{x+6}$



The two graphs intersect at $(-1, -0.4)$. The asymptotes are at $x = 4$ and $x = -6$. The graph of $y = \frac{x+3}{x-4}$ is above $y = \frac{x-1}{x+6}$ on $x < -6$ and on $-1 < x < 4$.

b) $x + 5 < \frac{x}{2x+6}$

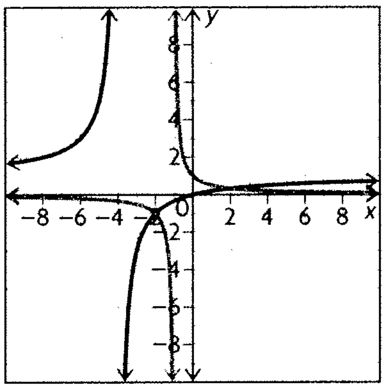
Graph each expression to determine when the inequality is true.



Notice that the graph of $y = x + 5$ is above the graph of $y = \frac{x}{2x + 6}$ after the vertical asymptote. The vertical asymptote occurs at $x = -3$. The solution is $x > 3$.

c) $\frac{x}{x + 4} \leq \frac{1}{x + 1}$

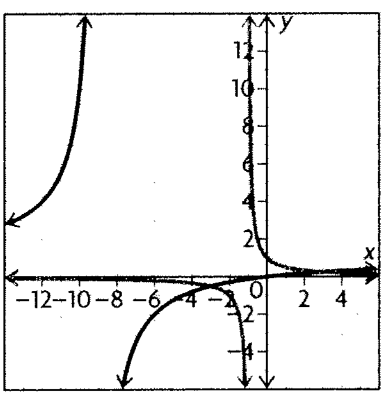
Graph each expression to determine when the inequality is true.



The graphs intersect at $(-2, -1)$ and $\left(2, \frac{1}{3}\right)$. The graph of $y = \frac{x}{x + 4}$ is below the other graph on $(-4, 2)$ and $(-1, 2)$.

d) $\frac{x}{x + 9} \geq \frac{1}{x + 1}$

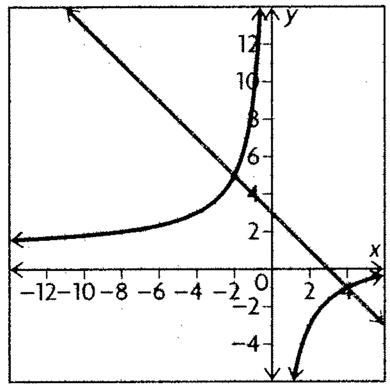
Graph each expression to determine when the inequality is true.



The graphs intersect at $(-3, -0.5)$ and $(3, 0.25)$. Because the inequality is $\geq$, the intervals that make the inequality true will include the points of intersection. The graph of $y = \frac{x}{x + 9}$ is above or intersecting with the other graph on $(-\infty, -9)$, $[-3, -1)$, and $[3, \infty)$.

e) $\frac{x - 8}{x} > 3 - x$

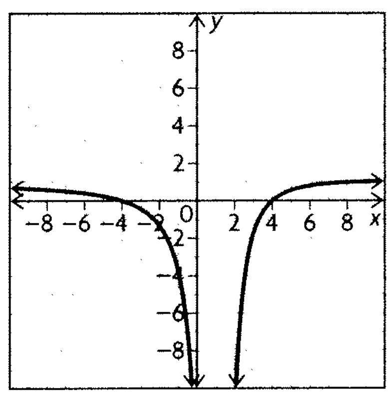
Graph each expression to determine when the inequality is true.



The two graphs intersect at $(-2, 5)$ and $(4, -1)$. The graph of $y = \frac{x - 8}{x}$ is above the other graph on $(-2, 0)$ and $(4, \infty)$.

f) $\frac{x^2 - 16}{(x - 1)^2} \geq 0$

Graph the expression and determine when the graph is above the x-axis.



The graph intersects the x-axis at $(-4, 0)$ and $(4, 0)$. The graph of $y = \frac{x^2 - 16}{(x - 1)^2}$ is above the x-axis on $(-\infty, -4)$ and $(4, \infty)$.

7. a)

$$\frac{3x-8}{2x-1} > \frac{x-4}{x+1}$$

$$\frac{3x-8}{2x-1} - \frac{x-4}{x+1} > 0$$

$$\left(\frac{x+1}{x+1}\right)\frac{3x-8}{2x-1} - \frac{x-4}{x+1}\left(\frac{2x-1}{2x-1}\right) > 0$$

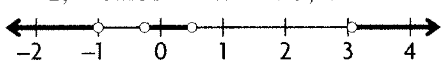
$$\frac{3x^2-8x+3x-8}{(x+1)(2x-1)} + \frac{2x^2-x-8x+4}{(x+1)(2x-1)} > 0$$

$$\frac{5x^2-14x-4}{(x+1)(2x-1)} > 0$$

$$\frac{(x-3.065)(x+0.2614)}{(x+1)(2x-1)} > 0$$

	$x<-1$	$-1<x<-0.2614$	$-0.2614<x<0.5$	$0.5<x<3.065$	$x>3.065$
$x-3.065$	−	−	−	−	+
$x+0.2614$	−	−	+	+	+
$x+1$	−	+	+	+	+
$2x-1$	−	−	−	+	+
$\frac{(x-3.065)(x+0.2614)}{(x+1)(2x-1)}$	+	−	+	−	+

The inequality is true on $x<-1$, $-0.2614<x<0.5$, and $x>3.065$.

b) 

Interval notation:
$(-\infty,-1)$, $(-0.2614, 0.5)$, $(3.065, \infty)$
Set notation:
$\{x \in \mathbf{R} | x < -1, -0.2614 < x < 0.5, \text{ or } x > 3.065\}$

8. a)

$$\frac{-6t}{t-2} < \frac{-30}{t-2}$$

$$\frac{-6t}{t-2} - \frac{-30}{t-2} < 0$$

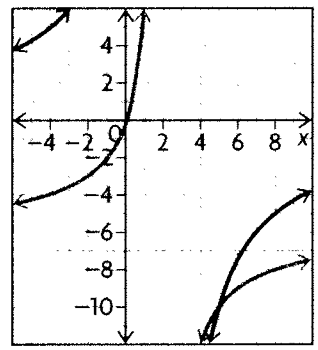
$$\frac{-6t+30}{t-2} < 0$$

$$\frac{-6(t-5)}{t-2} < 0$$

	$t<2$	$2<t<5$	$t>5$
$t-5$	−	−	+
-6	−	−	−
$t-2$	−	+	+
$\frac{-6(t+5)}{t-2}$	−	+	−

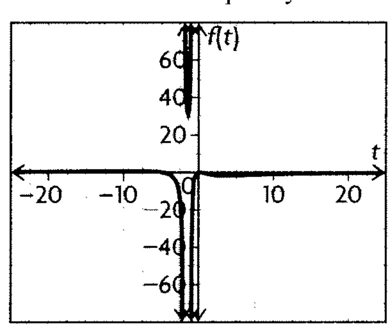
The inequality is true on $t<2$ and $t<5$.

b)



c) It would be difficult to find a situation that could be represented by these rational expressions because very few positive values of x yield a positive value of y.

9. The equation that gives the bacteria count over time for the tap water is $f(t) = \frac{5t}{t^2+3t+2}$. The equation that gives the bacteria count for the pond water over time is $g(t) = \frac{15t}{t^2+9}$. To see if the bacteria count in the tap water will ever exceed that of the pond water, set up the inequality $\frac{5t}{t^2+3t+2} > \frac{15t}{t^2+9}$. Solve this inequality graphically. Graph the expression $y = \frac{5t}{t^2+3t+2} - \frac{15t}{t^2+9}$ and determine when it is greater than 0 to find the solution to the inequality.



Notice that the only values that make the expression greater than 0 are negative. Because the values of t have to be positive, the bacteria count in the tap water will never be greater than that of the pond water.

10. a)
$$0.5x - 2 < \frac{5}{2x}$$
$$0.5x - 2 - \frac{5}{2x} < 0$$
$$\left(\frac{2x}{2x}\right)0.5x - \left(\frac{2x}{2x}\right)2 - \frac{5}{2x} < 0$$
$$\frac{x^2 - 4x - 5}{2x} < 0$$
$$\frac{(x-5)(x+1)}{2x} < 0$$

b)

	$x < -1$	$-1 < x < 0$	$0 < x < 5$	$x > 5$
$x - 5$	−	−	−	+
$x + 1$	−	+	+	+
$2x$	−	−	+	+
$\frac{(x-5)(x+1)}{2x}$	−	+	−	+

The inequality is true for $x < -1$ and $0 < x < 5$.

11. The profit would be the revenue minus the cost, $R(x) - C(x) = P(x)$. This is $P(x) = -x^2 + 10x - (4x + 5)$. Simplify.

$P(x) = -x^2 + 10x - (4x + 5)$
$= -x^2 + 6x - 5$
$= x^2 - 6x + 5$
$= (x - 1)(x - 5)$

Divide this by x to get the average profit.

$AP(x) = \frac{(x-1)(x-5)}{x}$

Substitute the factors of this equation into a table to determine when $AP(x) > \frac{(x-1)(x-5)}{x}$.

	$x < 0$	$0 < x < 1$	$1 < x < 5$	$x > 5$
$(x - 5)$	−	−	−	+
$(x + 1)$	−	−	+	+
x	−	+	+	+
$\frac{(x-1)(x-5)}{x}$	−	+	−	+

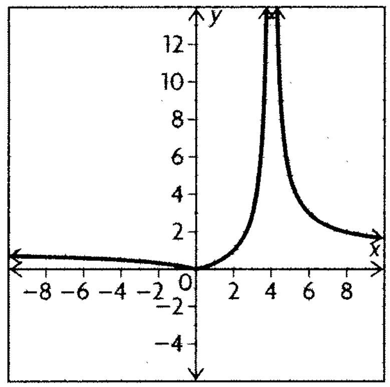
From the table, it can be determined that the average profit per snowboard is positive between $0 < x < 1$ and $x > 5$. Because you can't have a partial number of snowboards, the inequality is true on $x > 5$.

12. a) The first inequality can be manipulated algebraically to produce the second inequality.

b) You could graph the equation $y = \frac{x+1}{x-1} - \frac{x+3}{x+2}$ and determine when it is negative.

c) The values that make the factors of the second inequality zero are -5, -2, and 1. Determine the sign of each factor in the intervals corresponding to the zeros. Determine when the entire expression is negative by examining the signs of the factors.

13. You can graph the inequality to help you determine when the inequality is true.

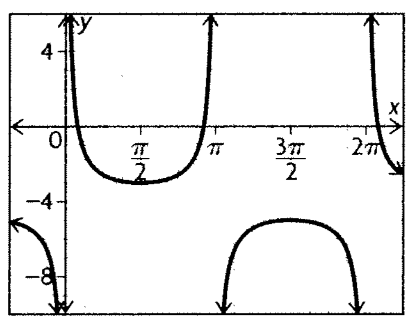


Notice that the function is greater than 1 on [2, 4) and $(4, \infty)$.

14. $\frac{1}{\sin(x)} < 4, 0° \leq x \leq 360°$

$\frac{1}{\sin(x)} - 4 < 0$

You can graph this inequality and then determine when the graph is negative.



The graph is negative on $14.48 < x < 165.52$ and $180 < x < 360$.

15. $\frac{\cos(x)}{x} > 0.5 \quad 0° < x < 90°$

Examine the inequality. You want the numerator to be greater than half of the denominator.

$\cos(x) > 0.5x$

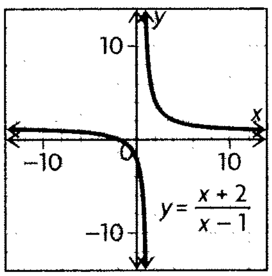
$x = 0$ would be the asymptote in this case, but values greater than zero could satisfy the inequality.

$\cos(1) > 0.5$

$0.999 > 0.5$
$\cos(x)$ becomes less than $0.5x$ after about 2.
$\cos(2) < 1$
$0.99 < 1$
So the inequality is true on $0 < x < 2$.

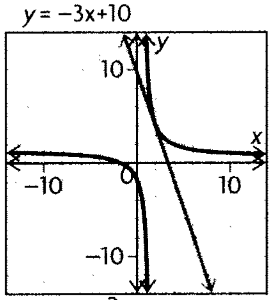
5.6 Rates of Change in Rational Functions, pp. 303–305

1. a)



Use the graph to determine $f(2)$ and $f(7)$.
$f(2) = 4$
$f(7) = 1.5$
$$y = \frac{1.5 - 4}{7 - 2}$$
$$= -0.5$$

b) $y = -3x+10$

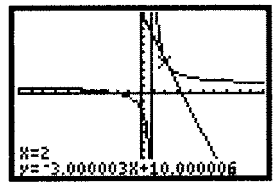


$y = \dfrac{x+2}{x-1}$

The slope of the tangent line is -3.

2. Use the difference quotient to help you estimate the average rate of change at $x = 2$. Use $h = 0.01$
$$\frac{f(a + h) - f(a)}{h}$$
$$= \frac{f(2) - f(2.01)}{0.01}$$
$$= \frac{3.97 - 4}{0.01}$$
$$= -3$$
The answers that I received match.

3. Use graphing technology to graph the function. Have the calculator draw a line tangent to point $x = 2$. Find the slope of that line.
The slope is -3, so the instantaneous rate of change at $x = 2$ is -3.



4. Use the difference quotient to determine the instantaneous rate of change of $f(x) = \frac{x}{x-4}$ at $(2, -1)$. The difference quotient is $f(x) = \frac{f(a+h) - f(a)}{h}$. You can use $h = 0.01$.
$$f(2.01) = \frac{2.01}{2.01 - 4}$$
$$f(2.01) = -1.01$$
Difference Quotient:
$$= \frac{-1.01 - (-1)}{0.01}$$
$$= -1$$

5. Use the difference quotient to determine the instantaneous rate of change for each function. The difference quotient is $f(x) = \frac{f(a+h) - f(a)}{h}$.
Use 0.01 for h.

a) $y = \dfrac{1}{25 - x}$, $x = 13$
$$f(13) = \frac{1}{25 - 13}$$
$$= \frac{1}{12}$$
$$f(13.01) = \frac{1}{25 - 13.01}$$
$$= \frac{1}{11.999}$$
Difference Quotient:
$$= \frac{\frac{1}{11.99} - \frac{1}{12}}{0.001}$$
$$= 0.01$$

b) $y = \dfrac{17x + 3}{x^2 + 6}$, $x = -5$
$$f(5) = \frac{17(-5) + 3}{(-5)^2 + 6}$$
$$= -2.645$$
$$f(-4.99) = \frac{17(-4.99) + 3}{(-4.99)^2 + 6}$$
$$= \frac{-81.83}{30.9001}$$
$$= -2.648$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{-2.648 - (-2.645)}{0.01}$$
$$= -\frac{0.003}{0.01}$$
$$= -0.3$$

c) $y = \dfrac{x+3}{x-2}$, $x = 4$

$$f(4) = \frac{4+3}{4-2}$$
$$= 3.5$$
$$f(4.01) = \frac{4.01+3}{4.01-2}$$
$$= 3.487$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{3.487 - 3.5}{0.01}$$
$$= -\frac{0.013}{0.01}$$
$$= -1.3$$

d) $\dfrac{-3x^2 + 5x + 6}{x+6}$, $x = -3$

$$f(-3) = \frac{-3(-3)^2 + 5(-3) + 6}{(-3) + 6}$$
$$= 6$$
$$f(-3.01) = \frac{-3(-3.01)^2 + 5(-3.01) + 6}{(-3.01) + 6}$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{6.06 - 6}{0.01}$$
$$= \frac{0.06}{0.01}$$
$$= 6$$

6. Use the difference quotient to determine the instantaneous rate of change for each function. The difference quotient is $f(x) = \dfrac{f(a+h) - f(a)}{h}$. Use 0.01 for h. The point where there is no tangent line would be any vertical asymptotes.

a) $f(x) = \dfrac{-5x}{2x+3}$, $x = 2$

$$f(2) = \frac{-5(2)}{2(2)+3}$$
$$= \frac{-10}{7}$$
$$= -1.429$$
$$f(2.01) = \frac{-5(2.01)}{2(2.01)+3}$$
$$= -1.432$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{1.432 - (-1.429)}{0.01}$$
$$= 286.1$$

The vertical asymptote would occur at $x = -1.5$.

b) $f(x) = \dfrac{x-6}{x+5}$, $x = -7$

$$f(-7) = \frac{(-7) - 6}{(-7) + 5}$$
$$= \frac{-13}{-2}$$
$$= 6.5$$
$$f(-7.01) = \frac{(-7.01) - 6}{(-7.01) + 5}$$
$$= \frac{-13.01}{-2.01}$$
$$= 6.4726$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{6.4726 - 6.5}{0.01}$$
$$= -2.74$$

The vertical asymptote would occur at $x = -5$.

c) $f(x) = \dfrac{2x^2 - 6x}{3x+5}$, $x = -2$

$$f(-2) = \frac{2(-2)^2 - 6(-2)}{3(-2) + 5}$$
$$= -20$$
$$f(-2.01) = \frac{2(-2.01)^2 - 6(-2.01)}{3(-2.01) + 5}$$
$$= -19.5535$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{-19.5535 - (-20)}{0.01}$$
$$= 44.65$$

The vertical asymptote would occur at $x = -\frac{5}{3}$.

d) $f(x) = \frac{5}{x - 6}, x = 4$

$$f(4) = \frac{5}{(4) - 6}$$
$$= -2.5$$
$$f(4.01) = \frac{5}{(4.01) - 6}$$
$$= -2.5126$$

Difference Quotient:

$$f(x) = \frac{f(a + h) - f(a)}{h}$$
$$= \frac{-2.5126 - (-2.5)}{0.01}$$
$$= -1.26$$

The vertical asymptote occurs at $x = 6$.

7. The function that models the concentration of the pollutant in the water is $c(t) = \frac{27t}{10\,000 + 3t}$, where the units of t is minutes.

a) There are sixty minutes in 1 hour so find $c(60)$.

$$c(60) = \frac{27(60)}{10\,000 + 3(60)}$$
$$= \frac{1620}{10\,180}$$
$$= 0.1591$$
$$c(60.01) = \frac{27(60.01)}{10\,000 + 3(60.01)}$$
$$= \frac{1620.27}{10\,180.03}$$
$$= 0.1592$$

Difference Quotient:

$$f(x) = \frac{f(a + h) - f(a)}{h}$$
$$= \frac{0.1592 - 0.1591}{0.01}$$
$$= 0.01$$

b) There are 10 080 minutes in one week. So determine $c(10\,080)$.

$$c(10\,080) = \frac{27(10\,080)}{10\,000 + 3(10\,080)}$$
$$= \frac{272\,160}{40\,240}$$
$$= 6.76$$
$$c(10\,080.01) = \frac{27(10\,080.01)}{10\,000 + 3(10\,080.01)}$$
$$= \frac{272\,160.27}{40\,240.03}$$
$$= 6.7634$$

Difference Quotient:

$$f(x) = \frac{f(a + h) - f(a)}{h}$$
$$= \frac{6.7634 - 6.76}{0.01}$$
$$= 0.34$$

8. a) The demand function is $p(x) = \frac{15}{2x^2 + 11x + 5}$. This function tells you the price of the cakes for every 1000 cakes. The revenue function would be the number of cakes sold, x, times the price of the cakes for that number of cakes sold $\frac{15}{2x^2 + 11x + 5}$. So the revenue function is $p(x) = \frac{15x}{2x^2 + 11x + 5}$

b) Use the difference quotient to determine the marginal revenue for $x = 0.75$ and $x = 2.00$.

$$f(x) = \frac{15x}{2x^2 + 11x + 5}$$
$$f(0.75) = \frac{15(0.75)}{2(0.75)^2 + 11(0.75) + 5}$$
$$= \frac{11.25}{14.375}$$
$$= 0.7826$$
$$f(0.751) = \frac{15(0.751)}{2(0.751)^2 + 11(0.751) + 5}$$
$$= \frac{11.265}{14.389}$$
$$= 0.7829$$

Difference Quotient:

$$f(x) = \frac{f(a + h) - f(a)}{h}$$
$$= \frac{0.7829 - 0.7826}{0.001}$$
$$= 0.3$$

The marginal revenue at $x = 0.75$ is 0.3.

Now find the marginal revenue for $x = 2.00$.

$$f(2.00) = \frac{15(2.00)}{2(2.00)^2 + 11(2.00) + 5}$$
$$= \frac{30}{35}$$
$$= 0.8571$$
$$f(2.01) = \frac{15(2.01)}{2(2.01)^2 + 11(2.01) + 5}$$
$$= \frac{30.15}{35.1902}$$
$$= 0.8568$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{0.8568 - 0.8571}{0.01}$$
$$= -0.03$$

The marginal revenue at $x = 2.00$ would be -0.03.

9. a) Since x is measured in thousands, find $C(3)$.

The function is $C(x) = \frac{x^2 - 4x + 20}{x}$.

$$C(x) = \frac{x^2 - 4x + 20}{x}$$
$$C(3) = \frac{(3)^2 - 4(3) + 20}{(3)}$$
$$= 5.67$$

The average cost per T-shirt is $5.67.

b) Determine $C(3.01)$ and use this value in the difference quotient to help you estimate the average rate of change of the average price of a T-shirt when the factory is producing 3000 of them.

$$C(3.01) = \frac{(3.01)^2 - 4(3.01) + 20}{(3.01)}$$
$$= 5.65$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{5.65 - 5.67}{0.01}$$
$$= -2$$

10. The function is $N(t) = \frac{100t^3}{100 + t^3}$.

a) $N(6) = \frac{100(6)^3}{100 + (6)^3}$

$$= \frac{21\,600}{316}$$
$$= 68.35$$
$$N(6.01) = \frac{100(6.01)^3}{100 + (6.01)^3}$$
$$= \frac{21\,708.18}{317.08}$$
$$= 68.46$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{68.46 - 68.35}{0.01}$$
$$= -11$$

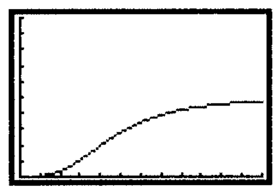
b) $N(12) = \frac{100(12)^3}{100 + (12)^3}$

$$= \frac{172\,800}{1828}$$
$$= 94.53$$
$$N(12.01) = \frac{100(12.01)^3}{100 + (12.01)^3}$$
$$= \frac{173\,232.36}{1832.32}$$
$$= 94.54$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{94.54 - 94.53}{0.01}$$
$$= 1$$

c)



The number of houses that were built increases slowly at first, but rises rapidly between the third and sixth months. During the last six months, the rate at which the houses were built decreases.

11. Examine the interval $14 \le x \le 15$. Find the rate of change over the interval.

$$f(15) = \frac{15 - 2}{15 - 5} = \frac{13}{10} = 1.3$$
$$f(14) = \frac{14 - 2}{14 - 5} = \frac{12}{9} = 1.33$$
$$\frac{1.3 - 1.33}{15 - 14} = -0.03$$

The slope over the interval $14 \le x \le 15$ is $m = -0.03$, which is approximately 0. Now find the instantaneous rate of change at 14.5.

$$f(14.5) = \frac{14.5 - 2}{14.5 - 5} = \frac{12.5}{9.5} = 1.316$$
$$f(14.51) = \frac{14.51 - 2}{14.51 - 5} = \frac{12.51}{9.51} = 1.315$$
$$\frac{1.315 - 1.316}{14.51 - 14.5} = -0.1$$

The instantaneous rate of change at the point 14.5 is -0.01, which is approximately 0. The rate of change over the interval and at the specific point is about 0.

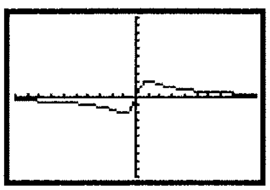
12. a) I would find $s(0)$ and $s(6)$ and would then solve $\frac{s(6) - s(0)}{6 - 0}$.

b) The average rate of change over this interval gives the object's speed.

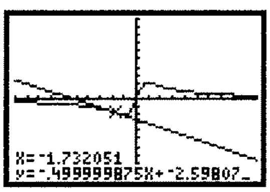
c) To find the instantaneous rate of change at a specific point, you could find the slope of the line that is tangent to the function $s(t)$ at the specific point. You could also find the average rate of change on either side of the point for smaller and smaller intervals until it stabilizes to a constant. It is generally easier to find the instantaneous rate using a graph, but the second method is more accurate.

d) The instantaneous rate of change for a specific time, t, is the acceleration of the object at this time.

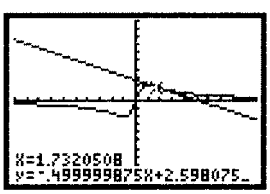
13. Use your graphing calculator to graph the equation $f(x) = \frac{4x}{x^2 + 1}$.



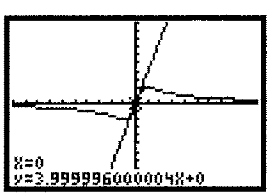
Find $x = -\sqrt{3}$ on the graph and use the calculator to find the line tangent to the graph at this point.



The equation of the line tangent to the function at $(-\sqrt{3}, -\sqrt{3})$ is $y = -0.5x - 2.598$.



The equation of the line tangent to the function at $(\sqrt{3}, \sqrt{3})$ is $y = -0.5x + 2.598$.



The equation of the line tangent to the function at $(0, 0)$ is $y = 4x$.

14. Examine the graph of the function. Use your calculator to zoom into points around the origin. The graph appears to be a straight line at this point, and so the instantaneous rates of change at $(0, 0)$ will likely be pretty close to the instantaneous rate of change at $(0, 0)$, which is 4.

Because the graph is almost a straight line at $(0, 0)$ the rate of change is neither increasing nor decreasing; it will remain constant. Therefore the rate of change at this rate of change will be 0.

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1. a) The function is $f(x) = 3x + 2$. The function will be a straight line and so the domain and range will be all real numbers—$\{x \in \mathbf{R}\}$ and $\{y \in \mathbf{R}\}$.

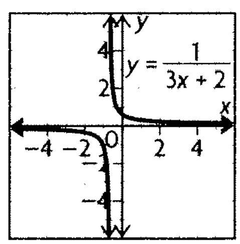
x-intercept:

$$0 = 3x + 2$$
$$-2 = 3x$$
$$\frac{-2}{3} = \frac{3x}{3}$$
$$-\frac{2}{3} = x$$

y-intercept:

$$y = 3(0) + 2$$
$$= 2$$

The slope of the equation is positive and so the function will always be increasing. Additionally, this means that the function will be negative on $\left(-\infty, -\frac{2}{3}\right)$ and positive on $\left(-\frac{2}{3}, \infty\right)$. Use this information to help you graph the function.



b) The function is $f(x) = 2x^2 + 7x - 4$. The function is a quadratic and so the domain will be $\{x \in \mathbf{R}\}$. The coefficient of the first term is positive and so the graph will be pointed up. The function factors to $f(x) = (2x - 1)(x + 4)$. Therefore, the x-intercepts would be $0 = 2x - 1$ and $0 = x + 4$, or $x = 0.5$ and -4.

Use the x-intercepts and direction that the graph is facing to determine the positive and negative intervals. The graph will be positive on $(-\infty, -4)$ and $(0.5, \infty)$. The graph will be negative on $(-4, 0.5)$. You can also use the x-intercepts to determine the range. The range corresponds to the vertex of the function and the vertex can be found halfway in between the two x-intercepts.

vertex $-\dfrac{0.5 + (-4)}{2} = -1.75$

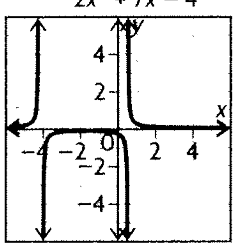
This is the x-value of the vertex. Now you can substitute -1.75 into the function to find the graphs minimum and its subsequent range.

$$\begin{aligned} f(-1.75) &= 2(-1.75)^2 + 7(-1.75) - 4 \\ &= -10.125 \end{aligned}$$

The range of the function would be $\{y \in \mathbf{R} | y > -10.125\}$.

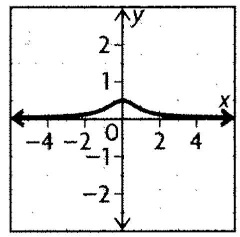
Because you know the vertex and the direction the graph is pointed, you can also find the increasing/decreasing intervals. The graph will be decreasing on $(-\infty, -10.125)$ and increasing on $(-10.125, \infty)$. Use this information to help you graph the function.

$y = \dfrac{1}{2x^2 + 7x - 4}$



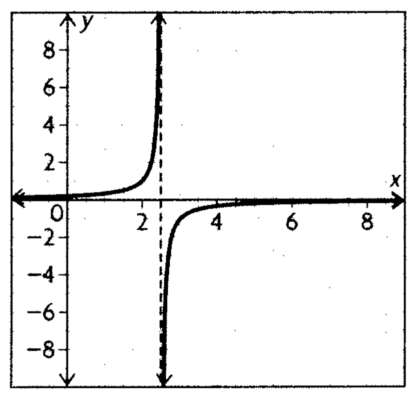
c) The function is $f(x) = 2x^2 + 2$. The function is a quadratic function. Therefore, the domain will be $\{x \in \mathbf{R}\}$. The graph will be facing upward because the coefficient of the first term is positive. Because there are no real solutions to $0 = 2x^2 + 2$, the graph will have no x-intercepts. The y-intercept is $y = 2(0)^2 + 2 = 2$. This means that the domain will be $\{y \in \mathbf{R} | y > 2\}$. The function will be decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$. Because the function has no x-intercepts and because the graph is facing up, the function will never be negative. Use this information to help you graph the reciprocal of the function.

$y = \dfrac{1}{2x^2 + 2}$



2. Examine each graph. Glean any information from these graphs that might help you to graph their function's reciprocals.

a) The function is linear. The domain and range are all real numbers. It is always decreasing. The x-intercept is $x = 2.5$. The y-intercept is $y = 5$. The function is positive on $(-\infty, 2.5)$ and negative on $(2.5, \infty)$. The graph of the reciprocal function would be:

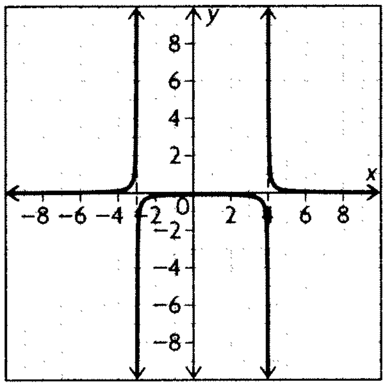


b) The function is a quadratic equation, which means that the domain of the function is $\{x \in \mathbf{R}\}$. The x-intercepts of the function are $x = -3$ and $x = 4$. The function is $f(x) = (x + 3)(x - 4) = x^2 - x - 12$. The y-intercept of the function is $(0, -12)$. The vertex can be found by finding the half-way point between the x-intercepts.

$$\frac{-3 + 4}{2} = 0.5$$

$$\begin{aligned} f(0.5) &= ((0.5) + 3)((0.5) - 4) \\ &= (3.5)(-3.5) \\ &= -12.25 \end{aligned}$$

The vertex is $(0.5, -12.25)$. This means that the range of the function is $\{y \in \mathbf{R} | y > -12.25\}$. The function is decreasing on $(-\infty, -12.25)$ and increasing on $(-12.25, \infty)$. Use this information to graph the reciprocal of the function.



3. a) The function is $y = \frac{1}{x + 17}$. To find the vertical asymptotes, find the zeros of the expression in the denominator.

$$0 = x + 17$$
$$0 - 17 = x + 17 - 17$$
$$-17 = x$$

Because the numerator of the rational function is a constant the horizontal asymptote would be $y = 0$.

b) The function is $y = \frac{2x}{5x + 3}$. To find the vertical asymptotes, find the zeros of the expression in the denominator.

$$0 = 5x + 3$$
$$0 - 3 = 5x + 3 - 3$$
$$-3 = 5x$$
$$\frac{-3}{5} = \frac{5x}{5}$$
$$-\frac{3}{5} = x$$

Divide the leading coefficients of the numerator and denominator to find the equation of the horizontal asymptote.

$$y = \frac{2x}{5x} = \frac{2}{5}$$

c) The function is $y = \frac{3x + 33}{-4x^2 - 42x + 22}$. To find the vertical asymptotes, find the zeros of the expression in the denominator.

$0 = -4x^2 - 42x + 22$

You can use the quadratic formula to help you factor the equation.

$0 = (x + 11)(x - 0.5)$

$x = -11$ and $x = 0.5$

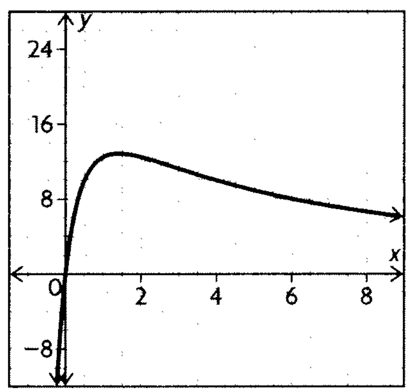
Notice that $(x + 11)$ is a factor in both the numerator and the denominator. This means that there will be a hole at $x = -11$. Because the degree of the expression in the denominator is 2 and the degree of the expression in the numerator is 1, the horizontal asymptote will be $y = 0$.

d) The function is $y = \frac{3x^2 - 2}{x - 1}$. To find the vertical asymptotes, find the zeros of the expression in the denominator.

$0 = x - 1$, so there is a vertical asymptote at $x = 1$.

The numerator does not factor, so no common factors from the numerator and denominator will cancel. Because the degree of the expression in the numerator is 2 and the degree of the expression in the denominator is 1, there will be an oblique asymptote. Divide the numerator by the denominator; the non-remainder part is the equation of the oblique asymptote: $y = 3x + 3$.

4. The function that models the population of the locusts is $f(x) = \frac{75x}{x^2 + 3x + 2}$. You can graph the function using a graphing calculator and then use the graph to describe the locust population over time.



The locust population increased during the first 1.75 years, to reach a maximum of 1 248 000. The population gradually decreased until the end of the 50 years, when the population was 128 000.

5. a) $f(x) = \frac{2}{x + 5}$

x-intercept:

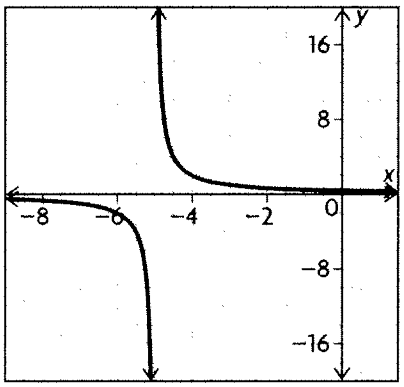
$$0 = \frac{2}{x + 5}$$
$$(x + 5)0 = \frac{2}{x + 5}(x + 5)$$
$$0 = 2$$

The function does not intercept the x-axis. This means that the horizontal asymptote of the function will be $y = 0$.

y-intercept:

$$f(0) = \frac{2}{0+5} = \frac{2}{5}$$

The function has a vertical asymptote at $x = -5$. This means that the domain of the function will be $\{x \in \mathbf{R} | x \neq -5\}$. The function will be negative for $x < -5$ and positive for $x > -5$.

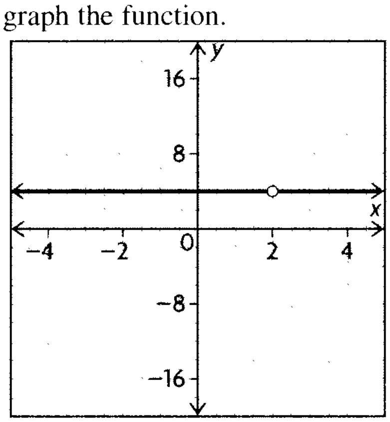


This function is never increasing and is decreasing on $(-\infty, -5)$ and $(-5, \infty)$.
D $= \{x \in \mathbf{R} | x \neq -5\}$;
negative for $x < -5$;
positive for $x > -5$

b) $f(x) = \dfrac{4x-8}{x-2}$

Notice that the function factors to $f(x) = \dfrac{4(x-2)}{(x-2)}$.

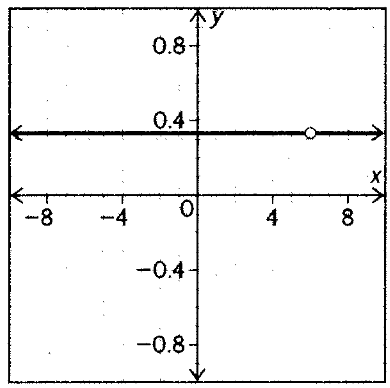
This means that there will be a hole at $x = 2$ and that the graph of the function will be the horizontal line $y = 4$. The function is positive everywhere except $x = 2$. There is no x-intercept, and the y-intercept is 4. Use this information to help you graph the function.



The function is not increasing or decreasing.

c) The function is $f(x) = \dfrac{x-6}{3x-18}$. Notice that the function factors to $f(x) = \dfrac{x-6}{3(x-6)} = \dfrac{1}{3}$.

This means that the graph will have a hole at $x = 6$ and will be a horizontal line at $y = \frac{1}{3}$. The function is positive everywhere except $x = 6$. There is no x-intercept, and the y-intercept is $\frac{1}{3}$.



The function is not increasing or decreasing.

d) The function is $f(x) = \dfrac{4x}{2x+1}$. To find the function's vertical asymptote, find the zero(s) of the expression in the denominator.

$$0 = 2x + 1$$
$$-1 = 2x$$
$$-0.5 = x$$

The function will have a vertical asymptote at $x = -0.5$. This means that the domain will be $\{x \in \mathbf{R} | x \neq -0.5\}$.

x-intercept:

$$0 = \frac{4x}{2x+1}$$
$$(2x+1)0 = \frac{4x}{2x+1}(2x+1)$$
$$0 = 4x$$
$$0 = x$$

The graph will intersect the x-axis at 0.

y-intercept:

$$\frac{4(0)}{2(0)+1} = 0$$

The y-intercept will also be 0.

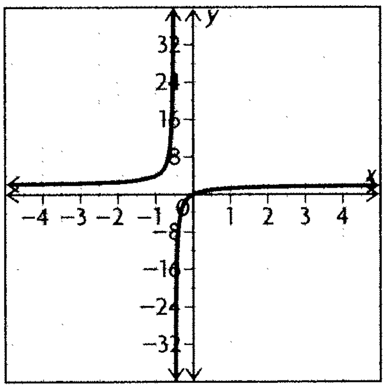
Divide the first terms of the expressions in the numerator and the denominator to find the equation of the horizontal asymptote.

$$y = \frac{4x}{2x} = 2$$

Use the following table to determine when the function is positive and negative.

	$x < -0.5$	$-0.5 < x < 0$	$x > 0$
$4x$	$-$	$-$	$+$
$2x + 1$	$-$	$+$	$+$
$\frac{4x}{2x+1}$	$+$	$-$	$+$

The function is positive on $x < -0.5$ and $x > 0$.
The function is negative on $-0.5 < x < 0$.
Use this information to help you graph the function.



The function is never decreasing and is increasing on $(-\infty, -0.5)$ and $(-0.5, \infty)$.

6. Answers may vary. For example, consider the function $f(x) = \frac{1}{x - 6}$. You know that the vertical asymptote would be $x = 6$. If you were to find the value of the function very close to $x = 6$ (say $f(5.99)$ or $f(6.01)$) you would be able to determine the behaviour of the function on either side of the asymptote.

$$f(5.99) = \frac{1}{(5.99) - 6} = -100$$

$$f(6.01) = \frac{1}{(6.01) - 6} = 100$$

To the left of the vertical asymptote the function moves towards $-\infty$. To the right of the asymptote the function moves towards ∞.

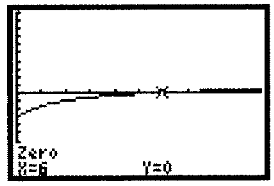
7. Make sure to use your graphing calculator to verify each solution.

a) $$\frac{x-6}{x+2} = 0$$

$$(x + 2)\frac{x-6}{x+2} = 0(x + 2)$$

$$x - 6 = 0$$

$$x = 6$$



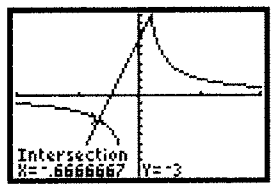
b) $$15x + 7 = \frac{2}{x}$$

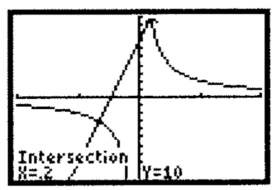
$$x\left(15x + 7 = \frac{2}{x}\right)$$

$$15x^2 + 7x = 2$$

$$15x^2 + 7x - 2 = 0$$

You can use the quadratic formula to help you solve this quadratic equation. The roots of the function are $x = 0.2$ and $x = -\frac{2}{3}$.





c) $$\frac{2x}{x - 12} = \frac{-2}{x + 3}$$

$$2x(x + 3) = -2(x - 12)$$

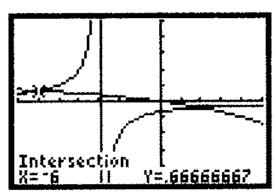
$$2x^2 + 6x = -2x + 24$$

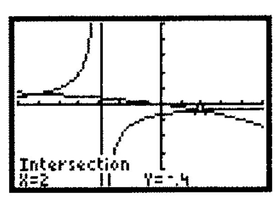
$$2x^2 + 8x - 24 = 0$$

$$2(x^2 + 4x - 12) = 0$$

$$2(x + 6)(x - 2) = 0$$

$$x = -6 \text{ or } x = 2$$



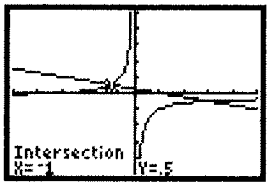


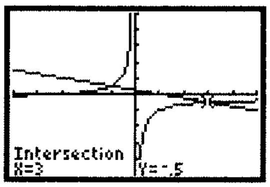
d) $$\frac{x + 3}{-4x} = \frac{x - 1}{-4}$$

$$-4(x + 3) = -4x(x - 1)$$

$$-4x - 12 = -4x^2 + 4x$$

$$0 = -4x^2 + 8x + 12$$
$$0 = -4(x^2 - 2x - 3)$$
$$0 = -4(x + 1)(x - 3)$$
$$x = -1 \text{ and } x = 3$$





8. Janet and Nick's rate would be $\frac{1}{m}$ and Rodriguez's rate would be $\frac{1}{m - 5}$. Working together their rate would be $\frac{2}{m} + \frac{1}{m - 5}$, which is equal to $\frac{1}{32.3}$.

$$\frac{2}{m} + \frac{1}{m - 5} = \frac{1}{3.23}$$
$$3.23m(m - 5)\left(\frac{2}{m} + \frac{1}{m - 5} = \frac{1}{3.23}\right)$$
$$3.23(m - 5)(2) + 3.23m = m(m - 5)$$
$$6.46m - 32.3 + 3.23m = m^2 - 5m$$
$$0 = m^2 - 14.69m + 32.3$$

Use the quadratic formula to determine the roots of this quadratic equation.

$m = 12$ and 2.69

The possible answers are 12 minutes and 2.69 minutes. But because Janet's time has to be greater than 5 minutes, the answer must be 12 minutes. It takes Janet about 12 minutes to wash the car.

9. The function that represents the concentration of a toxic chemical is $c(x) = \frac{50x}{x^2 + 3x + 6}$.

To determine when the concentration is 6.16 g/L solve the equation $6.16 = \frac{50x}{x^2 + 3x + 6}$.

$$6.16 = \frac{50x}{x^2 + 3x + 6}$$
$$6.16(x^2 + 3x + 6) = \frac{50x}{x^2 + 3x + 6}(x^2 + 3x + 6)$$
$$6.16x^2 + 18.48x + 36.96 = 50x$$
$$6.16x^2 - 31.52x + 36.96 = 0$$

Use the quadratic formula to solve this equation.

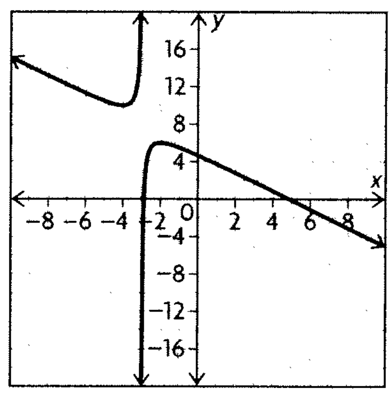
$x = 3.297$ and 1.82

The concentration of the chemical will be 6.16 g/L at 1.82 and 3.297 days.

10. a)
$$-x + 5 < \frac{1}{x + 3}$$
$$-x + 5 - \frac{1}{x + 3} < 0$$
$$\left(\frac{x + 3}{x + 3}\right)(-x) + \left(\frac{x + 3}{x + 3}\right)5 - \frac{1}{x + 3} < 0$$
$$\frac{-x^2 - 3x}{x + 3} + \frac{5x + 15}{x + 3} - \frac{1}{x + 3} < 0$$
$$\frac{-x^2 + 2x + 14}{x + 3} < 0$$
$$\frac{(x + 2.873)(x - 4.873)}{x + 3} < 0$$

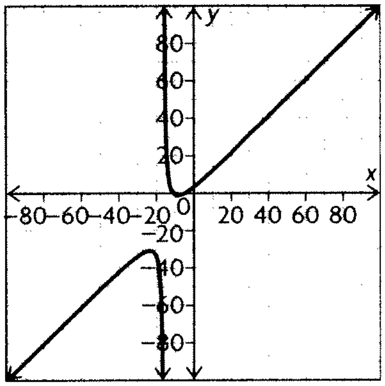
	$x < -3$	$-3 < x < -2.873$	$-2.873 < x < 4.873$	$x > 4.873$
$x + 2.873$	−	−	+	+
$x - 2.873$	−	−	−	+
$x + 3$	−	+	+	+
$\frac{(x + 2.873)(x - 4.873)}{x + 3}$	−	+	−	+

The inequality is true on $x < -3$ and $-2.873 < x < 4.873$.



b)
$$\frac{55}{x + 16} > -x$$
$$\frac{55}{x + 16} + x > 0$$
$$\frac{55}{x + 16} + \left(\frac{x + 16}{x + 16}\right)x > 0$$
$$\frac{x^2 + 16x + 55}{x + 16} > 0$$
$$\frac{(x + 5)(x + 11)}{x + 16} > 0$$

	$x < -16$	$-16 < x < -11$	$-11 < x < -5$	$x > -5$
$x + 5$	$-$	$-$	$-$	$+$
$x + 11$	$-$	$-$	$+$	$+$
$x + 16$	$-$	$+$	$+$	$+$
$\frac{(x+5)(x+11)}{x+16}$	$-$	$+$	$-$	$+$



c)

$$\frac{2}{3x+4} > \frac{x}{x+1}$$

$$\frac{2x}{3x+4} - \frac{x}{x+1} > 0$$

$$\left(\frac{x+1}{x+1}\right)\frac{2x}{3x+4} - \frac{x}{x+1}\,\frac{3x+4}{3x+4} > 0$$

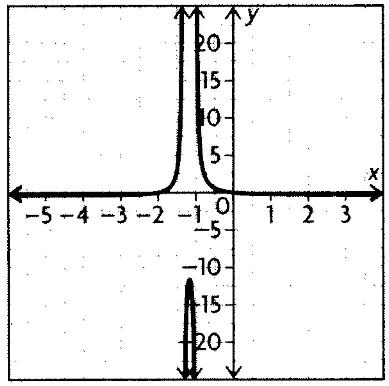
$$\frac{2x^2+2x}{(x+1)(3x+4)} - \frac{3x^2+4x}{(x+1)(3x+4)} > 0$$

$$\frac{-x^2-2x}{(x+1)(3x+4)} > 0$$

$$\frac{-x(x+2)}{(x+1)(3x+4)} > 0$$

	$x < -2$	$-2 < x < -1.33$	$-1.33 < x < -1$	$-1 < x < 0$	$x > 0$
$-x$	$+$	$+$	$+$	$+$	$-$
$x + 2$	$-$	$+$	$+$	$+$	$+$
$x + 1$	$-$	$-$	$-$	$+$	$+$
$3x + 4$	$-$	$-$	$+$	$+$	$+$
$\frac{-x(x+2)}{(x+1)(3x+4)} > 0$	$-$	$+$	$-$	$+$	$-$

The inequality is true on $-2 < x < -1.33$ and $-1 < x < 0$.



d)

$$\frac{x}{6x-9} \le \frac{1}{x}$$

$$\frac{x}{6x-9} - \frac{1}{x} \le 0$$

$$\left(\frac{x}{x}\right)\frac{x}{6x-9} - \frac{1}{x}\left(\frac{6x-9}{6x-9}\right) \le 0$$

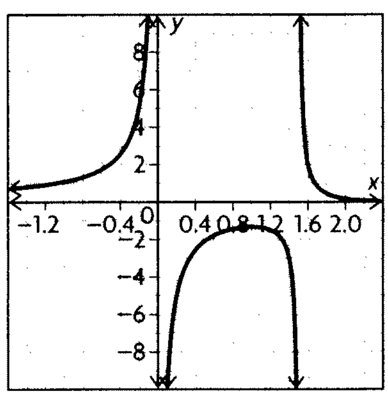
$$\frac{x^2}{(x)(6x-9)} + \frac{-6x+9}{(x)(6x-9)} \le 0$$

$$\frac{x^2-6x+9}{(x)(6x-9)} \le 0$$

$$\frac{(x-3)(x-3)}{(x)(6x-9)} \le 0$$

	$x \le 0$	$0 \le x \le 1.5$	$1.5 \le x \le 3$	$x \ge 3$
$x - 3$	$-$	$-$	$-$	$+$
$x - 3$	$-$	$-$	$-$	$+$
x	$-$	$+$	$+$	$+$
$6x - 9$	$-$	$-$	$+$	$+$
$\frac{(x-3)(x-3)}{(x)(6x-9)} \le 0$	$+$	$-$	$+$	$+$

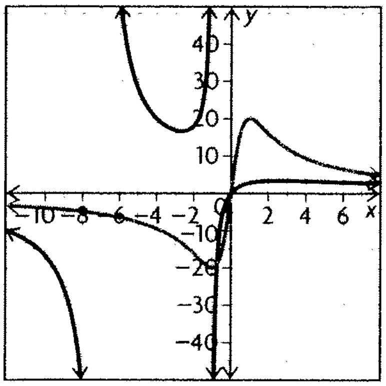
The inequality is true on $0 < x < 1.5$.



11. The function that models the biologist's prediction of the population of the tadpoles in the pond is

$f(t) = \frac{40t}{t^2 + 1}$. The actual population is modelled by

$g(t) = \frac{45t}{t^2 + 8t + 7}$. Graph $\frac{45t}{t^2 + 8t + 7}$ and $\frac{40t}{t^2 + 1}$ to determine where $g(t) > f(t)$.



$g(t)$ appears to be greater than $f(t)$ on
$-6.7 < x < -1.01$ and $-0.73 < x < 0$.

12. The slope of the line tangent to the graph for the given point is equal to the instantaneous rate of change at that point. Use the difference quotient to determine the instantaneous rate of change. The point where a vertical asymptote occurs is the point where no tangent line could be drawn.

a) $\frac{x+3}{x-3}, x = 4$

$$f(4) = \frac{4+3}{4-3} = 7$$

$$f(4.01) = \frac{4.01 + 3}{4.01 - 3} = 6.94$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h} = \frac{6.94 - 7}{0.01} = \frac{-0.06}{0.01} = -6$$

The vertical asymptote occurs at $x = 3$.

b) $f(x) = \frac{2x - 1}{x^2 + 3x + 2}, x = 1$

$$f(1) = \frac{2(1) - 1}{(1)^2 + 3(1) + 2} = 0.167$$

$$f(1.01) = \frac{2(1.01) - 1}{(1.01)^2 + 3(1.01) + 2} = 0.169$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h} = \frac{0.169 - 0.167}{0.01} = \frac{0.002}{0.01} = 0.2$$

The vertical asymptotes occur at $x = -2$ and $x = -1$.

13. a) To find the average rate of change during the first two hours of the drug's ingestion, find $c(2)$ and $c(0)$.

$$c(2) = \frac{5(2)}{(2)^2 + 7} = \frac{10}{11} = 0.91$$

$$c(0) = \frac{5(0)}{(0)^2 + 7} = 0$$

Average Rate of Change:

$$\frac{0.91 - 0}{2 - 0} = 0.455 \text{ mg/L/h}$$

b) Use the difference quotient to help you determine the drug's instantaneous rate of change at $t = 3$.

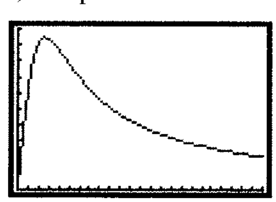
$$c(3) = \frac{5(3)}{(3)^2 + 7} = 0.9375$$

$$c(3.01) = \frac{5(3.01)}{(3.01)^2 + 7} = \frac{15.05}{16.0601} = 0.9371$$

Difference Quotient:

$$f(x) = \frac{f(a+h) - f(a)}{h} = \frac{0.9371 - 0.9375}{0.01} = \frac{-0.004}{0.01} = -0.04 \text{ mg/L/h}$$

c) Graph the function $c(t) = \frac{5t}{t^2 + 7}$.



The concentration of the drug in the blood stream appears to be increasing most rapidly during the first hour and a half; the graph is steep and increasing during this time.

14. The slope of the tangent line is going to be $\frac{4-10}{8-5} = -2$. This means that the equation of the line would be $y = -2x + 20$. You are looking for the point of intersection between the two functions, and so you need to solve $-2x + 20 = \frac{2x}{x-4}$.

$$\begin{aligned} -2x + 20 &= \frac{2x}{x-4} \\ (-2x+20)(x-4) &= 2x \\ -2x^2 + 8x + 20x - 80 &= 2x \\ -2x^2 + 26x - 80 &= 0 \\ x^2 - 13x + 40 &= 0 \\ (x-5)(x-8) &= 0 \end{aligned}$$

The two points of intersection would be $x = 5$ and $x = 8$. The point with the line parallel to the secant line would lie half-way in between these two points, $x = 6.5$.

15. a) As the x-coordinate approaches the vertical asymptote of a rational function, the line tangent to the graph will get closer and closer to being a vertical line. This means that the slope of the line tangent to the graph will get larger and larger, approaching positive or negative infinity depending on the function, as x gets closer to the vertical asymptote.

b) As the x-coordinate grows larger and larger in either direction, the line tangent to the graph will get closer and closer to being a horizontal line. This means that the slope of the line tangent to the graph will always approach zero as x gets larger and larger.

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1. a) The graph indicates that there is a vertical asymptote at $x = 0.5$ and a horizontal asymptote at $y = 0$. This matches equation B.

b) The graph indicates that there is a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 5$. This matches equation A.

2. a) If $f(n)$ is very large, then that would make $\frac{1}{f(n)}$ a very small fraction.

b) If $f(n)$ is very small (less than 1), then that would make $\frac{1}{f(n)}$ very large.

c) If $f(n) = 0$, then that would make $\frac{1}{f(n)}$ undefined at that point because you cannot divide by 0.

d) If $f(n)$ is positive, then that would make $\frac{1}{f(n)}$ also positive because you are dividing two positive numbers.

3. The horizontal asymptote of the function can be found by finding the zeros of the expression in the denominator.

$0 = x - 2$

$2 = x$

The horizontal asymptotes of the function can be found by dividing the first two terms of the expressions in the numerator and denominator.

x-intercept:

$$\begin{aligned} 0 &= \frac{2x+6}{x-2} \\ 0 &= 2x + 6 \\ -6 &= 2x \\ -3 &= x \end{aligned}$$

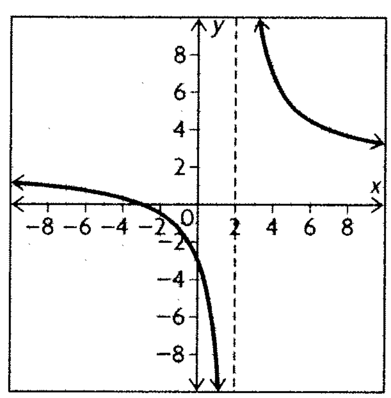
y-intercept:

$$\begin{aligned} y &= \frac{2(0)+6}{0-2} \\ &= -3 \end{aligned}$$

Use a table to determine when the graph is positive and negative.

	$x < -3$	$-3 < x < 2$	$x > 2$
$2x + 6$	−	+	+
$x - 2$	−	−	+
$\frac{2x+6}{x-2}$	+	−	+

Because the expression in the denominator is always increasing, this function will always be decreasing. Use all of this information to sketch the graph.



4. The average cost for a kilogram of steel before it has been processed would be $\frac{2249.52}{x}$.
The company has made \$2 profit on each pound of steel. So the price of steel after it has been processed would be $\frac{2249.52}{x} + 2$. The mass of the steel has lost 25 kilograms. The value of the steel would be the amount left multiplied by the current price.

$$\left(\frac{2249.52}{x} + 2\right)(x - 25) = 10\,838.52$$
$$(2249.52 + 2x)(x - 25) = 10\,838.52x$$
$$2249.52x - 56\,238 + 2x^2 - 50x = 10\,838.52x$$
$$2x^2 - 8639x - 56\,238 = 0$$
$$x = 4326$$

The original weight was 4326 kg. The original cost would be \$0.52/kg.

5. a) Algebraic

$$\frac{-x}{x - 1} = \frac{-3}{x + 7}$$
$$-x(x + 7) = -3(x - 1)$$
$$-x^2 - 7x = -3x + 3$$
$$-x^2 - 4x - 3 = 0$$
$$x^2 + 4x + 3 = 0$$
$$(x + 1)(x + 3) = 0$$
$$x = -1 \text{ and } x = -3$$

b)

$$\frac{2}{x + 5} > \frac{3x}{x + 10}$$
$$\frac{2}{x + 5} - \frac{3x}{x + 10} > 0$$
$$\left(\frac{x + 10}{x + 10}\right)\frac{2}{x + 5} - \frac{3x}{x + 10}\left(\frac{x + 5}{x + 5}\right) > 0$$
$$\frac{2x + 20}{(x + 10)(x + 5)} - \frac{3x^2 + 15x}{(x + 10)(x + 5)} > 0$$
$$\frac{-3x^2 - 13x + 20}{(x + 10)(x + 5)} > 0$$
$$\frac{(x + 5.5)(x - 1.2)}{(x + 10)(x + 5)} > 0$$

	$x < -10$	$-10 < x < -5.5$	$-5.5 < x < -5$	$-5 < x < 1.2$	$x > 1.2$
$(x + 5.5)$	−	−	+	+	+
$(x - 1.2)$	−	−	−	−	+
$(x + 10)$	−	+	+	+	+
$(x + 5)$	−	−	−	+	+
$\frac{(x + 5.5)(x - 1.2)}{(x + 10)(x + 5)} > 0$	+	−	+	−	+

The inequality is true on $x < -10$, $-5.5 < x < -5$, and $x > 1.2$.

6. a) To find the vertical asymptotes of the function, find the zeros of the expression in the denominator. To find the equation of the horizontal asymptotes, divide the first two terms of the expressions in the numerator and denominator.

b) This type of function will have a hole when both the numerator and the denominator share the same factor $(x + a)$.